The Limit of Generalization in International Trade Models

Chang Min Shin*

Introduction

It is interesting to investigate how far we can go in generalizing an international free trade model. For this purpose, two free trade models are selected; one by Takayama and another one by Dixon.

These selections are somewhat arbitrary; but not without reasons. Takayama has linear technology in his model, whereas Dixon can have nonlinear technology although what he has apparently in mind is a linear input-output type. Dixon assumes differentiability taken as granted and tries to have first order conditions, whereas Takayama assumes Slater's condition and concavity of the utility functions with linear technology, hence concave constraint functions, in order to have saddle point characterizations. Dixon, who claims to have been a student of Leontief and Houthakker, makes fairly many mistakes in some basic concepts, whereas Takayama displays a relatively neat free trade model.

Before we go into a brief introduction to Takayama and Dixon, it is also interesting to review very briefly the historical development in the international free trade models. Some of the important evolutionary versions of those may be the ones by Ricardo, Mill, and Graham. In stretching some concepts in the models into

* Assistant Professor, College of Business Administration, Chung-Ang University.

a more generalized model we will encounter the fundamental limitation of generalization of free trade model as will be revealed later.

Generalization in this context is possible by increasing number of variables in the classical and ultra-classical models and by relaxing the assumptions in those models: more specifically,

1. more than 2 countries and more than 2 commodities as was initiated by Graham,
2. introduction of the demand side into the picture in a rigorous way,
3. non-zero transportation costs,
4. more than one factor of production,
5. international factor mobility,
6. nonlinear technology which can include linear technology,

in addition to free movement of goods and services taken as granted in a free trade model.

We may add such features as

7. joint production
8. intermediate goods

for further generalization.

In the subsequent sections the followings will be discussed.

I. Classical, ultra-classical models
   A. Ricardo
   B. Mill
   C. Graham
II. Takayama
III. Dixon
IV. Generalization of a free trade model.

I. Classical, Ultraclassical Models

A. Ricardo

In a two country two commodity model, assuming free trade, no transportation costs, factor mobility within a country and factor immobility between countries, the production functions with constant costs in terms of labor, which is the sole productive factor, gives the following model of comparative advantages.

$$\max_{x,\, y} V = p_x \cdot x + p_y \cdot y$$

where x,y: commodities,

subject to
$$\frac{x}{a_1} + \frac{y}{b_1} \leqq \frac{b}{b_1}$$

$$\frac{x}{a_2} + \frac{y}{b_2} \leqq \frac{a}{a_2}$$

$$x \geqq 0, \quad y \geqq 0$$

given a, b, a_1, b_1, a_2, b_2.

That is, the problem is to maximize the value of outputs subject to the feasibility constraints which are in the form of linear and linearly homogeneous production functions of the two countries under consideration.

B. Mill

J.S. Mill introduces the demand side of the picture whereas the Ricardo model merely sets the boundaries of the price range. This may be shown as follows:

$$\max_{x,\,y} \quad u(x,y)$$

subject to
$$\frac{x}{a_1} + \frac{y}{b_1} \leqq \frac{b}{b_1}$$

$$\frac{x}{a_2} + \frac{y}{b_2} \leqq \frac{a}{a_2}$$

$$x \geqq 0, \quad y \geqq 0.$$

That is, the problem is to maximize the utility of the two countries combined subject to the feasibility constraint which is the same as the Ricardo's.

C. Graham

Criticizing the limited scope of the classical models, Graham shows a multi-country multi-commodity model, emphasizing the supply side.

This can be formulated as follows.

$$\max_{x} z = p \cdot x$$

p: n dimensional vector of prices
x: n dimensional vector of products

subject to $Ax \leqq \bar{z}$

$$\alpha Z \underline{1} = p \cdot x, \quad \alpha \cdot \underline{1} = 1$$

$$x \geqq 0$$

where

- z: the total value of world transactions,
- A: (m x n) matrix determined by the opportunity cost ratios of the various countries $j = 1, \ldots, m$
- $\bar{z}$: (m x l) vector of productive capacities of the countries of the countries in terms of labor,
- α: (l x n) vector of the proportion of the total value of world transactions devoted to the consumption of product i, $i = 1, \ldots, n$, hence $\alpha \cdot \underline{1} = 1$, where $\underline{1} = [1, \ldots, 1]^T$ with n elements in it.

