Two-Gap Models of Foreign Aid: A Survey

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The interrelationship between growth and foreign resource requirements in the context of economic development has been the focus of a great deal of research efforts in recent years. The outcome of the research has been what has come to be known in the development parlor as the two-gap models. In the present paper, we attempt to review these research efforts. Since the literature in this area is enormous, we will make no attempt to quote and summarize all contributions but rather distinguish among different approaches-indicating their essential differences, and their possibilities and limitations--and attempt to illuminate how those research studies are linked with one another in terms of their analytical essence. With this end in view, we first present, in Section I, an overview of what Micksell has aptly described as the 'macroeconomics of foreign aid'--encompassing essentially those aggregative models in the Harrod-Domar tradition, including the so-called two-gap models. Then in Section II, we proceed on to the multi-sector models that focus on the interactions between economic development and the flow of external resources, most of these models being cast within a programming framework. Section III adds some concluding remarks, indicating briefly the principal shortcomings of the past research efforts discussed in the preceding sections.

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I. Aggregative Models of Foreign Resource Requirements

In what follows we describe the major aggregative models of foreign resource requirements. The common features that run through all these models are that they are in some sense modifications of the basic Harrod-Domar model and that most of the relations defining the models are linear in different variables.

Before going on to the comprehensive theories of aid as enunciated by Chenery and Strout and others, one can distinguish three basic approaches to estimating the foreign resource requirements of a developing country:

(i) the savings-investment gap approach,
(ii) the export-import (or foreign exchange) gap approach, and
(iii) the capital absorption approach.
Let us discuss these approaches briefly.

The Savings-Investment Gap Approach: In its simplest form, the savings-investment gap—of which one early, systematic exposition can be found in Rosenstein-Rodan—states that the foreign resource requirements of a country to sustain a target rate of growth should be measured by the difference between domestic savings and the rate of investment necessitated by the growth-goal of the society.

The basic assumptions underlying this approach are:

(a) There is a linear relationship between savings and income—with the marginal savings rate (s') being higher than the average savings rate (s).
(b) There is a constant capital-output ratio (k) in production (thus this formulation ignores the role of labor and other factors of production).
(c) There is a pre-specified target rate of growth (r) for the economy.

Thus, for a given target rate of growth r, the foreign resource requirements of the country at the base year can be stated as

\[ F(0) = I(0) - S(0) = Y(0) \cdot k \cdot r - Y(0) \cdot s = Y(0) (k \cdot r - s) \]  \hspace{1cm} (1)

where I(0), S(0), Y(0) are investment, savings and GNP at the base year.

Now savings at year t is given by \[ S(t) = s \cdot Y(t) + s' (Y(t) - Y(0)) \].
Similarly, investment in year t is given by \[ I(t) = Y(t) \cdot k \cdot r \].
Thus the net inflow of foreign resources required at time \( t \) is given by:

\[
F(t) = Y(t) \cdot k \cdot r - ((s - s') Y(0) + s'Y(t))
\]

\[
= (kr - s') Y(t) + (s' - s) Y(0).
\]

Further, from the above eqs. (1) and (2) one can obtain:

\[
F(t) - F(0) = (kr - s') (Y(t) - Y(0))
\]

(3)

It can be easily seen from the above that foreign resource requirements will decline with time if the following condition holds: \( kr < s' \); implying that the marginal savings rate be greater than the product of capital-output ratio and the target rate of growth.

Again, manipulating the above relation, we can derive the rate of growth that will be sustained with a given inflow of aid:

\[
r(t) = \frac{F(t)/Y(t) + (s - s') \cdot Y(0)/Y(t) + s'}{1/k}
\]

(4)

Now taking the derivative of \( r(t) \) with respect to \( F(t) \), one can easily see that \( dr(t)/dF(t) > 0 \), implying that with a larger inflow of foreign aid, the rate of growth increases, given of course that \( k \) is constant.

From the above relations, one can derive further insight with respect to the date of termination of foreign aid and necessary analytical conditions. In order for \( F(t) \) to become zero, the following must hold:

\[
(kr - s') Y(t) - (s - s') Y(0) = 0
\]

(5)

implying, \( Y(t) = (s - s') Y(0)/(kr - s') \)

(6)

But \( Y(t) = Y(0) \cdot (1 + r)^t \). Substituting this in the above relation, one can deduce

\[
(1 + r)^t = (s - s')/(kr - s')
\]

(7)

which yields: \( t = \ln (s - kr) / \ln (1 + r) \).

