Optimal Foreign Borrowing and Altruism in an Overlapping Generations Model

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I. Introduction and Summary

Recently, external debt-related problems have emerged as one of the major political and economic issues in the developing countries that compels attention, especially because international financial market conditions have become tight. The core of the problem lies in the developing countries' "overborrowing" of foreign capital according to the criterion of Pareto optimality.

The problem of overborrowing is not a novel experience to the developing countries. Many of them have experienced it one way or another during the colonial period in the 19th century and early in this century. The major difference between the developing countries' overborrowing in the colonial period and today is the reasons for such overborrowing.

During the colonial period these countries experienced the problem of overborrowing in the form of unwanted borrowing from the colonial power in the process of being reorganized as enclaves of the ruling countries.

More recently, now sovereign developing countries have borrowed excessively for a number of reasons. Among them are a government's desire to pursue rapid economic growth and development, domestic firms' pursuit of ambitious expansion through the aid of foreign capital and technology (growth and ex-

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pansion are usually perceived as being identical to long-run profit maximization by many firms in developing countries), and domestic firms' arbitrage activities which try to exploit the interest rate differential between the foreign borrowing rate and the domestic rate. At the same time, such countries are often waging nationwide campaigns to raise their current low national saving ratios in order to self-finance their ambitious investment projects.

The purpose of this paper is to analyze optimal foreign borrowing and resource allocation by incorporating an omnipotent planner into an overlapping generations model and subsequently replacing him with altruistic agents. This paper shows that the above-mentioned growth-centered attitudes and arbitrage activities acturally lead to overborrowing (i.e., foreign borrowing above the optimal level) and thus non-optimal resource allocation. It also shows that a nationwide campaign to self-finance ambitious development projects with national saving will not easily succeed since foreign borrowing crowds out national saving and the current low national saving ratio is attributed to overborrowing.

In discussing foreign borrowing, the Pareto optimality criterion should be a dynamic one, since borrowing affects not only living generations but also future generations. Almost all developing countries proclaim that economic growth enchances the welfare of both present and future generations, and that foreign borrowing, if any, is beneficial to economic growth. Hence, we imagine a simple production economy where agents live finite lives, while the planning horizon of their government is beyond the life span of living generations. It is assumed that the government, represented by an omniscient planner, can costlessly maximize the weighted summation of the welfare of living and future generations, subject only to the economy's intertemporal resource constraint. In this way, optimal foreign borrowing and resource allocation are derived in this planner's economy. This is illustrated in the next section by using a variant of the Diamond model (1965) as a basic model. Section II also discusses the properties of such optimal resource allocations.

Section III discusses a competitive model which is closely related to the basic model. It is easily shown that overborrowing will arise if competitive firms were allowed to introduce foreign debt at their own discretion. A government's overall foreign borrowing plan like the planner's, as discussed in the previous section, is thus necessary.

Even if the government controls firms in order to guarantee optimal borrowing, optimal intertemporal resource allocation is not achieved under agents' egoistic preference. Under the circumstances, an alternative "altruistic"[1] preference is considered.

The main content of Section III is that the optimal resource allocation derived in section II will be attained as a competitive equilibrium if agents are concerned about the welfare of their immediate descendants.

In economics we have almost always relied on the behavioral assumption that agents are rational and egoistic, as if rationality and self-interest were indispensable to each other. Sometimes it is a fruitful approach to assume that agents are rational yet non-self-regarding, especially in studying developing economies. This is because in many developing societies, which closely approximate a gemeinschaft-type motivational model, the framework and focal point of personal ambition is the group rather than the unattached individual of tradional economic theory.

Section IV presents concluding remarks.

II. The Basic Model: Planner's Economy

1. *The Environment*

The model, a variant of Diamond model (1965), is a discrete time, one-good production economy with population consisting of overlapping generations. At each time $t \geqq 1$, the population consists of N_t young, the member of generation t, and N_{t-1} old, the member of generation t-1, where $N_t/N_{t-1} = 1+n$, $n \geqq 0$ for all $t \geqq 1$. Each two-period-lived agent is endowed with one unit of homogeneous labor (supplied inelastically) while young, and none while old.