That is, the problem is to maximize the total value of outputs subject to constant cost production functions, maintaining labor as the sole productive factor, and predetermined ratios of consumption of products which are identical in all countries, i.e., a type of unitary elastic consumptions with respect to income.

It seems that the consumption pattern is excessively restrictive in this model.

II. Takayama

The notations used in this section are as follows.

country: $j = 1, \ldots, m$
primary resources: $1, \ldots, l$
goods and services: $i = 1, \ldots, n$
international transportation services: $1, \ldots, \tilde{n}$
production activities in country j: $k = 1, \ldots, k_j$
or $k = 1, \ldots, s_{j-1}, s_j, s_{j+1}, \ldots, k_j$

where $k = 1, \ldots, s_{j-1}$ for regular goods and services,
$k = s_j, \ldots, k_j$ international transportation services available to country j.

a^{jk} : an (ℓ x 1) vector of the amounts of the primary resources involved in a unit level operation of the kth activity in country j.

b^{jk} : an (n x 1) vector of which ith element b_{ijk} denotes the amount of the ith good or service produced in country j (or transported to country j) if $b_{ijk} > 0$, or the amount of the ith good or service required as an intermediate good or service in country j (or transported from country j) if $b_{ijk} < 0$, for a unit level of operation of the kth activity,

$\tilde{b}^{jk}$: an ($\tilde{n}$ x 1) vector whose ith element $\tilde{b}_{ijk}$ denotes the ith international transportation service produced ($\tilde{b}_{ijk} > 0$) or required ($\tilde{b}_{ijk} < 0$) for a unit level operation of the kth activity,

t : the activity level vector,

x : the vector of commodity outputs,

$\tilde{x}$: the vector of international transportation services supplied,

r : the resource supply vector.

Using these notations, the following matrices are defined.

$$A^j_G \equiv [a^{j1}\ a^{j2} \ldots\ldots a^{js_j-1}],\quad A^j_T \equiv [a^{js_j}\ a^{js_j+1} \ldots\ldots a^{jk_j}]$$

$$A^j \equiv \begin{pmatrix} 0 & A^1_T \\ \vdots & \vdots \\ 0 & A^{j-1}_T \\ A^j_G & A^j_T \\ 0 & A^{j+1}_T \\ \vdots & \vdots \\ 0 & A^m_T \end{pmatrix}$$

$$B^j_G \equiv [b^{j1}\ b^{j2} \ldots\ldots b^{js_j-1}],\qquad B^j_T \equiv [b^{js_j}\ b^{js_j+1} \ldots\ldots b^{jk_j}]$$

$$B^j \equiv \begin{pmatrix} 0 & B_T^1 \\ \vdots & \vdots \\ 0 & B_T^{j-1} \\ B_G^j & B_T^j \\ 0 & B_T^{j+1} \\ \vdots & \vdots \\ 0 & B_T^m \end{pmatrix}$$

$$\widetilde{B} \equiv \begin{pmatrix} 0 & \cdots\cdots & 0 & \widetilde{b}_{1js_j} & \cdots\cdots & \widetilde{b}_{1jk_j} \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & & 0 & \widetilde{b}_{\tilde{n}js_j} & \cdots\cdots & \widetilde{b}_{\tilde{n}jk_j} \end{pmatrix}$$

$$\equiv [\ \underline{0} \ \cdots\cdots\ \underline{0} \quad \widetilde{b}^{js_j} \ \cdots\cdots\ \widetilde{b}^{jk_j}\ ]$$

$$\equiv [\qquad 0 \qquad\qquad \widetilde{B}_T^j \qquad]$$

$$A \equiv [A^1 \ \cdots\cdots\cdots\cdots\cdots\cdots \ A^m]$$

$$B \equiv [B^1 \ \cdots\cdots\cdots\cdots\cdots\cdots \ B^m]$$

$$\widetilde{B} \equiv [\widetilde{B}^1 \ \cdots\cdots\cdots\cdots\cdots\cdots \ \widetilde{B}^m]$$

