

The Heterogeneity of Family and Hired Labor in Agricultural Production: A Test Using District-Level Data from India*

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I. Introduction

There is no dearth of production function studies on the agricultural sectors of less-developed countries.¹ However, with only a few exceptions, most studies have failed to distinguish between family labor and hired labor inputs, thus implicitly maintaining the questionable assumption of homogeneity of the two types of labor in agricultural production. Since family labor, unlike hired labor, is often entrusted with managerial tasks on the family farm, it is quite likely that the two kinds of labor are heterogeneous and may have different effects on agricultural output. The central point of this paper is that it is incorrect to simply assume away the heterogeneity of family and hired labor by

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1 Examples of such studies are Heady and Dillon (1961), Rao (1965), Yotopoulos, Lau and Somel (1970), Huang (1971), Bardhan (1973), Nath (1974), Brown and Salkin (1974), Barnum and Squire (1979).

treating them as identical and perfectly substitutable inputs in the production function, as previous studies have done.

District-level data from India are used in this paper to test the hypothesis of homogeneity of labor in agricultural production. The production function we employ is general enough to permit family and hired labor to have different effects on output as well as any constant elasticity of substitution between each other. Nested within the general model are several other more restrictive models, including the Cobb-Douglas production function having total labor as one input and the Cobb-Douglas production function having family and hired labor as two separate inputs. This makes it possible for us to test the general model against the conventional production functions that have been commonly estimated in the literature.

To anticipate our findings, we generally reject the conventional Cobb-Douglas production function which does not distinguish between family and hired labor. On the other hand, we accept the Cobb-Douglas production function with family and hired labor as two *separate* inputs. Further, family labor is consistently observed to have a larger impact on output than hired labor. This suggests that family and hired labor are heterogeneous both in the sense of being imperfect substitutes for each other and in the sense of having different effects on agricultural output. This finding has important implications for the interaction of labor demand and labor supply in the agricultural sector of LDCs as well as for fertility among farm households, as is pointed out later in the paper.

In addition, although this is not a central concern of this paper, we also test the heterogeneity of irrigated and unirrigated land using the same models.² Most empirical studies normally treat the two types of land as separate inputs in production functions. The result of our test of this assumption indicates that irrigated and unirrigated land are perfect rather than imperfect substitutes, but that they still are heterogeneous inputs in that they have a differential effect on output. The Cobb-Douglas production function with irrigated and unirrigated land as two separate inputs is thus rejected in favor of the Cobb-Douglas production

2 To our knowledge, no study has attempted to test alternative functional forms for the relationship between the two types of land.

function, in which a weighted sum of irrigated and unirrigated land is entered as a single input.

The plan of the paper is as follows. Section 2 discusses previous evidence on the heterogeneity issue. Section 3 elaborates on some implications of heterogeneous labor. In section 4 we specify the functional forms of the production function, with which we test for heterogeneity of labor. Results of the tests are reported in section 5. Section 6 investigates the heterogeneity of land, while section 7 concludes.

II. Homogeneity Versus Heterogeneity of Labor in Previous Research

Previous researchers have typically estimated Cobb-Douglas production functions having total (hired plus family) labor as a single input. The assumptions underlying such a production function are (i) family and hired labor are symmetric in terms of their effect on output, and (ii) family and hired labor are perfect substitutes (in the sense of having an infinite elasticity of substitution between them) in agricultural production. To see this, let the agricultural production function be:

$$(1) \quad Y = CL \prod_{i=1}^n X_i^{\beta_i},$$

where i indexes non-labor inputs, Y = output, and L = labor services. C , β , and β_i are parameters to be estimated. Labor services are assumed to be "produced" according to a linear production function:

$$(2) \quad L = FL + HL,$$

where FL and HL are quantities of family and hired labor used, respectively. As is obvious from equation (2), the coefficients on hired and family labor are identical (and equal to one), implying equal effects of the two inputs on output. Further, since the elasticity of substitution is always infinite in a linear production function, the relationship between family and hired labor in equation (2) is that of perfect substitutability.

To our knowledge, Bardhan (1973) is among the few researchers to have statistically tested for the heterogeneity of family and hired labor in an agricultural production function. However, his test was neither rigorous nor complete. His specification of the production function was

$$(3) \quad Y = C L^\alpha L_h^{*\gamma} \prod_{i=1}^n X_i^{\beta_i}$$

where L = total labor used on the farm (equation (2)), and L_h^* = proportion of total labor that is hired, i.e., $L_h^* = HL/(HL + FL) = HL/L$. Equation (3) can be rewritten as

$$(4) \quad \ln Y = \ln C + (\alpha - \gamma) \ln L + \gamma \ln HL + \sum_{i=1}^n \beta_i \ln X_i$$

Thus, Bardhan's specification is equivalent to including both total labor and hired labor as separate inputs in the production function.

The parameter of interest is γ . For most of his samples, (Indian farms), Bardhan obtained estimates of γ that were not significantly different from zero, but for two samples γ was estimated positive, implying heterogeneity of labor such that hired labor is more productive than family labor (Bardhan, 1973, p. 1381).