1 "Altruism" in this paper is not used in its popular sense of "disinterested generosity," but in the sense of behavior oriented toward maximizing the welfare of a given group, of which, however, the altruistic agent is generally a member.

All individuals are alike in preference within a generation and across generations. The preference of a member of generation t is represented by $U(C_{1t}, C_{2t})$, where C_{it} is age i consumption of the single good of a member of generation t, satisfying assumptions (A1.1) to (A1.2), collectively denoted by (A1):

(A1.1) U is concave and twice differentiable with

$$U_i(C_1,C_2) = \frac{\partial U(C_1,C_2)}{\partial C_i} > 0,$$

$$U_{ii}(C_1,C_2) = \frac{\partial^2 U(C_1,C_2)}{\partial C_i^2} < 0$$

and $U_{12}(C_1, C_2)=U_{21}(C_1, C_2)\gtreqq 0$ for $i=1,2$ (t subscript will be omitted when no confusion would result);

(A1.2) $U_1(0, C_2)=U_2(C_1, 0)= \infty$ and $U_1(\infty, C_2)=U_2(C_1, \infty) = 0$ for $C_1, C_2 > 0$.

On the production side, cost-minimizing firms produce the single output according to a constant-returns-to-scale production function, f, with capital K and labor N as inputs. The production function $f(K_t/N_t) = f(k_t)$ is a "well-behaved" neoclassical per capita production function satisfying the following assumption (A2):

(A2.1) $f(0)=0$ and $f(k)>0$ for all $k>0$;

(A2.2) f is concave and twice differentiable with $f'(k)>0$ and $f''(k)<0$ for all $k>0$; $f'(0)=\infty$ and $f'(\infty)=0$.

The output can be either consumed or used as a capital good. Capital, once put in the production process, depreciates completely. Thus, in the absence of international transactions the economy's resource constraint at time t is

$$(1) \quad N_t C_{1t} + N_{t-1} C_{2t-1} + K_{t+1} \leqq N_t f(k_t) \quad \text{or}$$

$$C_{1t} + \frac{C_{2t-1}}{1+n} + (1+n)k_{t+1} \leqq f(k_t) \text{ where } k_{t+1} = \frac{K_{t+1}}{N_{t+1}}.$$

At the beginning t = 1 an omniscient planner is present in this economy. His sole function is to maximize the following social welfare function:[2]

$$(2) \quad \sum_{t=1}^{T} \beta^{t-1}\, U(C_{1t}, C_{2t}) + \beta^{-1}\, U(C_{10}, C_{20}),\ 0<\beta<1,\ T \geqq 2.$$

Money and other financial assets do not exist in the economy.

The only policy instrument available to the planner is foreign borrowing. Foreign borrowing takes the form of borrowing the physical capital (the only good in the rest of the world[3] as well as in this economy) from abroad. All foreign capitals are one-period debts. Foreign debt D_t borrowed at time t is repaid with interest at a rate r^f_{t+1} in time t+1 where r^f_{t+1} is the borrowing rate at t in the international capital market. Domestic capital and foreign capital are perfect substitutes, both in production and consumption.

In the presence of such external transactions, resource constraint facing the planner becomes

$$(3) \quad N_t C_{1t} + N_{t-1} C_{2t-1} + (1 + r^f_t) D_{t-1} + K_{t+1} \leqq N_t f(k_t) + D_t,$$

for all $t \geqq 1$.

Denoting $\frac{D_t}{N_t} = d_t$, we can express equation (3) in per capita terms:

$$(4) \quad C_{1t} + \frac{C_{2t-1}}{1+n} + \frac{1+r^f_t}{1+n}\, d_{t-1} + (1+n)\, k_{t+1} \leqq f(k_t) + d_t,$$

for all $t \geqq 1$.

[2] In equation (2) β combines intergenerational preference with the usual time preference. The planner assigns greater weight to the welfare of living generations than that of future generations. The weight attached to the welfare of a representative member of generation 0 (the old at t = 1) can be any positive real number. β^{-1} in equation (2) is used for convenience in order not to carry a separate first order condition for generation 0 in equation (6) or (7).