(8)
Thus from the above relation, one can derive the termination date of foreign resource requirements. One can see that $t$ depends negatively on $s$ and positively on $r$.

The Export-Import (Foreign-Exchange) Gap Approach: This approach focuses on foreign-exchange earnings as the principal constraint on domestic investment and growth.\(^1\) It states that foreign aid should fill in the gap between the required import expenditures and the actual export earnings. This approach is based on the following set of assumptions:

(i) Import is linearly dependent on income.
(ii) Export is linearly dependent on income.
(iii) There is a target rate of growth of income, given by $r$.

Given these assumptions, the foreign-exchange gap in the base year is given by:

$$ F(0) = M(0) - X(0) = Y(0) \cdot m - Y(0) \cdot e $$

(9)

where $m$ and $e$ denote the average rate of import and the average rate of export respectively.

Import in the $t$-th year is given by

$$ M(t) = m \cdot Y(0) + m' \{Y(t) - Y(0)\} $$

(10)

where $m'$ is the marginal rate of import.

Similarly, export in the $t$-th year is given by

$$ E(t) = e \cdot Y(0) + e' \{Y(t) - Y(0)\} $$

(11)

where $e'$ is the marginal rate of export.

Then the foreign-exchange requirements in year $t$ can be defined as follows:

$$ F(t) = m \cdot Y(0) + \{Y(t) - Y(0)\} - e \cdot Y(0) - e' \{Y(t) - Y(0)\} $$

(12)

$$ = (m - e) \cdot Y(0) + (m' - e') \{Y(t) - Y(0)\} $$

$$ = F(0) + (m' - e') \{Y(t) - Y(0)\} $$

Now $F(t) - F(0) = (m' - e') \{Y(t) - Y(0)\}$.  

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\(^1\) This approach is emphasized among others by Balassa.
One can easily see that \( F(t) < F(0) \), if and only if \( m' < e' \), that is, the marginal import is less than the marginal export rate. Here one should note that though the assumption of a constant marginal import may be valid as a first approximation, the assumption of a constant marginal export is hard to maintain in reality, as one expects the value of this parameter to fall with income. On this ground, the above condition is hard to fulfill empirically.

**Absorptive Capacity Approach:** There is a widespread belief among economists that developing countries at their initial stages of growth cannot absorb all possible amounts of investment; there is an upper limit to it, often caused by, among other things, a shortage of skill and technical know-how. Under these circumstances, the less-developed countries may not be able to achieve the rate of investment called for by the target rate of growth, but have to settle for the level of investment permitted by the upper limit of absorptive capacity.\(^2\) In other words, this approach regards capital requirements as being determined by the ability of a country to employ both domestic and foreign capital productively in the sense that they earn a basic minimum rate of return. This rough-and-ready approach has a special appeal for many practical administrators who have only limited resources to ration. However, the capital absorptive capacity constraint has also been employed as a special constraint attached to the savings-investment gap approach or in combination with both the savings and foreign exchange constraints.

**Chenery and Strout Model:** It was to the credit of Chenery and Strout (1968) to combine these three basic approaches to foreign aid into a comprehensive model. They contend that a country is likely to go through three phases of economic development each associated with a dominant inhibiting factor to development.\(^3\) However, we should emphasize this does not mean to imply that in each phase, there will be only one constraint operative and others will cease to be binding, but rather that only one of the constraints will be dominant. Thus, in the first phase, the dominant constraint will be that of the shortage of skills, or what is otherwise known as the absorptive capacity constraint; in the second phase, it will be

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\(^2\) For an elaboration of the absorptive capacity approach to foreign resource requirements, see Adler and Resenstei-Rodan.