$$t \equiv \begin{pmatrix} t^1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ t^m \end{pmatrix} \qquad \text{where} \qquad t^j \equiv \begin{pmatrix} t^{j1} \\ \cdot \\ \cdot \\ \cdot \\ t^{js_j-1} \\ t^{js_j} \\ \cdot \\ \cdot \\ t^{jk_j} \end{pmatrix}$$

$$r \equiv \begin{pmatrix} r^1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ r^m \end{pmatrix} \qquad \text{where} \qquad r^j \equiv \begin{pmatrix} r^{j1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ r^{jl} \end{pmatrix}$$

$$X \equiv \begin{pmatrix} X^1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ X^m \end{pmatrix} \qquad \text{where} \qquad X^j \equiv \begin{pmatrix} X^{j1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ X^{jn} \end{pmatrix}$$

$$\tilde{X} \equiv \begin{pmatrix} \tilde{X}^1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \tilde{X}^m \end{pmatrix} \qquad \text{where} \qquad \tilde{X}^j \equiv \begin{pmatrix} \tilde{X}^{j1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \tilde{X}^{j\tilde{n}} \end{pmatrix}$$

Now, the world production system can be described as follows:

$$\begin{aligned} At + \ r &\geqq 0\,, & t &\geqq 0, \\ X &= Bt\,, & t &\geqq 0, \text{ and} \\ \tilde{X} &= \tilde{B}t\,, & t &\geqq 0. \end{aligned}$$

Or

$$z = Dt, \quad z + \bar{z} \geqq 0, \quad t \geqq 0$$

where

$$z \equiv \begin{pmatrix} x \\ \tilde{x} \\ At \end{pmatrix} \qquad D \equiv \begin{pmatrix} B \\ \tilde{B} \\ 0 \end{pmatrix} \text{ and } \qquad \bar{z} \equiv \begin{pmatrix} 0 \\ 0 \\ r \end{pmatrix}$$

The world production possibilities set is defined to be:

$$Y_w \equiv [z : z = Dt, \quad z + \bar{z} \geqq 0, \quad t \geqq 0].$$

Consumption and utility come into the picture as follows in the model.

Let c_{hj} denote the consumption vector for individual h of country j.

Then, $\sum_h c_{hj} = c_j$ and the world aggregate consumption vector,

$$(n + \tilde{n} + 1) \times 1, \text{ is } \sum_{j=1}^{m} c_j.$$

Let C_{hj} be the consumption possibilities set of the individual h in country j, and assume that C_{hj} is a convex subset of the nonnegative orthant of the commodity space.

Let $u_{hj}\,(c_{hj})$ be his utility function.

Then we have $\sum_{j=1}^{m} c_j \leqq z + \bar{z}, \ \ \bar{z} \in Y_w.$

Next, the following definitions are in order.

Definition (Feasibility) An array of consumption vectors $\{c_{hj}\}$ is said to be feasible if there exist production vectors z such that $z + \bar{z} \geqq \sum_{j=1}^{m} c_j$ with $c_{hj} \in C_{hj}$ for all h and j, and $z \in Y_w^{j=1}$.

Definition (Pareto Optimality) A feasible array of consumption vectors $\{\hat{c}_{hj}\}$ is said to be Pareto optimal if there does not exist a feasible array of consumption vectors $\{c_{hj}\}$ such that $u_{hj}(c_{hj}) \geqq u_{hj}(\hat{c}_{hj})$ for all h and with a strict inequality for at least one h.