[3] This "one-good world" assumption exempts us from worrying about trade balance.

In order to make the problem an interesting one, the international borrowing rate is assumed to be country-specific. It is assumed to be an increasing function of per capita borrowing[4]:

$$(5)\quad r^f_{t+1} = r^f_{t+1}(d_t), \text{ where } \frac{dr^f_{t+1}}{d\,d_t} > 0 \text{ and}$$

$$\frac{d^2 r^f_{t+1}}{d\,d_t^2} \geqq -\frac{1}{2d_t}\,\frac{dr^f_{t+1}}{d\,d_t} \text{ for } d_t \geqq 0 \text{ for all } t \geqq 1.$$

Equation (5) can be defended on the ground that either international capital market is imperfectly competitive, or even if it is perfectly competitive, increased foreign borrowing is perceived as an increase in the risk of default. (Bardhan, 1967, Hamada, 1969, Hanson, 1974). The restriction $2d_t \frac{d^2 r^f_{t+1}}{d\,d_t^2} \geqq -\frac{d\,r^f_{t+1}}{dd_t}$ is a technical condition which guarantees the existence of the unique per capita debt level in a steady state.

There is no uncertainty in this economy. The planner has a perfect foresight.

2. *The Planner's Choice Problem*

At time $t=1$, the planner has two options. He can solve his optimization problem either on an open economy basis or on a self-sufficient closed economy basis.

Let's first consider the planner's problem on an open economy basis.

The planner tries to maximize equation (2) subject to equations (4) and (5). His choice variables are

[4] Diamond (1965) studies the case where foreign and domestic capital receive the same rate of return. It can easily be shown that, under the objective function (2) and constraint (3), the case leads to no foreign borrowing as an optimal solution.

$$\left\{ C_{1t}, C_{2t-1}, k_{t+1}, d_t \right\}_{t=1}^{T} \quad \text{where } C_{1t}, C_{2t-1}, k_{t+1} > 0$$

and $d_t \geqq 0$.

As initial conditions, $k_1 > 0$, $C_{10} > 0$, $d_0 \geqq 0$ and $r_1^f > 0$ are given. As a developing economy, k_1 is assumed to be sufficiently low to warrant nonnegative foreign borrowing. The first order conditions constitute necessary and sufficient conditions of this choice problem. They are summarized as the following equation (6):

$$(6.1) \quad \frac{U_1(C_{1t}, C_{2t})}{U_2(C_{1t}, C_{2t})} = f'(k_{t+1})$$

$$(6.2) \quad U_2(C_{1t-1}, C_{2t-1}) = \beta \frac{1}{1+n} U_1(C_{1t}, C_{2t}) \text{ or}$$

$$\frac{U_1(C_{1t}, C_{2t})}{U_2(C_{1t-1}, C_{2t-1})} = \frac{1+n}{\beta}$$

$$(6.3) \quad f'(k_{t+1}) = 1 + r^f_{t+1}(d_t) + d_t \cdot \frac{d\, r^f_{t+1}}{d\, d_t}$$

$$(6.4) \quad C_{1t} + \frac{C_{2t-1}}{1+n} + \frac{1+r^f_t}{1+n} d_{t-1} + (1+n)\, k_{t+1} = f(k_t) + d_t \,,$$

$$C_{1t}, C_{2t-1}, k_{t+1} > 0 \text{ and } d_t \geqq 0 \quad \text{for all} \quad t \geqq 1 .$$

(6.5) For $T = \infty$, transversality condition

$$\lim_{T\to\infty} \beta^{T-1} U_1(C_{1T}, C_{2T}) = 0.$$

Equation (6.1) says that intertemporal marginal rate of substitution should be equal to intertemporal marginal rate of transformation.

Equation (6.2) is the condition of optimal intergenerational consumption allocation that utility gained and lost between generations should be equal at the margin. If one unit of consumption good is taken from each member of generation t-1, his lost utility is $U_2(C_{1t-1}, C_{2t-1})$. A representative member of generation t gets $1/1+n$ units of consumption good, which yields utility gain of $\frac{1}{1+n}U_1(C_{1t}, C_{2t})$. This is discounted at the rate of β.