However, production function (4) has some undesirable characteristics. If γ is negative, the production function is not everywhere concave, and when HL approaches zero, Y approaches infinity. If γ is positive, no production occurs without hired labor (i.e., $Y = 0$ for $HL = 0$). Thus it would seem more likely that one estimates γ positive. In addition, if HL rises relative to FL , the productivity difference between family and hired labor decreases, for any value of γ . The direction of a possible bias caused by this assumption is uncertain, but the specification becomes rather restrictive.

The test is not complete in the sense that heterogeneity consists of a differential effect of the two inputs on output and/or imperfect substitutability between the inputs. The elasticity of substitution between family and hired labor is not constant in

equation (4). Bardhan does not calculate this elasticity of substitution, and thus does not provide evidence on this aspect of heterogeneity.³ Therefore, his rejection of heterogeneity of labor cannot be accepted as conclusive.

Other evidence on this issue is rather sketchy. Huang (1971) includes family and hired labor as separate inputs in an agricultural production function (Cobb-Douglas). Using farm data from Malaysia he observes a simultaneous occurrence of overuse of family labor and underuse of hired labor, with the latter being more productive. Substitutability is not further investigated.

Brown and Salkin (1974) also find, using farm data from South Vietnam, that hired labor is more productive than family labor. Not all evidence presented by them is convincing, however. Some of it is obtained by comparing the labor coefficients of two subsamples separated according to whether hired labor constitutes more or less than twenty percent of total labor used. This selection criterion is an endogenous aspect of the heterogeneity issue and this biases the result. This criticism can also be leveled against the same finding by Desai and Mazumdar (1970). They separate the sample of (Indian) farms according to whether labor is hired, and then compare estimated labor coefficients.

Frequently the productivity gap between family and hired labor is explained with dual labor market arguments. Nath (1974) provides another insight as to why family labor may be less productive. He distinguishes a busy season and a slack season, and assumes that all casual labor is employed during the busy season in addition to a certain fraction of permanent labor (which is estimable in principle, although Nath only tries certain values). Among Indian farms, the productivity of busy season labor is positive, while the productivity of slack season labor is zero. Aggregating all labor together in the production function yields an estimated productivity equal to zero as well. Therefore failing to

³ Since Bardhan has not presented his complete estimation results or the sample means of the variables in his data set, it was not possible for us to calculate the elasticity of substitution between family and hired labor implied by his model. However, when we estimated equation (3) with our data set, we obtained a negative elasticity of substitution between the two types of labor. The isoquants between family and hired labor were thus observed to be concave, not convex (as is required by theory), to the origin. See the Appendix for more details.

distinguish the contribution of family labor during the various seasons may have led to underestimating the marginal product of family labor and, as a consequence, to overestimating the marginal product of hired labor.

In this context a finding by Ahmed (1981) is interesting as well. One usually assumes that all farmers employ the same production technology, although at a different scale. Ahmed found that small farmers in Bangladesh produce more labor-intensive crops than large farmers. Estimates of labor productivity based on the assumption of identical production technology over the full scale of operation will be biased downward. Since smaller farms are more family-labor oriented, this downward bias will be expressed mainly through the coefficient on family labor. Viewing the results reported above, this prediction is borne out quite well.

In summary, heterogeneity of labor consists of two elements. Evidence of the first, a differential effect of family and hired labor on agricultural output, exists, but the tests have not been clean and should be performed more accurately in view of seasonality (Nath) and potential supply responses (Ahmed; see also Brown and Salkin, and Desai and Mazumdar). Evidence of the second element of heterogeneity — imperfect substitutability between family and hired labor — is only implied by use of Cobb-Douglas production functions. Imperfect substitutability has not been tested as such. Therefore we will proceed in section 4 with a test of the heterogeneity of labor.

III. Implications of Labor Heterogeneity

The most immediate implication of heterogeneous family and hired labor is for the growing literature on empirical applications of the 'theory' of the household (Lau, Lin, and Yotopoulos, 1978; Barnum and Squire, 1979). These models have typically involved separate estimation of consumption and production models of a farm household and subsequent 'integration' of the estimated models to calculate the net (final) impact of prices, wage rates, and policy variables on a representative farm household. Separate estimation of consumption and production decisions has generally been justified on the grounds that there is a perfectly competitive market for labor in LDCs and that family and hired labor are

homogeneous. Farm households are thus assumed to make their family labor supply decisions independently of the demand for on-farm labor, since the competitive market and homogeneous labor assumptions imply that excess family labor can always be sold in the casual labor market, or excess demand can be met by hiring in casual labor from the market, at a fixed wage rate.

If family and hired labor are heterogeneous, the labor demand and labor supply decisions of farm households cannot be so easily separated. To take an extreme example, if the elasticity of substitution between family and hired labor is zero, the supply of labor by family members cannot be determined independently of the on-farm demand for managerial and supervisory tasks, since the latter can never be performed by hired labor. Even for more plausible elasticities of substitution (i.e., greater than zero but less than infinity), the conventional models of the farm household, which assume separability of the household's production and consumption decisions, will have to be substantially revised.