Definition (Free Trade Equilibrium)
$[\hat{p}, \{\hat{c}_{hj}\}, \hat{z}]$ with $\hat{p} \geqq 0$, $c_{hj} \in C_{hj}$ and $\hat{z} \in Y_w$ is called a free trade equilibrium if:

1. $u_{hj}(\hat{c}_{hj}) \geqq u_{hj}(c_{hj})$ for all $c_{hj} \in C_{hj}$ with $\hat{p} \cdot c_{hj} \leqq \hat{p} \cdot \hat{c}_{hj}$ for all h, j.
2. $\hat{p} \cdot \hat{z} \geqq \hat{p} \cdot z$ for all $z \in Y_w$
3. $z + \bar{z} \geqq \sum_{j=1}^{m} \hat{c}_j$ and $\hat{p}\,(z + \bar{z} - \sum_{j=1}^{m} \hat{c}_{hj}) = 0.$

Definition (Satiation) Individual h of country j is satiated at c'_{hj} if $u_{hj}(c'_{hj}) \geqq u_{hj}(c_{hj})$ for all $c_{hj} \in C_{hj}$.

Consider the following assumptions.

1. (Slater's condition) There exists $c^\circ_{hj} \in C_{hj}$ for all h and j and $z^\circ \in Y_w$ such that $\sum_{h,j} c^\circ_{hj} \leqq z^\circ + \bar{z}$.
2. (Concavity) $u_{hj}(c_{hj})$ is a continuous and concave function for all h and j.
3. (Compactness) C_{hj} is compact for all h and j.
4. (Nonsatiation) For every $c_{hj} \in C_{hj}$, there is a c°_{hj} which is preferred to c_{hj}.
5. (Survival without trade) Every consumer can survive in the absence of trade on the basis of commodities he holds.

Under these assumptions there exists a free-trade equilibrium with a nonzero price vector.

There exists a $p' \geqq 0$, such that

$$u' + p'\,(z' + \bar{z} - \sum_{h,j} c_{hj}) \geqq u' + p'\,(z' + \bar{z} - \sum_{h,j} c_{hj}) \tag{1}$$

for all $c_{hj} \in C_{hj}$ and $z \in Y_w$, where $u' \equiv \sum_{h,j} \alpha_{hj}\, u_{hj}\,(c'_{hj})$, and

$$p'\,(z' + \bar{z} - \sum_{h,j} c_{hj}) = 0, \;\; z' + \bar{z} - \sum_{h,j} c_{hj} \geqq 0 \tag{2}$$

Before we proceed, let us consider the following assumption.
6. (Cheaper point) Given a point c'_{hj} and a prevailing price vector p', there exists a $c^{\circ}_{hj} \in C_{hj}$ such that $p' \cdot c'_{hj} \geqq p' \cdot c^{\circ}_{hj}$ for all h and j.

Now, condition 3 of the free trade equilibrium follows directly from the equation (2).

Let $c_{hj} = c'_{hj}$ for all h and j in (1). Then condition 2 of the free trade equilibrium follows.

Finally let $c_{hj} = c'_{hj}$ for all h except $h = h_\circ$ and let $z = \hat{z}$. Then $\alpha_{h_\circ j}\, u_{h_\circ j}\,(c'_{h_\circ j}) - \alpha_{h_\circ j}\, u_{h_\circ j}\,(c'_{h_\circ j}) \geqq p'c'_{h_\circ j} - p'c_{h_\circ j}$ for all $c_{h_\circ j} \in C_{h_\circ j}$.

If $\alpha > 0$, then condition 1 of the free trade equilibrium is satisfied for individual $h_\circ$ of country j. If $\alpha_{h_\circ j} = 0$, then $p'\, c'_{h_\circ j} < p'\, c_{h_\circ j}$ for all $c_{h_\circ j} \in C_{h_\circ j}$, which contradicts assumption 6. Hence $\alpha_{h_\circ j} > 0$. Since the choice of $h_\circ\, j$ is arbitrary, this holds for all h and j.

$p' \neq 0$, since nonsatiation is assumed. Thus, we can normalize p' such that it is in a unit simplex.

Let α be a vector whose typical element is α_{hj}.

We suppose that in the above maximization problem, α is in a unit simplex.

Takayama and El-hodiri, following Negishi, show the following.

Since C_{hj} and Y_w are bounded, there exists a number M such that $\sum_{h,j} |M_{hj}\,(p, z) - p \cdot c_{hj}| < M$ for all $c_{hj} \in C_{hj}$ and $z \in Y_w$.