Equation (6.3) is the condition of optimal foreign borrowing. In determining the amount of foreign borrowing it addresses a certain relationship which should be held between marginal product of capital at home and international borrowing rate.

Equation (6.4) is the resource constraint (4) with equality.

Let's consider now the planner's problem on a self-sufficient closed economy basis. This is somewhat simpler. The planner tries to maximize equation (2) subject to equation (1). Given $k_1 > 0$, $C_{10} > 0$, and $d_0 \geqq 0$ his choice variables are $\left\{ C_{1t}, C_{2t-1}, k_{t+1} \right\}_{t=1}^{T}$. Necessary and sufficent conditions of this choice problem are summarized as the following equation (7):

$$(7.1)\ \frac{U_1\ (C_{1t}, C_{2t})}{U_2\ (C_{1t}, C_{2t})} = f'(k_{t+1})$$

$$(7.2)\ \frac{U_1\ (C_{1t}, C_{2t})}{U_2\ (C_{1t-1}, C_{2t-1})} = \frac{1+n}{\beta}$$

$$(7.3)\ C_{1t} + \frac{C_{2t-1}}{1+n} + (1+n)\ k_{t+1} = f(k_t),$$

$$C_{1t}, C_{2t-1}, k_{t+1} > 0 \quad \text{for all } t \geqq 1.$$

(7.4) For $T=\infty$, transversality condition

$$\lim_{T\to\infty} \beta^{T-1}\ U_1(C_{1T}, C_{2T}) = 0.$$

Equations (7.1) and (7.2) are identical to (6.1) and (6.2), respectively. Equation (7.3) is the resource constraint (1) with equality.

3. Steady State Analysis

A perfect foresight equilibrium of this planned economy is defined as a sequence $\left\{\bar{C}_{1t}, \bar{C}_{2t-1}, \bar{k}_{t+1}, \bar{d}_t\right\}_{t=1}^{T}$ which satisfies equation (6) or a sequence $\left\{\hat{C}_{1t}, \hat{C}_{2t-1}, \hat{k}_{t+1}, \hat{d}_t=0\right\}_{t=1}^{T}$ which satisfies equation (7). A steady state is a perfect foresight equilibrium where $C_{1t}=\bar{C}_1>0$, $C_{2t-1}=\bar{C}_2>0$, $k_{t+1}=\bar{k}>0$, and $d_t=\bar{d}\geqq 0$ for $1\leqq t\leqq T$.

A steady state in a closed economy will be called an autarky steady state and that in an open economy will be called an open economy steady state.

Following results are straightforward.

Proposition 1

There exists a unique open economy steady state and a unique autarky steady state in this economy.

Proof

In an open economy steady state equations (6.1) and (6.2) reduce to $f'(k)=\frac{1+n}{\beta}$, thereby determining a unique k. Then, both $d>0$ and r^f are uniquely determined from equations (5) and (6.3), as shown in Figure 1. Due to assumptions (A1) on preference, both

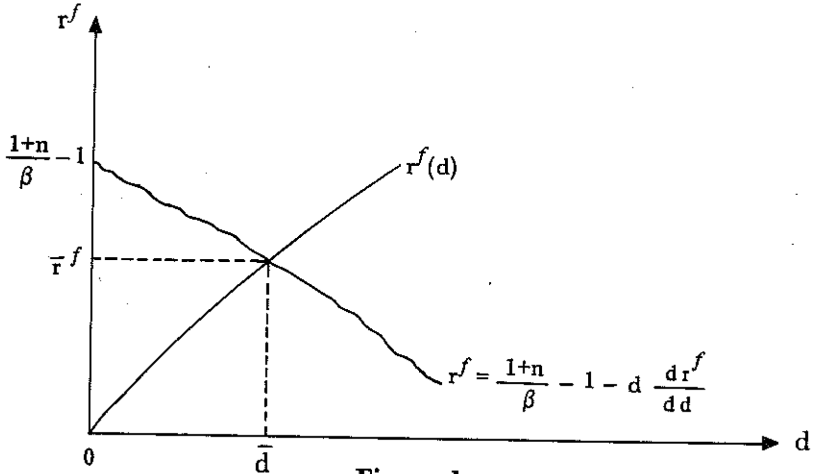

Figure 1

C_1 and C_2' are subsequently determined from equations (6.1) and (6.4). Similar for an autarky steady state.//

Let r_{t+1} be the imputed domestic interest rate between time t and t + 1. From 100% capital depreciation $f'(k_{t+1}) = 1 + r_{t+1,}$ and proposition 2 is immediate.