A second implication of the imperfect substitutability between family and hired labor is at the labor market level and relates to rural-urban migration. Abstracting from household consumption decisions, if family and hired labor are perfect substitutes and if the former migrates to the city, the demand for hired labor will go up by the amount of family labor migrated.⁴ As a result, one would expect wages paid to hired labor to rise. Thus migration would benefit the population that stays behind in the agricultural sector. However, this conclusion is not so clear when family and hired labor are imperfect substitutes. Taking the extreme case in which substitutability is zero, the demand for hired labor may decrease when family labor migrates, depending on the substitutability with other inputs. Therefore, the landless agricultural population may actually be impoverished due to the migration and its effect on rural poverty and income distribution.

A third implication, especially over a longer run, of the imperfect substitution between family and hired labor is that

4. When one would integrate this implication with the existence of household consumption decisions, one has to make assumptions about the nature of migration: does the migrant remit part of his earnings; does the household now consist of two units and how does that affect consumption? Alternatively one might assume that the migrant leaves permanently and therefore that the family size decreases. Each of these assumptions affects the implications of migration somewhat.

variables such as farm size, irrigation, or technical change, which increase the demand for family labor on the farm, may be expected to affect fertility rates among farm households. If family and hired labor are identical and perfect substitutes for each other, fertility among farm families should not be related to these factors,⁵ since the greater demand for family labor on large or irrigated farms can always be met by hiring in casual agricultural labor at a fixed wage rate.

Our central argument is that the conventional concept of 'labor demand' is invalid if family and hired labor are heterogeneous in the sense of having different efficiencies and being imperfect substitutes for each other. Instead, we need to talk about a demand function for family labor and a demand function for hired labor arising out of constrained profit maximization by farm households. In general, the wage rate paid to hired labor will not be the correct price of family labor. The latter will be wage rate received by family workers while working away from the family farm. Hardly any study has bothered to check whether the wage rate paid by cultivators to hired workers is different from that received by them when working on other people's farms, although such information is generally available from most household surveys. Instead, most studies have simply assumed that the two wages are equal.

IV. The Model

We assume that the agricultural production function facing farms is of the Cobb-Douglas type⁶:

$$(5) \quad Y = C L^{\beta} \prod_{i=4}^n X_i^{\beta_i}$$

5 Of course, the positive relationship between fertility and farm size or irrigation may well be due to a positive income effect of the latter variables on fertility. The positive income effect would imply that children are normal goods.

6 A Cobb-Douglas relationship between labor and other inputs and among non-labor inputs is assumed, since there is considerable empirical evidence for a unitary elasticity of substitution between land and labor and between land and capital in agriculture (Yotopoulos, Lau, and Somel, 1970). Besides, we are primarily interested in this paper in exploring the 'best' relationship between family and hired labor; as such, the relationships among other inputs are not of central concern to us. More general testing procedures (e.g., Denny and Fuss, 1977) do not permit nested testing of labor heterogeneity.

where Y = output, L = labor services, and X_i = quantity of the i th nonlabor input used. We assume that labor services L are produced using family labor FL and hired labor HL :

$$(6) \quad L = L(FL, HL).$$

As discussed in the previous sections, the most common functional specification of L is additive: $L = FL + HL$. In this section, we examine two specifications which nest the additive form as a special case and which have a variety of interesting implications.

The first specification is the generalized CES production function, which contains the parameters α_1 and ρ_1 . By appropriately restricting these parameters, we get the following five models:

$$(7) \quad L = (\alpha_1 FL^{-\rho_1} + (1-\alpha_1) HL^{-\rho_1})^{-1/\rho_1}, \rho_1 \geq -1, \text{ (model A.1)}$$

$$(8) \quad L = (0.4 FL^{-\rho_1} + 0.5 HL^{-\rho_1})^{-1/\rho_1}, \rho_1 \geq -1, \text{ (model A.2)}$$

$$(9) \quad L = \alpha_1 FL + (1-\alpha_1) HL, \text{ (model A.3)}$$

$$(10) \quad L = 0.5 FL + 0.5 HL = 0.5 (FL + HL), \text{ (model A.4)}$$

$$(11) \quad L = FL^{\alpha_1} HL^{1-\alpha_1} \text{ (model A.5)}$$

Model (A.1) represents the most general form. In model (A.2), α_1 is restricted to be equal to 0.5, while in model (A.3) ρ_1 is restricted to be -1 . Model (A.4) is the commonly-estimated additive form in which $\alpha_1 = 0.5$ and $\rho_1 = -1$ are imposed. Finally, in model (A.5), ρ_1 is constrained to be zero (Arrow, Chenery, Minhas, and Solow, 1961); this implies that the production function of Y is Cobb-Douglas in all inputs, including family and hired labor. Clearly, this is an interesting special case, since the Cobb-Douglas production function is easier to estimate than models (A.1)-(A.3).