\(^3\) Actually the Chenery-Strout thesis of three-phase growth is based not so much on the internal logical structure of the model, as on the intuitive view of the process of development over a long run historical perspective. Ranis and Fei have indeed rightly pointed out: "... the C-S three-phase growth (is) stimulating and provocative but rather unconvincing on a priori grounds." (p. 898)
the shortage of domestic savings; and in the third phase, it will be
the shortage of foreign exchange. In different phases of develop-
ment, foreign aid can help relax these dominant constraints by
augmenting the supply of skills, the supply of investible resources
and the supply of imported commodities and services. The basic
building blocks of the model are very similar to those of the other
models discussed before. It assumes:

(i) Production is completely described by a constant capital-
output ratio (k).
(ii) There is a linear savings function.
(iii) In the absorptive-capacity constraint phase, there is a con-
tant growth in the rate of investment (b).
(iv) In other phases, the country pursues a target rate of growth
(r).
(v) Exports grow at an exogenous rate, e.

Let us turn briefly to the three phases of growth. As has been
mentioned before, in the first phase it is the absorptive capacity
constraint that becomes binding. In this phase investment can
grow at a maximum rate of b. Thus, the total investment in the
time interval (0, t-1) is given by

\[ I(O) \cdot \left\{ \frac{(1 + b)^t - 1}{b} \right\} \quad (14) \]

where I(0) is the investment in the base year.
Assume that income has in this time interval changed from Y(o) to
Y(t). Now by the assumption of constant capital-output ratio, the
following relation must hold:

\[ I(O) \left\{ \frac{(1 + b)^t - 1}{b} \right\} = (Y(t) - Y(O)) \cdot k \quad (15) \]

With slight algebraic manipulations, one can derive:

\[ I(t) = I(O) + b \cdot k \cdot (Y(t) - Y(O)) \quad (16) \]

Savings at time t is given by

\[ S(t) = S(O) + s' \cdot (Y(t) - Y(O)) \quad (17) \]
Thus the level of capital inflow is given by

\[ F(t) = I(t) - S(t) = I(O) - S(O) + (b \cdot k - s') (Y(t) - Y(O)) \] (18)

The last expression defines the external capital inflow in the absorptive-capacity phase. Phase I ends when investment reaches the level adequate enough to sustain the target rate of growth.

In phase II, GNP and investment rise at a constant rate with external assistance being determined by the difference between \( k \cdot r \) and \( s (APS) \). From the Harrod-Domar model, one can derive the growth equation as follows.

\[ r(t) = \frac{s(t) + f(t)}{k} \] (19)

where \( s(t) = APS = ((s(0) - s'). Y(0)/Y(t)) + s' \) and \( f(t) = F(t)/Y(t) \), which is the total aid expressed as a proportion of total income.

By elementary calculations, one can easily derive:

\[ F(t) = r(t) \cdot k \cdot Y(t) - s' \cdot Y(t) - (s(O) - s') \cdot Y(O). \] (20)

By differentiating the above expression with respect to time, one can verify that \( F(t) \) declines with time if and only if \( (r(t) \cdot k - s') \) is negative. In other words, it can be stated that foreign resource requirements will decline with time if the product of the target rate of growth (\( r \)) and the capital-output ratio (\( k \)) is less than the marginal rate of savings (\( s' \)). Similarly, by manipulating the above expression, one can derive the termination date of foreign aid.

In phase III, as is mentioned before, a shortage of foreign exchange becomes the constraining factor. They postulate that the level of import \( M(t) \), required to sustain a given level of income at time \( t \): \( Y(t) \), is given by a linear function of the following form:

\[ M(t) = M(O) + m' (Y(t) - Y(O)) \] (21)

where \( M(0) \) is as usual the base year import. The export is assumed
to grow exponentially. Thus

\[
\dot{E}(t) = E(O) \cdot (1 + e)^t
\]  
\[ (22) \]

The trade gap is defined as follows

\[
F(t) = M(t) - E(t) = M(O) + m' (Y(t) - Y(O)) - E(O) \cdot (1 + e)^t
\]  
\[ (23) \]

From the above expression, one can derive, as Chenery and Strout did, the condition for the elimination of the trade gap within \( t \) periods. Thus

\[
E(O) \cdot (1 + e)^t - M(O) - m' (Y(t) - Y(O)) > 0
\]  
\[ (24) \]

if, as one can derive by elementary manipulations,

\[
\frac{E(O)}{M(O)} \cdot (1 + e)^t - \frac{m'}{m} (1 + r)^t > 1 - \frac{m'}{m}
\]  
\[ (25) \]

Thus, for the trade gap to be eliminated either the export growth \( e \), must exceed the target rate of growth \( r \), or the marginal import ratio \( m' \), must be substantially below the initial average import ratio \( m \). This condition establishes the trade criterion for progress toward a given rate of self-sustaining growth.