Proposition 2

A modified golden rule prevails in the steady state: The steady state gross interest rate is $1 + r = \frac{1}{\beta}(1 + n)$.

Proposition 2 shows that this model is basically in line with optimal growth models advanced by Cass (1965) and Koopmans (1965). It is well-known that consumption allocation under a modified golden rule is Pareto optimal.

Let ε be the interest rate elasticity of international capital: $\varepsilon_t = \frac{r^f_{t+1}}{d_t} \frac{d\,d_t}{dr^f_{t+1}}$. Then from equation (6.3), proposition 3 is immediate.

Proposition 3

In the open economy steady state $f'(k) = 1 + r^f(1 + \frac{1}{\varepsilon})$.

Proposition 3 is a dynamic extension of Kemp (1964).

Since both types of steady states lead to $f'(k) = \frac{1}{\beta}(1 + n)$, we have

Proposition 4

Capital-labor ratio $\bar{k}$ in the open economy steady state is the same as that in the autarky steady state.

Proposition 4 shows that foreign borrowing does not affect capital accumulation in the steady state: In the steady state foreign borrowing crowds out domestic saving by the same amount. What, then, is good for foreign borrowing in this economy? The following proposition answers the question.

Proposition 5

If the steady state r^f is less (greater) than n, then the open economy steady state is superior (inferior) to the autarky steady state.

Proof

Equation (6.4) becomes $f(k) + \frac{n - r^f}{1+n} d = C_1 + \frac{C_2}{1+n} + (1+n)k$ in the steady state. According to proposition 4, $f(k) - (1+n)k$ is identical in both steady states. Thus, if $r^f < n$, $d > 0$ gives higher level of C_1 and C_2 than the case $d = 0$. //

Proposition 5 shows that the planner will choose the open economy steady state if he sees with perfect foresight that the steady state r^f is less than n and choose the autarky steady state otherwise.

4. *Analysis of Dynamic Paths*

What happens if an initial condition does not coincide with the steady state, namely, $(k_1, C_{10}, d_0) \neq (\bar{k}, \bar{C}_1, \bar{d})$? For $T = \infty$, it can be shown that there exists a unique perfect foresight equilibrium path which satisfies equation (6) or (7) for all $t \geqq 1$ and converges to the steady state, if the initial condition lies in a small neighborhood of the steady state. Using a specific example of Buiter (1980)-Carmichael (1982) model, where there exists government bond instead of foreign capital, Ahn (1982) has established such result.

If the planning horizon is finite, we cannot stick to the steady state. Given an initial condition, a dynamic equilibrium path $\left\{ C_{1t}, C_{2t-1}, k_{t+1}, d_t \right\}_{t=1}^{T}$ will be uniquely determined once k_{T+1} is set as a policy target. If k_{T+1} were set high, consumption stream during the planning period would, in general, be reduced. The implication here is that a rapid capital accumulation and high economic growth in a given period will be achieved at the expense of consumption during the period and an attempt to avoid such consumption sacrifice through foreign borrowing will violate the optimality condition (6).

III. The Modified Model: Competitive Economy under Altruism

In the previous section we obtained optimal intertemporal resource allocation conditions (6) or (7) when the planner cares about not only living generations but also all future unborn generations. This section deals with whether these optimality conditions can be supported by a competitive mechanism.