We call α_1 the symmetry parameter: it determines whether the function L is symmetric in family and hired labor. ρ_1 is the curvature parameter, since the curvature of the isoquants of the labor services production function becomes sharper with increas-

ing ρ_1 . In fact, the elasticity of substitution of the function L (σ_{23}^L) is related to ρ_1 by

$$(12) \quad \sigma_{23}^L = \frac{1}{1 + \rho_1}$$

There is, however, another measure of the elasticity of substitution between family and hired labor. This is the Allen-Uzawa partial elasticity of substitution σ_{23}^Y , which measures the substitutability between FL and HL in the context of the production of Y (not L) (Allen, 1938, p. 504).⁷ The relation between σ_{23}^L and σ_{23}^Y is not obvious. However, when σ_{23}^L approaches infinity (as in models (A.3) and (A.4)), σ_{23}^Y goes to infinity as well.

The second general specification of the labor services production function is the generalized linear production function (Diewert, 1971, p. 503), which has two parameters α_{21} and α_{22} . Appropriately restricting these parameters yields four models:

$$(13) \quad L = \alpha_{21} FL + 2 \alpha_{22} FL^{1/2} HL^{1/2} + (1 - \alpha_{21}) HL, \quad (\text{model B.1})$$

$$(14) \quad L = 0.5 FL + 2 \alpha_{22} FL^{1/2} HL^{1/2} + 0.5 HL, \quad (\text{model B.2})$$

$$(15) \quad L = \alpha_{21} FL + (1 - \alpha_{22}) HL, \quad (\text{model B.3})$$

$$(16) \quad L = 0.5 FL + 0.5 HL = 0.5 (FL + HL). \quad (\text{model B.4})$$

Model (B.1) represents the most general specification here. Models (B.2) and (B.4) restrict α_{21} to 0.5, while in models (B.3) and (B.4) α_{22} is restricted to be zero. Note that models (B.3) and (B.4) are identical to models (A.3) and (A.4), respectively.

In models (B.1)–(B.4), α_{21} is the symmetry parameter, and α_{22} the curvature parameter. When α_{22} is positive (negative), the isoquants of the labor services production function are convex (con-

7 If the production function of Y with inputs Z_i is written as $Y = f(Z_1, \dots, Z_n)$ then σ_{23}^Y is defined as

$$\sigma_{23}^Y = (F_{23} \sum_i f_i Z_i) / (Z_2 Z_3 F),$$

where $f_i = \partial Y / \partial Z_i$, $f_{ij} = \partial^2 Y / \partial Z_i \partial Z_j$, F is the bordered Hessian, and F_{ij} is the cofactor of f_{ij} .

cave) to the origin. In this sense, the parameters α_{21} and α_{22} are analogous to the parameters α_1 and ρ_1 of models (A.1)-(A.5), respectively. The elasticity of substitution of the function L and be expressed as:

$$(17) \quad \sigma_{23} L = \frac{2 FL^{1/2} HL^{1/2}}{L \alpha_{22}} (\alpha_{21} + \alpha_{22} FL^{1/2} HL^{1/2})$$

$$(1 - \alpha_{21} + \alpha_{22} FL^{1/2} HL^{1/2}).$$

We thus have seven distinct models that can be tested against each other in order to obtain evidence on the labor homogeneity issue. Those models that are nested can be tested with the standard F-test. Models that are not nested can be compared using a recently developed test by Davidson and McKinnon (1981).

A final note concerns the occurrence of zero hired labor inputs observed in samples of farm level data. The additive labor models (A.3) and (A.4) (and equivalently (B.3) and (B.4)) are consistent with the observation that some farms merely use family labor and do not hire outside labor. On the other hand, the CES specification does not permit zero values of inputs for positive values of ρ_1 , nor does the Cobb-Douglas specification in model (A.5). For values of ρ_1 in the open interval $(-1, 0)$, the isoquants of the production function of labor services are tangent to the FL- and HL-axes. This implies that zero inputs are consistent but will be chosen only if the price of such inputs approaches infinity. For any finite wage rate of hired labor, each profit-maximizing farm will always hire some outside labor. The generalized linear production function employed in models (B.1) and (B.2) suffers from the same problem.

This problem is not serious for the estimations reported in this paper, since we use an aggregated community-level data set which does not contain any zero values for any of the inputs. Even at the farm level, zero inputs for family or hired labor are rarely observed, at least in Indian agriculture. For instance, Rosenzweig (1978, pp. 847-848) reports, on the basis of a 1970-71 all-India

survey of over 5,000 rural households, that 88 per cent of small farm households in India hire in outside labor and 85 per cent of large farm households use family labor on the family farm.

V. Estimation and Results

We have estimated the various models presented in the previous section with district-level data on 268 districts from all over India.⁸ The data are for the agricultural year 1970-71, except in the case of gross value of agricultural output, which is averaged over three years (1969-70, 1970-71, and 1971-72) to eliminate short-term fluctuations arising because of abnormal weather. In calculating district output, constant all-India prices have been used to value each crop. Since the district-level variables are totals over varying numbers of farms in each district, all variables have divided by the number of holdings (or farms) in a district before estimation.⁹ Each observation is thus assumed to represent an "average" farm in a district.