\textit{Fei-Paauw Model:} Fei and Paauw analyze the relationship between external resources and the mobilization of domestic savings. They construct a simple, aggregative dynamic model to analyze the problem, and then apply it to project the termination date for foreign resource requirements for a sample group of countries.\(^4\)

Although the model they construct is in the basic Harrod-Domar tradition, their savings function is very different from others—they postulate that incremental per capita savings is a constant fraction \( u \), of the increment of the per capita income. Another interesting feature of this model is that it incorporates the rate of

\(^4\) The Fei-Paauw model and Chenery-Strout models are very similar in spirit. However, some of the differences are: one is worked out in per capita terms (Fei-Paauw), while the other is done in aggregative terms; one (Fei-Paauw) focuses on the savings gap, while the other focuses on both savings and foreign exchange gaps.
growth of population as a variable in the model structure. Other assumptions of the model include: a constant target rate of growth of per capita income, \( h \); a constant rate of population growth, \( n \).

Now the investment required to achieve a given rate of growth \( (h+n) \) is given by

\[
I = Y.K \cdot (h+n)
\]

where as usual \( I \) denotes investment, \( k \) capital-output ratio, \( Y \) income.

As a first approximation, they postulate \( S = s \cdot Y \). Given \( A \) the total aid requirements and \( a = A/Y \), we can write

\[
k \cdot (h+n) = s + a
\]

This relationship can be simplified to:

\[
h = (s - nk + a) / k
\]

Thus one can see from the above that, given \( n,k,s \), the relationship between \( h \) and \( a \) \((= A/Y)\) is a linear one. Here we can distinguish two cases:

(i) \((s-n.k)\) is positive, implying that the intercept term is positive. This is what Fei-Paauw characterize as the favorable case. This case is distinguished by a high value of \( s \) or a low value of the rate of population growth or both.

(ii) \((s-n.k)\) is negative, implying that the intercept term is negative. This is the unfavorable case. This is characterized by a high rate of population growth, a low value of the savings rate or a combination of both.

The above model is highly simplified. 'To construct a more realistic model', Fei-Paauw replace the savings assumption with the following: They postulate that, as has been mentioned before, per capita savings is a constant fraction \( u \), of the increment in per capita income. From that they derive the average propensity to save \( s \), which follows the pattern:

\[
s = u - ((u-s \cdot O) / \exp (h.t))
\]
Substituting this in the preceding relation, one obtains:

\[ a = A/Y = k_1 (h+n) - (u- (u-s(0))/\exp (h.t)) \] (30)

Note that the function \( s \), which is denoted by the expression in the parenthesis, has the following properties:
(i) it is increasing and concave; (ii) the limiting values of \( s \) are \( s(0) \) and \( u \) as \( t \) tends to zero and infinity respectively. Further note that \( k_1(h+n) \) is constant in the model.

Thus one can distinguish three sets of possibilities. Case I is portrayed in Figure 1 (a). It shows what Fei-Paauw call the favorable case. In this case, no foreign capital is necessary. The distinguishing characteristics of this case is that \( (h+n) \) is less than the initial average savings rate, \( s(0) \).

Case II is portrayed in Figure 1 (b). It shows what Fei-Paauw would call the intermediate case. In this case the country is initially importing capital, but with time the country starts exporting capital. The distinguishing mark of this case is that the initial savings rate is less than the long-run rate of growth of capital, \( u/k \).

Case III is the unfavorable case. In this case, foreign capital will be required for ever to sustain a given rate of growth of per capita income. The distinguishing property of this case is that \( (h+n) \) exceeds the long-run rate of growth of capital, \( u/k \).

Both the Chenery-Strout and Fei-Paauw models were empirically applied. But while the former does not calculate the termination date of foreign resource inflows to various countries, the latter does not attempt to project capital requirements on an aggregative basis. Moreover, the set of countries to which the models were applied are not exactly the same though overlapping. The results derived from either model do not completely match with those of the other. For example, three successful countries in the Fei-Paauw model--Colombia, Tunisia and Mexico--fail to pass the roughly equivalent savings criterion of Chenery-Strout; again, eight countries which pass the test of Chenery-Strout--Argentina, Brazil, Honduras, India, Israel, Nigeria, Panama and Peru--were among the set of unsuccessful countries of Fei-Paauw.