Define $u_{hj} \equiv \{0, \alpha_{hj} + [M_{hj}\,(p, z) - p \cdot c_{hj}]\,/M\}$, and $\hat{\alpha}_{hj} = \dfrac{u_{hj}}{\Sigma u_{hj}}$.

Construct the following mappings.

(a) $\alpha \rightarrow [\{c'_{hj}\}, z', p']$

(b) $[\{c'_{hj}\}, z', p'] \rightarrow [\alpha', \{c'_{hj}\}, z', p']$

The point-to-set mapping (a) is the mapping from α to the saddle point of $u + p\,(z + \bar{z} - \sum_{h,j} c_{hj})$.

It's image is nonempty due to assumption 5 and the Weierstrass theorem, and the mapping is uppersemicontinuous with compact and convex images.

The mapping (b) is a point-to-point mapping and continuous. Let c_{hj} is an arbitrary point in C_{hj} and z be an arbitrary point in z. Let p be an arbitrary point in (n - 1) unit simplex.

Combing (a) and (b), the mapping $[\alpha, \{c_{hj}\}, z, p] \rightarrow [\alpha' \{c'_{hj}\}, z' p']$ is an uppersemicontinuous mapping from compact convex set into itself whose image is nonempty and convex.

Hence, due to Kakutani's fixed point theorem, there exists a fixed point $[\hat{\alpha}, \{\hat{c}_{hj}\}, \hat{z}, \hat{p}]$.

They, then, show that at this point all of the conditions of a free trade equilibrium are satisfied.

III. P.B. Dixon

Dixon states that one way to generate points on the utility possibilities frontier(*) is by solving the following joint maximization problems (J.M.P.):

(J.M.P. 1)

$$\max_{c_j, z_j} \sum_{j=1}^{m} w_j \; u_j\,(c_j) \qquad j = 1, \ldots\ldots, m$$

$$\text{subject to} \quad f_j\,(z_j) \geqq 0 \qquad j = 1, \ldots\ldots, m$$

$$\sum_{j=1}^{m} c_j \leqq \sum_{j=1}^{m} z_j$$

$$c_j \geqq 0 \qquad j = 1, \ldots\ldots, m$$

where

w_j: a set of weights, i.e., $\sum_j w_j = 1$, $w_j \geqq 0$ for all j.
u : utility

* We will discuss whether this is correct.

c : consumption of goods and services $i = 1, \ldots, n$
f : constraints in the production sector
z : vector of all goods and services
j : country

Dixon has the following two ways in finding the utility possibilities frontier in addition to the (J.M.P. 1)

(J.M.P. 2)

$$\max_{c_j, z_j} u_m(c_m) \qquad j = 1, \ldots\ldots, m$$

$$\text{subject to} \quad f_j(z_j) \geqq 0, \quad j = 1, \ldots\ldots, m$$

$$\sum_{j=1}^{m} c_j \leqq \sum_{j=1}^{m} z_j$$

$$u_j(c_j) \geqq u_j^{\circ}(c_j) \quad j = 1, \ldots\ldots, m-1,$$

$$c_j > 0 \qquad j = 1, \ldots\ldots, m$$

where

u_j° : predetermined minimum levels of utilities for countries $j = 1, \ldots, m-1$,

(J.M.P. 3)

$$\max_{c_j, z_j} u_m(c_m) \qquad j = 1, \ldots\ldots, m$$

$$\text{subject to} \quad f_j(z_j) \geqq 0 \quad j = 1, \ldots\ldots, m$$

$$\sum_{j=1}^{m} c_j \leqq \sum_{j=1}^{m} z_j$$

$$u_j(c_j) \geqq a_j u_m(c_m) \quad j = 1, \ldots\ldots, m-1,$$

$$c_j \geqq 0 \qquad j = 1, \ldots\ldots, m$$

where

a_j : predetermined minium values for

$u_j\ (c_j) / u_m\ (c_m)$.