The assumption maintained implicitly in estimating an aggregate agricultural production function,¹⁰ and the problems inherent therein, have been described by Timmer (1970), who has estimated production functions for U.S. agriculture using state-level data. Although such estimates are beset with serious theoretical complications, they serve a useful policy purpose in that they describe the aggregate response of output to changes in input levels.

The definitions of the variables used are given in Table 1. To estimate equations (A.1)-(A.5) and (B.1)-(B.4), we have added an i.i.d. disturbance term multiplicatively to each equation. Equations (A.4) (= B.4) and (A.5) have been estimated by

8 The data have been compiled from a number of sources, including the various state reports of the *Agricultural Census of India 1970-71*, a joint Jawaharlal Nehru University-Planning Commission study entitled *Foodgrains Growth: A Districtwise Study* (for data on gross value of agricultural output), and *Fertilizer Statistics 1972* (for data on fertilizer use).

9 As a contrast, consider the fact that model (C.2) has a RSS, which is only 14 percent lower than the RSS of the model with the *highest* RSS among all other specifications.

10 One particularly strong assumption is that the number of days worked in agricultural production activities by a cultivator or agricultural laborer does not vary systematically across regions. This assumption is necessary because of the nature of the data available at the district level; in particular, only data on numbers of cultivators and agricultural laborers, and not on days or hours worked, are available.

Table 1
VARIABLE DICTIONARY: INDIAN DISTRICTS, 1970-71

<u>Variable</u>	<u>Mean</u>	<u>Std. Dev.</u>
Y (Gross Rupee value, at constant all-India prices, of output of 22 major crops per operational holding in district)	2743.30	8214.10
FL (Number of cultivators in district per operational holding)	1.28	0.55
HL (Number of agricultural laborers in district per operational holding)	0.71	0.45
X ₄ (Hectares of irrigated cropped land in district per operational holding)	0.68	2.36
X ₅ (Hectares of unirrigated cropped land in district per operational holding)	2.12	2.43
X ₆ (Kilograms of fertilizer used in district per operational holding)	39.20	52.73

Note : Number of observation is 268.

ordinary least squares, while equations (A.1)-(A.3) and (B.1)-(B.2) have been estimated by non-linear least squares. In estimating equations (A.1) and (A.2), a lower bound of $\rho_1 = -1$ (corresponding to an infinite elasticity of substitution) has been set to assure concavity (positive elasticity of substitution) of the production function. For models (B.1) and (B.2), the analogous bound was $\alpha_{22} = 0$.

Results of the least squares estimation of models (A.1)-(A.5) and (B.1)-(B.4) are reported in table 2. The boundary limits on ρ_1 and α_{22} were binding in models (A.2) and (B.2), respectively. Therefore the estimated models (A.2), (A.4), (B.2) and (B.4) are all equivalent.

Of interest in Table 2 are the parameters α_1 and α_{21} , which indicate the relative weight to be attached to family labor vis-a-vis

Table 2

PRODUCTION FUNCTION ESTIMATES, MODELS
A.1.-A.5. AND B.1-B.4 (WITH LABOR AS A HETEROGENEOUS
INPUT): INDIAN DISTRICTS, 1970-71.

(asymptotic t-statistics in parentheses)

Parameter	A.1	A.3 B.3	A.2, A.4, B.4	A.5	B.1	B.2
C	8.215	8.237	7.969	8.225	8.238	7.969
β_1	0.450 (3.177)	0.449 (4.311)	0.380 (3.846)		0.471 (4.226)	0.380 (3.846)
α_1	0.901 (4.733)	0.758 (6.052)	0.500 ^a	0.821 ^b		
ρ_1	0.701 (0.413)	-1.000 ^a	-1.000 ^c	0. ^a		
α_{21}					0.740 (6.047)	0.500 ^a
$\alpha_{22}(\times 10^3)$					0.135 (0.454)	0. ^d
β_2				0.381 (4.307)		
β_3				0.083 (2.052)		
β_4	0.220 (6.484)	0.220 (8.933)	0.236 (10.043)	0.221 (9.120)	0.219 (8.852)	0.236 (10.043)
β_5	0.252 (4.607)	0.251 (6.361)	0.273 (7.185)	0.249 (6.386)	0.248 (6.227)	0.273 (7.185)
β_6	0.133 (3.331)	0.141 (4.962)	0.122 (4.453)	0.134 (4.649)	0.142 (4.969)	0.122 (4.453)
σ_{23}^L	0.588	∞	∞	1,000	2,402.3	∞
σ_{23}^Y	0.034	∞	∞	1,000	5,466.3	∞
RSS	45.430	45.833	46.579	45.509	45.760	46.579

Notes: ^aImposed value. ^bValue calculated using formula: $\alpha = \beta_2/\beta_2 + \beta_3$

^cBoundary value in the case of model A.2; imposed value in the case of model A.4.