The differences in the empirical results of the two models can of course be traced to the differences in their formulations--different, though related, ways of conceptualizing the problem.

*Mckinnon Model:* The McKinnon model (1966) focuses on
foreign-exchange constraints to growth and proceeds to show how foreign resources can play a dynamic role in the early stages of economic development.

The basic assumptions and approach of the study are very similar to those of other models in this area. However, one novel innovation of this model is that McKinnon draws a distinction between domestically produced and foreign produced capital goods. While this distinction is always there in the background in all of
the gap models, McKinnon (1966) made it more explicit than did the other authors mentioned above. He assumed that total output \( Y \) is a fixed-coefficient function of domestic and foreign capitals--\( K_d \) and \( K_f \). Thus, he postulates

\[
Y = \min (\alpha K_d, \beta K_f)
\]  

(31)

He further assumes a proportional savings function: \( S = sY \) with all savings being invested.

From the above, one can derive:

\[
\dot{Y} = \min (\alpha I_d, \beta I_f) = \min (\alpha I_d, \beta I_f)
\]  

(32)

where \( \dot{Y} \) is the time derivative of \( Y \) etc., and \( I_d \) and \( I_f \) denote domestic and foreign type investment respectively. Assuming that all savings can be translated into export--that is, there is no export constraint, one can derive:

\[
\dot{Y} = W \cdot I = W \cdot s \cdot Y
\]  

(33)

where \( W = I / \{(1/\alpha) + (1/\beta)\} \).

Now intergrating, one gets: \( Y(t) = Y(0) \cdot \exp(W \cdot s \cdot t) \).  

(34)

Next we drop the assumption that all savings can be translated into export, but rather assume: \( E = eY \), implying that total export earnings are linearly related to total GNP. Now calculating backwards, the required foreign investment for \( Y(0) \cdot \exp(W \cdot s \cdot t) \) is given by

\[
I_f = (1/\beta) \cdot Y = (1/\beta) \cdot W \cdot s \cdot Y(0) \cdot \exp(W \cdot s \cdot t) = W \cdot (1/\beta) \cdot Y(t)
\]  

(35)

while the actual export is \( e \cdot Y(t) \). Now the foreign-exchange constraint will not exist if the following holds:

\[
W \cdot (s/\beta) \cdot Y(t) < e \cdot Y(t)
\]  

(36)

or expressed alternatively, \( W \cdot (s/\beta) < e \).

(37)
From the above conditions, we can easily see that the foreign-exchange constraint (or bottleneck constraint, as McKinnon calls it) will not be binding for large values of $\beta$ (implying small requirements for foreign capital) or for large values of $e$ (implying high export capacity).

Suppose that the bottleneck constraint is bindings, in that case the growth profile of income is given by

$$Y = I = eY$$  \hspace{1cm} (38)

implying, $Y(t) = Y(0). \exp (\beta et) < Y(0). \exp (Wst)$.

From the above discussion, it is evident that foreign capital inflows have a different impact on growth depending on whether the savings constraint or the foreign-exchange constraint is binding. Denoting $f = F/Y$, the foreign resource transfer as a fraction of na-

Figure 2

**Effects of Foreign Resource Transfer on Growth**

![Diagram showing the effects of foreign resource transfer on growth](image)
tional income, we can define the sustainable rate of growth $G$ as follows:

$$G = \beta (e+f) \text{ if } \beta (e+f) < W (s+f) \text{ (bottleneck constraint)} \quad (39)$$

$$G = W (s+f) \text{ if } \beta (e+f) > W (s+f) \text{ (savings constraint)} \quad (40)$$

The impact of foreign resource transfer on economic growth under different constraints can be beautifully illustrated by a diagram used by McKinnon. From the following figure (Figure 2), one can easily see that the impact of aid on growth is greater if the bottleneck constraint is binding rather than the savings constraint.