Dixon's most general model employs (J.M.P.1) method with tariffs, quotas, and transportation costs. Deleting the terms that are related with tariffs and quotas, the Dixon's model can be transformed into a free trade model with transportation costs.

$$\max_{c_j,\, c_j^m,\, c_j,\, m_{jj'}} \quad \sum_j w_j\ u_j\,(c_j) + w_j^m\ u_j^m\,(c_j^m)$$

subject to

$$f_j\,(z_j) \geqq 0$$

$$\sum_{j'} \sum_{j \neq j'} s_{jj'} \cdot m_{jj'} = \sum_j z_j^s$$

$$c_j + c_j^m + \sum_{j' \neq j} m_{j'j} = z_j + \sum_{j \neq j'} m_{jj'}$$

$$j, j' = 1, \ldots\ldots, m$$

$$c_j \geqq 0\,, \qquad j = 1, \ldots\ldots, m$$

where the superscripts m in w_j, u_j, and c_j denote those involved in imports,

$m_{jj'}$) : imports of country j from country j'

$s_{jj'}$: a vector giving the ton-miles of shipping necessary to transport a unit of each commodity from j' to j.

z_j^s : the output of shipping service by country j.

Implicitly assuming differentiability, the following Lagrangian gives the first order conditions shown below.

$$Q = \sum_j \left[w_j\ u_j\,(c_j) + w_j^m\ u_j^m (c_j^m)\right] - \sum_j \lambda_j\, f_j\,(z_j)$$

$$+ \pi_j \left(z_j + \sum_{j \neq j'} m_{jj'} - c_j - c_j^m - \sum_{j' \neq j} m_{j'j}\right)$$

$$+ \pi^s \left(\sum_j z_j^s - \sum_{j'} \sum_{j \neq j'} s_{jj'} \cdot m_{jj'}\right)$$

$$w_j \frac{\partial u_j}{\partial c_{ji}} - \pi_{ji} \leqq 0 \qquad i = 1, \ldots\ldots, n \qquad j = 1, \ldots\ldots, m$$

$$(w_j \frac{\partial u_j}{\partial c_{ji}} - \pi_{ji})\ c_{ji} = 0 \quad i = 1, \ldots\ldots, n \qquad j = 1, \ldots\ldots, m$$

$$w_j^m \frac{\partial u_j^m}{\partial c_j^m} - \pi_{ji} \leqq 0 \qquad i = 1, \ldots\ldots, n \qquad j = 1, \ldots\ldots, m$$

$$(w_j^m \frac{\partial u_j^m}{\partial c_{ji}^m} - \pi_{ji})\ c_{ji}^m = 0 \quad i = 1, \ldots\ldots, n \qquad j = 1, \ldots\ldots, m$$

$$-\lambda_j \frac{\partial f_j}{\partial z_{ji}} + \pi_{ji} \leqq 0 \qquad i = 1, \ldots\ldots, n \qquad j = 1, \ldots\ldots, m$$

$$(-\lambda_j \frac{\partial f_j}{\partial z_{ji}} + \pi_{ji})\ z_{ji} = 0 \quad i = 1, \ldots\ldots, n \qquad j = 1, \ldots\ldots, m$$

$$\pi_{ji} - \pi_i^s\ s_{jj'i} \leqq 0 \qquad i = 1, \ldots\ldots, n \qquad j, j' = 1, \ldots\ldots, m$$

$$(\pi_{ji} - \pi_i^s s_{jj'i})\ m_{jj'i} = 0 \quad i = 1, \ldots\ldots, n \qquad j, j' = 1, \ \ldots\ldots, n$$

$$f_j(z_j) \geqq 0 \qquad \lambda_j\ f_j(z_j) = 0 \qquad j = 1, \ldots\ldots, m$$

$$z_j + \sum_{j \neq j'} m_{jj'} - c_j - c_j^m - \sum_{j'=j} m_{j'j} = 0 \quad \sum_j z_j^s - \sum_j \sum_{j \neq j'} s_{jj'} \cdot m_{jj'} = 0$$

$$j = 1, \ldots\ldots, m$$

When Dixon expands from the version (J.M.P.1) to the expanded international trade model, he does not explain that the production set includes the international transportation services. This should have been included explicitly in the production set.