^dBoundary value.

hired labor. Both α_1 and α_{21} are consistently greater than one-half (except in models where they are constrained to be equal to one-half), implying that, in adding up family and hired labor, the former should be weighed anywhere from three to nine times as much as the latter.

In contrast to the consistency of the symmetry parameters, the curvature parameters differ dramatically across models. For instance, the elasticity of substitution implied by the estimated curvature parameter in model (A.1) is 0.6, while in model (B.1) the implied elasticity of substitution between family and hired labor is 2402.

There are a total of five specifications of the agricultural production function (A.1, A.3, A.4, A.5, B.1 and their equivalents), that can be tested against each other. We compare nested models by a standard F-test and use to Davidson-McKinnon (1981) test to compare non-nested models.¹¹ Table 3 shows the results of these tests. Model (A.4) is rejected in favor of all other specifications. Model (B.1) is dominated by model (A.3), which in turn is rejected in favor of (A.1). Model (A.5) dominates (A.1). Thus, by any yardstick, model (A.5), a Cobb-Douglas relationship between family and hired labor, emerges as the "best" specification¹² of the agricultural production function.¹³

In view of the past literature this is an important result. Past research has concentrated either on production function (A.4) or on (A.5), without verifying the choice with statistical testing. As noted in section 2, an important reason for choosing model (A.5) was to make a distinction between season and off-season labor, with the idea in mind that labor is hired to a large extent at the peak times of the production process, and that substitutability of labor between the various stage of this process is limited (Nath, 1974). Our results are consistent with this argument for two reasons. First, substitutability of family and hired labor appears to be limited. Second, the ratio of the marginal product of family

11 The Davidson-McKinnon test consists of two steps in which each of the models involved in the test perform as the null hypothesis. A significant test statistic indicates rejection of the null hypothesis rather than acceptance of the alternative hypothesis.

12 In what follows, the word "best" is used to describe a model that cannot be rejected in favor of any other model on the basis of a standard F-test or the Davidson-McKinnon test.

13 Our results illustrate an ambiguity of the Davidson-McKinnon test: model (A.5) is accepted with (A.3) as the alternative hypothesis, but model (A.3) is at most weakly rejected at the 13.2 percent significance level when tested with (A.5) as the alternative hypothesis.

Table 3

MODEL SPECIFICATION TESTS, MODELS A.1-A.5 AND
B.1-B.4: INDIAN DISTRICTS, 1970-71^a

Alternative Hypothesis (H_1)	Null Hypothesis (H_0)		
	A.3 ^e	A.4 ^f	A.5
A.1	F = 2.315 ^b (0.129)	F = 3.301 (0.038)	F = 0.454 (0.501)
A.3 ^e		F = 4.264 (0.040)	P = -0.606 (0.544)
A.4 ^f			J = -0.733 (0.464)
A.5	P = 1.507 ^c (0.132)	J = 2.524 ^d (0.011)	
B.1	F = 0.416 (0.520)	F = 2.336 (0.099)	

Notes: ^aSignificance levels in parentheses.

^bStandard F-Statistic.

^cDavidson-McKinnon's t_{α} of the P-test.

^dDavidson-McKinnon's t_{α} of the J-test.

^eEquivalent model: B.3

^fEquivalent models: A.2, B.2, and B.4

labor to the marginal product of hired labor in model (A.5) equals 2.54 at the mean value of the variables. Since labor is measured as number of individuals (cultivators or agricultural laborers) per farm, either family labor is that much more productive than hired labor (if the number of man-days of work per individual is equal), or, more likely, the number of man-days of work per family member exceeds the number of man-days of hired workers. The latter may be the case if the hiring of workers shows a seasonal pattern (i.e., the argument above), or if there is

significant unemployment among agricultural laborers; this possibility is a result of the aggregate nature of our data. Unfortunately, because we do not have more accurate measures of work effort (days or hours of work and unemployment rates), we cannot decompose the difference in the marginal products of the two kinds of labor into more detail.

As a final note, our results on the heterogeneity of labor are consistent with the finding of Huffman (1976) based on U.S. data using an entirely different methodology. He estimated the elasticity of substitution between family (distinguishing between the farm husband and the farm wife) and hired labor by regressing labor ratios on wage ratios. He concluded that, given the computational cost of more non-linear specifications of the production function, the Cobb-Douglas specification was optimal.

VI. A Similar Test on The Heterogeneity of Land

In the analysis of the preceding sections we have treated irrigated and unirrigated land as two separate inputs in the production process, as has been common practice among studies on agricultural production. This assumption is supported in the estimates of model (A.5): the ratio of the marginal products of the two types of land is 2.77 in favor of irrigated land. However, the assumption of a unitary elasticity of substitution, inherent to the Cobb-Douglas specification, has never been questioned. It is possible that the two types of land, although very different in their effects on agricultural output, may be perfectly substitutable for each other. In that case it is better to aggregate the two types of land after weighing them differently.