Several complexities arise. First, if competitive firms were allowed to introduce foreign debt at their own discretion, equation (6.3) will not hold, but more foreign capital than the optimal level dictated by equation (6.3) will be introduced. This can be seen from the fact that $r^f_{t+1} < r_{t+1}$ in (6.3) and (5):

$$1 + r_{t+1} = f'(k_{t+1}) = 1 + r^f_{t+1} + d_t \frac{dr^f_{t+1}}{dd_t} > 1 + r^f_{t+1}.$$

Competitive firms will do arbitrage and incur foreign debt up to the level where $r^f_{t+1} = r_{t+1}$. In determining the amount of foreign borrowing, individual firms do not take into account that their borrowing activities will increase borrowing costs for the whole of the national economy. In order to get around this over-borrowing problem, it is assumed that the government still determines the total amount of foreign borrowing.[5]

Second, even if the optimal level of foreign borrowing were provided, the competitive mechanism does not necessarily achieve optimal intergenerational consumption allocation, expressed as equation (6.2) or equation (7.2). Especially, in the usual egoistic preference $U(C_{1t}, C_{2t})$, there is no guarantee for equation (6.2) to hold.

Thirdly, the budget constraint facing a representative member of a generation will not be the same as the economy's resource constraint. The budget constraint together with market equilibrium conditions will imply the economy's resource constraint.

[5] The usual optimal tax formula for international investment can be proposed. But the formula presupposes knowledge of the total amount of foreign borrowing in our dynamic framework.

Thus, both individual's budget constraint and market equilibrium conditions should be properly defined.

This section shows that if every agent cares about the welfare of his immediate descendants, the planner's optimality condition can be mimicked by a competitive equilibrium. This illustration is made in a simple setting where the sequence of optimal foreign debt is given as in the planner's economy.

1. The Environment

The environment is basically the same as in II. 1 except for the following: The preference V_t of a member of generation t is expressed as a nested utility function. Specifically, it is additively separable with respect to own utility level and his descendant's welfare level:[6]

$$(8) \quad V_t = U(C_{1t}, C_{2t}) + \beta \ V^*_{t+1}, \quad 0 < \beta < 1$$

where $U(C_{1t}, C_{2t})$ has the same properties as in (A1) and V^*_{t+1} is defined as the maximized indirect utility function of a representative member of generation $t+1$ perceived by a member of generation t. Each member of generation t can affect V^*_{t+1} by leaving bequests b_t. $V^*_{t+1}(b_t)$ is assumed to be a differentiable function.

Each agent is a utility-maximizer. He has perfect foresight with respect to all aggregate parameter values and other generation's behavior. Bequests b_{t-1} left to generation t by a member of generation t-1 are shared equally among descendants. Each agent treats his received intergenerational transfer as parametric. All markets are competitive.

The government adopts the policy that any excess of debt service requirement over new foreign borrowing is financed by lump-sum taxes on the young.

[6] The additive separability of the utility function is not restrictive. We can work with a more general utility function $V_t = V(C_{1t}, C_{2t}, V^*_{t+1})$ to obtain the same results provided $0 < \frac{\partial V(\cdot)}{\partial V^*_{t+1}} < 1$. This restriction or the restriction on β in (8) is necessary for the steady state utility level to be well-defined.

2. Competitive Equilibrium

The budget constraint of a member of generation t is

$$(9)\quad C_{2t} + b_t \leqq (W_t - \tau_t - C_{1t} + \frac{b_{t-1}}{1+n})\ (1+r_{t+1})$$

for all $t \geqq 1$

where W_t is the real wage rate at time t and τ_t is the lump-sum tax levied on a member of generation t at time t. Individual savings at time t are used as capital at t+1 by a competitive firm and are repaid with interest at a rate r_{t+1} in time t+1.

The optimization problem of a member of generation t is the maximization of (8), with respect to C_{1t}, C_{2t} and b_t subject to the budget constraint (9).