By so doing, direct consumption of international transportation services can be included in the utility fuction; an example, travel by tourists as in Takayama's model.

In this nonlinear programming model, Dixon makes some serious mistakes.

First, the objective utility function is seperated into two groups; one for domestic goods and services, another for imports.

Since the two utility functions are seperate and additive, domestic goods and services are independent of foreign goods and services unless identical functions are present in both groups of functions. At any rate, it is unrealistic if we assume a Korean car can not start when its fuel is from abroad, or foreign spice does not change the taste of home-country food.

Second, as noted above, (J.M.P.1) does not yield all the points in the utility frontier directly. It gives a Bergson contour, which is an envelope curve derived from Scitovsky contours, which, in turn, are based on utility frontiers.

Third, in an attempt to prove the existence of general equilibruim, Dixon counts the number of equations and the number of unknowns in the first order conditions (Dixon p.5), and this is implicitly carried on in his trade models. However, the equality of these two numbers is neither necessary nor sufficient for the existence of an equilibrium (eg. see Intriligator p.227).

IV. Generalization of a Free Trade Model

As shown above, Takayama has the following features in his generalized model.

His model has multi-country, multi-commodity, multi-factor-inputs, multi-intermediate goods.

A country may have zeros in some of the elements of the vector for primary resources.

Factor mobility between countries can be allowed as well as within a country.

A country may be unable to produce some products before trade. He does not distinguish between intermediate goods and final goods since the same commodity may well belong to both categories. International transportation services are dealt with as seperate services from other goods and services. They are the only services whose prices need to be the same between countries. Takayama does not assume that every country has the same number of production activities, thus country j selects from among k_j different production activities. And joint production is allowed.

Consider the following list of assumptions for factor price equalization in international trade:

1 free competition in all markets,
2 absence of transportation costs, hence equality of all commodity prices as between different countries or regions,
3 specialization in production is incomplete,
4 the production functions are indentical in countries and homogeneous of degree one,
5 the production function must have different intensities of factors of production in some of commodities,
6 the factors of production are qualitatively the same in all countries,
7 the number of factors is not greater than number of commodities.

Comparing these standard assumptions for the factor price equalization theorem with the assumptions of the above Takayama's model, the important difference in these two sets of assumptions is found to be the inclusion of the transportation cost in Takayama. The product prices will be equalized except to the extent of transportation costs. Accordingly the factor prices will be equalized except to the extent of the differences in the product prices due to transportation costs and to the extent of direct transportation costs of factors of production, primary goods and intermediate goods.

In proving the existence of a free trade equilibrium, Takayama maximizes the uitilities of individuals regardless of the countries they belong to.

Takayama, in effect, arrives a model of a world nation, similar to one of regional spatial economic theories.

This can be considered as the fundamental limitation of generalization of free trade model in the sense that the last possible stage of generalization puts the economies of the different countries into one economy without any artificial barriers; beyond which stage we cannot go; and at which stage the essential significance of international trade disappears.

It is nothing particular that this type of one of the most general model includes classical, ultra-classical, and the Heckscher-Ohlin model as special cases.

Since he arrived virtually a state of the world nation in his generalization of a free trade model, Takayama can use mutadis mutandis the proof for the existence of general equilibrium under autarky in proving that of a free trade equilibrium, and can show that the same theorems about the relationships between Pareto optimality and competitive equilibrium used in a closed economy can also be applied to his free trade model:

Theorem 1. Under assumptions 1 and 2 in the section of Takayama above, if $[\{\hat{c}_{hj}\}, \hat{z}]$ is a Pareto optimal point, then there exists a $\hat{p} \geqq 0$ such that $[\hat{p}, \{\hat{c}_{hj}\}, \hat{z}]$ is a free trade equilibrium.

Theorem 2. If $[\{\hat{c}_{hj}\}, \hat{z}, \hat{p}]$ is a free-trade equilibrium at an efficient point in the production set, then $[\{\hat{c}_{hj}\}, \hat{z}]$ is a Pareto optimum.