To explore the appropriate relationship between irrigated and unirrigated land, we take our result on the heterogeneity of labor as given (i.e., the Cobb-Douglas relation between family and hired labor), and we assume that the overall agricultural production function facing farms is:

$$(18) \quad Y = C FL^{\beta_2} HL^{\beta_3} X_6^{\beta_6} A^{\beta_7}$$

where A = services from land, and other variables as defined

previously. Land services A are produced using irrigated (X_4) and unirrigated (X_5) land according to the following technology:

$$(19) \quad A = \left[\alpha_3 X_4^{-\rho_2} + (1-\alpha_3) X_5^{-\rho_2} \right]^{-1/\rho_2} \quad (\text{Model C.1})$$

where $\rho_2 \geq -1$. Similar to the A models appropriate restrictions generate four additional C models: $\alpha_3 = .5$ (model C.2); $\rho_2 = -1$. (model C.3); $\alpha_3 = .5$ and $\rho_2 = -1$ (model C.4); $\rho_2 = 0$ (model C.5), equivalent to model A.5). Note that models (C.3) and (C.4) imply an infinite elasticity of substitution between irrigated and unirrigated land. In models (C.1) ~ (C.5) α_3 is the symmetry parameter and ρ_2 is the curvature parameter for the land services production function.

Results of the least squares estimation of models (C.1)–(C.5) are presented in Table 4. The boundary value (of -1) for ρ_2 was binding in the case of model (C.1), which made the latter equivalent to model (C.3). The estimate for the symmetry parameter α_3 in these two models (C.3) is 0.793, which suggests that in adding up irrigated and unirrigated land, the former should be weighed roughly four times as much as the latter. The elasticity of substitution implied by the parameter in model (C.2) (which is the only model where the curvature parameter is freely estimated) is 1.616.

When we compare the C models (table 5), we find that model (C.5) does not fare too well. In fact, all models are rejected in favor of (C.3).¹⁴ The superiority of this model is reflected in the fact that it has a residual sum of squares (RSS) that is 16 percent lower than the RSS of the model with the next lowest RSS (viz., model C.2).¹⁵

To conclude, the best functional form for an agricultural production function in which labor and land are disaggregated is the Cobb-Douglas form with family and hired labor as separate inputs and irrigated and unirrigated land added together as a

14 Due to high computational costs and the poor performance of the generalized linear functional form in estimating the relationship between family and hired labor, we tried only the CES relationship between irrigated and unirrigated land.

15 In table 4 we again see the ambiguity of the Davidson-McKinnon test: for example, model (C.4) is rejected when model (C.5) is the alternative hypothesis and vice versa.

Table 4

PRODUCTION FUNCTION ESTIMATES, MODELS C.1-C.5
(WITH LAND AS A HETEROGENEOUS INPUT):
INDIAN DISTRICTS, 1970-71.

(asymptotic t-statistics in parentheses)

Parameter	C.1, C.3	C.2	C.4	C.5, A.5
C	8.287	8.320	8.028	8.225
β_2	0.190 (2.323)	0.172 (2.014)	0.135 (1.537)	0.381 (4.307)
β_3	0.044 (1.196)	0.029 (0.765)	-0.035 (-0.957)	0.083 (2.052)
β_4				0.249 (9.120)
β_5				0.249 (9.120)
β_6	0.138 (5.512)	0.175 (6.929)	0.242 (11.723)	
β_7	0.590 (13.468)	0.590 (12.678)	0.540 (11.778)	
α_3	0.793 (11.705)	0.500 ^b	0.500 ^b	0.470 ^c
ρ_2	-1.000 ^a	-0.381 (-4.876)	-1.000 ^b	0. ^b
σ_{45}^A	∞	1.616	∞	1.000
σ_{45}^Y	∞	2.008	∞	1.000
RSS	36.212	40.843	43.194	45.509

Notes: ^aBoundary value in the case of model C.1; imposed value in the case of Model C.3.

^bImposed value.

^cCalculated as $\alpha_3 = \beta_4 / \beta_4 + \beta_5$.

single input, but only after being weighed differently. Unfortunately, because of its non-linearity, this may not be the easiest form to estimate. One alternative for researchers is to add up irrigated and unirrigated land by weighing the former four times as much as the latter prior to estimation. This practice is often followed in adding up bullock hours and tractor hours to arrive at a single measure of draught animal input (Barnum and Squire, 1979). Once an aggregate measure of land is constructed, a conventional Cobb-Douglas production function with family labor, hired labor, aggregate land, and other inputs can be estimated.

Table 5
TESTS ACROSS MODELS C.1-C.5:
INDIAN DISTRICTS, 1970-71^a

Alternative Hypothesis (H_1)	Null hypothesis (H_0)			
	C.2	C.3 ^e	C.4	C.5 ^f
C.2		P = -0.932 (0.351)	F = 15.085 ^b (0.000)	P = 7.670 (0.000)
C.3 ^e	P = 5.625 ^c (0.000)		F = 50.522 (0.000)	P = 8.433 (0.000)
C.4				J = 9.315 (0.000)
C.5 ^f	P = 2.221 (0.026)	P = 0.977 (0.328)	J = 4.087 ^d (0.000)	

Notes: ^aSignificance levels in parentheses.

^bStandard F-statistic.