II. Multi-Sector Models

The preceding section was devoted to a discussion of aggregative models. In the present section we shall focus our attention on the multi-sector planning models which tend to highlight the interactions between growth and external resource inflow. While discussing these models, unlike our exposition in the preceding section, we shall, of necessity, desist from spelling out the mathematical argument of the models in detail.

So far as the models are concerned, they are all linear and they fall into two broad categories—consistency models of the input-output variety and optimizing models of the programming variety. Consistency models can provide projections of investment, output levels, balance of payments etc., consistent with specified final demands. Consistency models are, to a large extent, a tool of exploration—to highlight the growth options facing the economy and to illuminate the possible ways of exploiting these options to the best advantage of the economy. Optimizing models—to some extent natural complements of consistency models—on the other hand, seek to provide the 'best' pattern of final demands and resource allocation from among the set of possibilities—the 'best' being defined in the sense of optimizing some welfare function. Both types of models are highly useful and can provide valuable insights; and they have been profitably utilized to investigate the problem of interactions between domestic and foreign resources—in the empirical context of different economies.

Bergsman and Manne is an input-output model of India, encompassing the third and fourth plans of the country. Their objec-
tive had been to project balance of payments under alternative strategies of growth--defined with respect to aggregate growth targets and import substitution targets. The interesting result that follows from this exercise is that faster import substitution in the capital goods sector leads to larger deficits in the early years, but these deficits are more than offset by imports saved in later years--defined in terms of discounted sums (the rate of discount being defined at 10%). The policy implication that follows from the above is that a larger inflow of external resources in the early years decreases the total volume of aid needed to sustain the postulated growth targets. However, one must mention one disturbing feature of this model: it requires input-output consistency only for the terminal years of the plans; and for other years, it uses interpolation techniques to estimate the time paths of important variables. In that sense only, the Bergsman-Manne model remains an 'almost consistent' model.

Bruno (1967) is a static linear programming model, based essentially on an earlier work of the author (1966). The problem poses is to maximize the total consumption subject to the constraints on both skilled and unskilled labor as well as balance of payments and savings. By changing the volume of aid inflow, he attempts to trace the impact of aid on aggregative consumption. He finds that with increases in resource inflow consumption increases but at a decreasing rate, implying decreasing marginal productivity (shadow price) of foreign resource inflow. Secondly, by introducing savings constraints along with the foreign-exchange constraints, he finds that the marginal productivity of aid (or its shadow price) is higher if the foreign-exchange constraints are binding rather than the savings constraints. This finding seems to be consistent with the results of McKinnon (1964)--derived in the context of a simpler theoretical model.

Manne and Weisskopf is an intertemporal planning model of India. The feature that distinguishes this model from others is the innovation which Manne and Weisskopf call the 'gradualist consumption path'--gradualist in the sense that consumption should increase at a certain parametric rate over time. The way they pose the problem is as follows: The objective function is to maximize the increment in the first year consumption level, the initial year aggregate consumption level as well as the growth rate of increments in consumption in following years being defined exogenously.

5 Bruno (1967) is based on a 'reduced-form' of Bruno (1966). Except for some extra constraints, the two works are essentially the same so far the structure is concerned.
They experiment with three alternative patterns of external capital inflow—all starting at the same level but declining at different rates. Under the given assumptions they find that both aggregate capital-output ratio and the shadow price of foreign exchange increase with an increasing rate of decline in the inflow of aid.

Weisskopf—a static linear programming model—focuses on the trade-off between domestic and foreign resources in the empirical context of India. With the usual linear programming constraints set, Weisskopf defines the problem as to minimize the objective function, which is a weighted sum of domestic and foreign resource costs of achieving the target rates of development—the targets being defined in terms of aggregate consumption, exports and so on. The primary domestic cost is essentially the wage bill while foreign resource costs consist of the foreign exchange costs of import required to sustain the target rates. With growth targets remaining the same, by changing the relative weights between domestic and foreign resource costs in the objective function, Weisskopf derives trade-offs between external capital inflow and domestic savings. Not surprisingly, as the relative weight of external resource costs is increased, more domestic savings and less foreign capital are used to achieve the given target rates—implying greater opportunity costs of substitution of foreign resources by domestic resources. Further, he finds that as the relative weight of foreign resource costs is increased, the economy-wide incremental capital-output ratio increases, reflecting rising costs of import substitution; or viewed from a different point, it implies increasing marginal productivity of foreign capital.