One of the most important assumptions that characterize international trade may be factor immobility between countries. However, in the real world factors of production can cross borders of countries. On the contrary, within a country not every factor is mobile; some factors of production are locational constants for example (as the usage of the term defined in Richardson), and so on.

In this respect there does not exist any distinctive line between international economics and regional economics, when the unit entity is switched around. Factor mobility is therefore a matter of degree. In general, we expect less artificial barriers in factor movements within a country, and more of them in the case between countries.

On the consumption side of the picture, we can maximize the utilities of every individuals in the frame of the world economy as a whole as Takayama does, or we can maximize the utilities of every individual in each country where a country is the unit in which individuals maximize their utilities within the boundary of each country.

Even though we are discussing free trade models, anticipating some governmental roles in further analyses, it may make us more comfortable if the reference is given to each country in their respective maximization of utilities than if the whole world is dealt with as one huge world nation. Theoretically, under free trade the sum of the Bergson contours obtained in each country is supposed to be equivalent to the Bergson contour obtained in the framework of the world nation, if the eventual weighting schemes turn to be the same in both cases.

For analytical and practical purposes, a free trade model may be desired to be generalized to the point of relative factor immobility (as compared with the case of regional spatial economics) and to the point where the recognition of different countries is made, with respect to the utility maximization of individuals.

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NOTES AND ANNOUNCEMENTS

The International Ecnomic Association is organizing in 1981, 1982, and 1983 various roundtable conferences preliminary to a world congress on "Structural Change, Economic Interdependence, and World Development." Authors of recent contributions to aspects of this subject are invited to send abstracts to the Secretariat of the IEA (4 rue de Chevreuse, 75006 Paris, France) so that they may be forwarded to the different organizers of the roundtable conferences, and/or for the congress. A copy might also be sent to the President of the IEA, Professor Victor L. Urquidi, El Colegio de Mexico, Camino al Ajusco No. 20, Mexico 20, D.F., Apartado Postal 20-671, Mexico.

The Council for European Studies announces the Third International CES Conference of Europeanists to be held in Washington, D.C., April 29-May 1, 1982. The conference theme is "Periods and Cycles in Europe—Past and Present." Scholars are invited to submit suggestions for participants, panels, and papers by November 1, 1981. Panel and paper proposals should be accompanied by a short precis of proposed content and an abbreviated vita. Address: Council for European Studies, Conference 1982, 1429 International Affairs Bldg, Columbia University, New York, NY 10027.

Call for Papers: Montego Bay, Jamaica will be the site of the thirteenth International Atlantic Economic Conference, February 11-16, 1982. Those wishing to present papers should include a submission fee ($15.00 U.S.) for each paper plus two copies of a 500-word summary and a separate cover sheet giving location of conference; name(s), address, telephone number, institution or affiliation of author(s); number and name of the *JEL* category under which the article primarily belongs. Those wishing to be discussants or chairmen should also submit the above information to John M. Virgo, Program Chairman, Atlantic Economic Conference, Box 258, Worden, IL 62097.

Annual Meeting: The Southwestern Economics Association will hold its next meeting in San Antonio, Texas, March 17-20, 1982 in conjunction with the sixtieth annual meeting of the Southwestern Social Science Association. Proposals for papers or to chair or discuss can be submitted until November 15 to David E.R. Gay, President, Southwestern Economics Association, Department of Economics, BA 402, University of Arkansas, Fayetteville, AR 72701 (telephone 501+575-4002).

Call for Papers: An international statistics meeting will be held at the Hebrew University, June 14-18, 1982. It is organized by the Israel Statistical Association, and cosponsored by the Israeli Academy of Sciences and Humanities, the Hebrew University, and the International Association of Survey Statisticians. There will be two main subjects: Analysis of Sample Survey Data, including theoretical foundations and methods of analysis of data from sample surveys and their application, and the design of analytical surveys; and Sequential Analysis, including optimality of sequential procedures in estimation and testing, applications of sequential procedures, and theory of stopping rules. Short abstracts of papers and requests for further information should be sent to J. Yahav or G. Nathan, Department of Statistics, Hebrew University, Jerusalem, Israel 91904.

Economists who are strongly oriented toward the humanities, who use human-