Form a Lagrangian:

$$(10)\quad L(\cdot) = U\ (C_{1t}, C_{2t}) + \beta\, V^*_{t+1}\ (b_t) + \lambda_t \left\{(W_t - \tau_t - C_{1t} + \frac{b_{t-1}}{1+n})(1+r_{t+1}) - b_t - C_{2t}\right\}.$$

The first order conditions are:[7]

$$(11.1)\quad \frac{\partial L}{\partial C_{1t}} = U_1\ (C_{1t}, C_{2t}) - \lambda_t(1+r_{t+1}) = 0$$

$$(11.2)\quad \frac{\partial L}{\partial C_{2t}} = U_2\ (C_{1t}, C_{2t}) - \lambda_t \quad = 0$$

$$(11.3)\quad \frac{\partial L}{\partial b_t} = \beta\ \frac{\partial V^*_{t+1}}{\partial b_t} - \lambda_t \ = 0^{8}$$

[7] Using the Buiter-Carmichael model similar to this model, Ahn (1982) shows that the first order conditions are necessary and sufficient for the optimization problem if we limit our attention to the study of equilibrium paths converging to the steady state and if $U_{12}(C_1, C_2) = \frac{\partial^2 U(C_1, C_2)}{\partial C_2\, \partial C_1} \geqq 0$. Ahn (1982) also shows that the differentiability of $V^*_{t+1}(\cdot)$ can be justified.

[8] If there is an institutional constraint that bequests cannot be negative, the second equality of equation (11.3) is replaced by "$\leqq$" and we have to distinguish an interior solution from a corner solution. This complexity is conveniently ignored in this paper by assuming that bequests can take any value exceeding some lower bound $\underline{b}<0$, and this bound is not binding.

$$(11.4)\ \frac{\partial L}{\partial \lambda_t} = (W_t - \tau_t - C_{1t} + \frac{b_{t-1}}{1+n})\ (1+r_{t+1})$$

$$- C_{2t} - b_t = 0.$$

The optimization conditions are summarized by equation (12).

$$(12.1)\ \frac{U_1\ (C_{1t}, C_{2t})}{U_2\ (C_{1t}, C_{2t})} = 1+r_{t+1}$$

$$(12.2)\ \frac{U_1\,(C_{1t+1}, C_{2t+1})}{U_2\,(C_{1t}, C_{2t})} = \frac{1}{\beta}(1+n)$$

$$(12.3)\ \ (W_t - \tau_t - C_{1t} + \frac{b_{t-1}}{1+n})\ (1+r_{t+1}) = C_{2t} + b_t\ .$$

Equation (12.2) comes from (11.3), (11.2) and the following proposition.

Proposition 6

$$\frac{\partial V^*_{t+1}}{\partial\ b_t} = \frac{1}{1+n}\quad U_1\,(C_{1t+1}, C_{2t+1})$$

Proof

The optimization problem enables us to write

$$V^*_t\ (W_t - \tau_t, r_{t+1}, b_{t-1}) = \underset{\{C_{1t}, C_{2t}, b_t\}}{\text{Max}}\ \Big\{U\ (C_{1t}, C_{2t})$$

$$+ \beta\ V^*_{t+1}\ (W_{t+1} - \tau_{t+1}, r_{t+2}, b_t)\Big\}$$

By the envelope theorem,

$$\frac{\partial V_t^*}{\partial b_{t-1}} = \frac{\partial L}{\partial b_{t-1}} = \lambda_t \frac{1+r_{t+1}}{1+n} \quad \text{from} \quad (10)$$

$$= U_1 \ (C_{1t}, C_{2t}) \ \frac{1}{1+n} \quad \text{from (11.1)}.$$

Since this holds for all $t \geqq 1$,

$$\frac{\partial V_{t+1}^*}{\partial b_t} = \frac{1}{1+n} \ U_1 \ (C_{1t+1}, C_{2t+1}). /\!/$$

Other economy-wide equilibrium conditions are the following:

$$(13.1) \quad (W_t - \tau_t - C_{1t} + \frac{b_{t-1}}{1+n}) \ N_t = K_{t+1} \quad \text{or}$$

$$W_t - \tau_t - C_{1t} + \frac{b_{t-1}}{1+n} = (1+n) \ k_{t+1}$$

$$(13.2) \quad (1+r_t^f) \ D_{t-1} = D_t + N_t \ \tau_t \quad \text{or}$$

$$(1+r_t^f) \ d_{t-1} = (1+n) \ (d_t + \tau_t)$$

$$(13.3) \quad W_t = f(k_t) - k_t f'(k_t)$$

$$(13.4) \quad f'(k_{t+1}) = 1 + r_{t+1}, \quad t \geqq 1 .$$

Equation (13.1) is the asset market equilibrium condition: Total saving must equal total stock of assets consisting of real capital. Equation (13.2) is the government budget constraint. Government spending is assumed to be zero for convenience. Equations (13.3) and (13.4) are the familiar labor and capital market clearing conditions.