^cDavidson-McKinnon's t_{α} of the P-test

^dDavidson-McKinnon's t_{α} of the J-test

^eEquivalent model: C.1.

^fModel C.5 is identical to model A.5

VII. Concluding Observations

In this paper, we have tested the hypotheses of homogeneity of family and hired labor and of irrigated and unirrigated land, using district-level data from India. The evidence suggests that both inputs are heterogeneous. However, the nature of heterogeneity is different in the two cases. While family and hired labor are heterogeneous both in the sense of having different effects on output and in the sense of being imperfect substitutes for each other, irrigated and unirrigated land are heterogeneous only in the former sense (asymmetry). The hypothesis of perfect substitutability between irrigated and unirrigated land cannot be rejected. Hence, while it is valid to add up irrigated and unirrigated land (after attaching different weights to each) and include the weighted sum as a single input in a Cobb-Douglas production function, it is not valid to treat family and hired labor in the same way. Our results suggest that it is better to enter family and hired labor as separate inputs in a Cobb-Douglas production function, since the hypothesis of unitary elasticity of substitution between the two types of labor cannot be rejected.

Clearly, the hypothesis of homogeneity of labor in agricultural production needs to be further tested with household-level data sets from India and other LDCs before it is completely rejected. There are several important implications of the heterogeneity of family and hired labor that make it worthwhile to explore this issue further. For instance, the entire literature on labor demand in LDC agriculture needs revision to accommodate labor heterogeneity. For instance, if family and hired labor are neither symmetric nor perfect substitutes for each other, it can no longer be assumed, as it has been by previous studies on farm households, that family labor supply decisions by cultivator households are made independently of on-farm labor use decisions. This considerably complicates the existing "theory" of the farm household, since the assumptions of homogeneity of labor and perfectly competitive labor markets have been critical in the empirical applications of this theory. Furthermore, the heterogeneity issue has implications for the effect of migration patterns and policies on the economic welfare of the rural population. Finally, heterogeneity of labor implies that factors inducing an increase in the demand for family labor (such as farm size,

irrigation, and technical change) will, in the long run, increase the demand for fertility among farm households.

Appendix

Bardhan (1973) considers the following functional form for testing the assumption of heterogeneity of labor:

$$(i) \quad Y = cL^\alpha (HL/L)^\gamma \prod_{i=1}^n X_i^{\beta_i},$$

where HL is hired labor and L is total labor (which includes hired labor). As we show in the text, this can be rewritten as:

$$(ii) \quad \ln Y = \ln c + (\alpha - \gamma) \ln L + \gamma \ln HL + \sum_{i=1}^n \beta_i \ln X_i.$$

The elasticity of substitution between HL and FL (family labor) implicit in equation (i) and be derived analytically. It turns out to be*:

$$(iii) \quad \sigma_{23}^L = 1 + \frac{HL}{FL} \left[\frac{\alpha - \gamma}{\gamma} + 1 \right].$$

There if $\gamma < 0$ (i.e., hired labor is less efficient than family labor), it is possible for σ_{23}^L to be negative. This will occur if $|\alpha - \gamma / \gamma| > 1$ and $\frac{HL}{FL} \left[\frac{\alpha - \gamma}{\gamma} + 1 \right] - 1$. However, as long as $\gamma > 0$ (i.e., hired labor is more efficient than family labor), σ_{23}^L will always be positive, assuring concavity of the production function. Since the γ 's that Bardhan reports for two of his samples are positive, the implicit

* The definition for σ_{23}^L and σ_{23}^Y can be found in section 4 of this paper.

elasticity of substitution between family and hired labor in these samples is positive. However, Bardhan does not report the sign of γ for his other samples; he only mentions the lack of significance of the other estimates of γ . At any rate, it is important to restrict the value of σ_{23}^L to positive values in the estimation procedure. Due to the complicated formula for σ_{23}^L in equation (iii), it is not straightforward to impose such a constraint for all observations in the sample in estimating equation (i).

The expression for σ_{23}^Y is much more complicated. It seems impossible to formulate a restriction that guarantees a positive sign for σ_{23}^Y while permitting a negative estimate for γ .

We obtained the following results when we fitted equation (ii) to our data set:

$$\begin{aligned} \ln Y = & 7.908 + 0.418 \ln L - 0.064 \ln (HL/L) + 0.228 \ln X_4 \\ & (4.028) \quad (-1.188) \quad (9.370) \\ & + 0.261 \ln X_5 + 0.133 \ln X_6 \\ & (6.606) \quad (4.597) \end{aligned}$$

$$\begin{aligned} R^2 = & 0.613 \\ & (\text{t-statistics in parentheses}) \\ & (\text{all variables as described in Table 1}) \end{aligned}$$

Since the ratio of HL/FL at the sample mean was equal to 0.555 in our data set, the elasticity of substitution between family and hired labor (σ_{23}^L) at the sample mean is -2.625 . The estimated value of σ_{23}^Y equals -7.933 . So both measures of the elasticity of substitution are negative. However, they are based on an insignificant estimate of γ .

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