Combining equation (12.3) with (13) we get

$$f(k_t) + d_t = C_{1t} + \frac{C_{2t-1}}{1+n} + (1+n) \ k_{t+1} + \frac{1+r_t^f}{1+n} \ d_{t-1} .$$

Thus, for $\{d_t\}_{t=1}^{T}=\{0\}$ equations (12) and (13) become identical to equation (7). For an open economy equilibrium, if $\{d_t\}_{t=1}^{T}$ is given in such a way that equation (6.3) is met, then equations (12) and (13) also become identical to equation (6).

In proving above results, total foreign borrowing was assumed to be given at the optimal level. Instead, government schemes for regulating firms' overborrowing that take into account firms' optimization problem should have been incorporated. Given this limitation in our competitive model, what happens if foreign borrowing exceeded the optimal level? More borrowing from abroad corresponds to lower τ_t and larger disposable income in an agent's budget constraint (9). This leads to an increase in current consumption and a decrease in the current saving ratio. Thus, the current low saving ratio is attributed to overborrowing. This explains why a nationwide campaign by a high-debt developing country to increase its national saving ratio in order to self-finance its ambitious investment projects will not easily succeed. Proposition 4 and above results imply that foreign borrowing crowds out national saving.

IV. Concluding Remarks

If agents are assumed to have altruistic preferences expressed as equation (8) to begin with, then the planner's solution incorporating this preference would also be identical to the competitive equilibrium.

Most people are concerned about their children and the nested utility function (8) seems a reasonable specification of the preference of a representative individual in the real world. It is reassuring that such reasonable altruistic preference is conducive to achieving an optimal intertemporal resource allocation.

This paper can best be interpreted as an abstract modelling from the following historical event.

Early in this century, Japan attempted to make the Korean economy dependent on the Japanese economy by forcing the Chosun (the previous name of Korea under the Yi dynasty, 1392-1910) government to borrow Japanese capital on unfavorable terms.[9] Sensing the possible problem of the loss of Chosun's political and economic independence through the borrowing of Japanese capital, the Korean people reacted to Japan's lending policy with a nationwide campaign to liquidate the national debt.

Having originated among the Korean grass-roots, the national debt liquidation campaign was intensely waged[10] until it was suppressed under Japanese colonial rule. The major concern of the national debt liquidation campaign was preserving Korea's national independence, which in turn came mainly from the people's concern for the welfare of their descendants, as was expressed in the following statement:

> "If we cannot liquidate our national debt, our land will not be our land... Our nation will become subjugated to other nations and our children, then, will become their slaves. It is disheartening to think even the possibility that dignity and human rights of our children will be taken away due to our irresponsibility..."[11]

This campaign was a good example of altruism combining with nationalism to bring about the reduction of overborrowing of foreign debt. The basic thesis of this paper is that such altruism is fully compatible with agents' intertemporal optimization and thus it is a form of enlightened self-interest.

As for the recent external debt problems in many developing countries, this paper suggests that the growth-centered attitude shown by governments and business firms should be redressed in the first place.

Since the model used in this paper is so simple, its rule-of-the-thumb principle for optimum foreign borrowing can be summarized as follows: Use foreign borrowing when its cost is much lower than the domestic borrowing cost. Even though our model, as a one-good model, can not deal with foreign trade, we can con-

9 Chosun's annual budget at that time showed a deficit of about 800,000 Japanese yen. Government total revenue was about 13.2 million yen, while total expenditure was about 14 million yen. Total borrowings amounted to 13 million yen as of 1907.

10 Within three months after the campaign started, total voluntary contributions amounted to 2.3 million yen.

11 See Park (1968).

jecture that this straightforward principle will also hold by analogy for imports and symmetrically for exports. As for exports this will be: Export those commodities whose export prices are higher than their domestic prices. The implication here is that an export drive to sell commdities abroad at prices below their domestic prices is not optimal.

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