Tendulkar presents a static, multi-sector optimizing model for India. In this study, the objective is to maximize the terminal year consumption subject to constraints upon the availability of primary resources. He contrasts two versions of his model. In his open loop version, he takes the supply of foreign exchange as the only limiting factor. This is what he calls the pure trade limited process (PTL). In his closed loop version, Tendulkar adds a savings constraint along with the foreign exchange one. This is what he calls trade and savings limited growth (TSL). Tendulkar attempts to highlight interactions between domestic and foreign resources.

Like Bruno (1967), he varies the amount of foreign resource inflow and seeks to trace their impact on consumption. It may be mentioned that the shadow price is measured here by the increase in consumption due to an additional inflow of foreign resource.
And quite expectedly, he comes to the following conclusions: A closed loop variant yields a higher shadow price at each level of foreign resource inflow, but the differences seem to narrow with increasing inflow of foreign resource. The intuition behind this result seems to be that in the closed loop model, additional foreign resources play the dual role of increasing domestic investment capacity and the capacity to import; but, on the other hand, in the open loop model foreign resource relaxes only the import constraint. Further, Tendulkar also reports the impact of foreign resource inflow on incremental capital-output ratio. His finding is that the incremental capital-output ratio always remains higher in the closed loop variant and seems to be falling as the rate of inflow of foreign resource is increasing.

Chenery and MacEwan is a dynamic linear programming model which has as its object of study the optimal growth strategy for Pakistan. One important feature of this model is that it allows the total amount and the time pattern of resource inflow to be varied within limits. The objective function of the model is the sum of the following terms:
(i) the sum of discounted consumption over the plan period;
(ii) the weighted value of the terminal year output; and
(iii) the discounted value of foreign assistance which is multiplied by a constant term, signifying the price of foreign capital. The model is cast in terms of two sectors: traded and non-traded. Chenery and MacEwan postulate the following three alternative profiles of resource inflow: (a) Given the cost of foreign aid, the model is allowed to determine endogenously through the process of optimization the termination date of foreign aid. (b) In the second experiment they give the termination date of foreign resource inflow; the shadow price and the total foreign resource requirements are determined endogenously; (c) In the third experiment the discounted sum of foreign resource inflow is specified exogenously while the shadow price and the termination date of resource inflow are determined endogenously.

Chenery and MacEwan come to the following conclusions. The optimal time pattern of aid inflow seems to form a dome-shaped curve—first increasing, then decreasing finally reducing itself to zero. Aid is utilized in the first phase to the upper limit of the absorptive capacity, with investment being divided equally between the traded goods sector and the non-traded goods sector. In the second phase, growth and investment slow down while the direction of investment goes in favor of the traded sector. In the third phase, aid inflow goes down to zero and a balanced growth between the
pattern. As the volume of aid inflow increases, its shadow price declines. But on the other hand, as absorptive capacity or marginal propensity to save increases, the shadow price of aid also seems to be increasing.

A model similar to that of Chenery and MacEwan has been experimented with in the case of Israel by Bruno, Dougherty and Frankel. The objective function of this study consists of two components: the sum of discounted consumption and the discounted value of the terminal capital stock. They consider three alternative variants of the model. In the first variant, they assume no savings constraint; the total discounted sum of aid is fixed but the profile of aid inflow is flexible and subject to choice. In the second variant, they impose the savings constraint but the profile of aid inflow is flexible. In the third variant, again there is no savings constraint but aid inflows are specified for each period with no intertemporal transfers permitted.

The results of this study seem to be in conformity with other models. In the case where there is the flexibility of choosing the time profile of aid, it is found that optimality requires more aid in the initial years and repayment of it in the later years by export promotion and import substitution. Similarly, the shadow price falls as the level of aid inflow in increased, but decreases with the rise of domestic savings—a reflection of the complementarity of domestic savings and foreign resource inflow.

III. Some Concluding Remarks

The foregoing account has highlighted the main strands of research efforts in this area. While all the works discussed above have contributed useful insights, none of them has in any sense reached the ideal—in fact, they can be subject to a great deal of criticism. Here we intend to point briefly to some major shortcomings—as we perceive them—of these works.

Although the aggregative models of foreign resource requirement as outlined in Section II differ a great deal among themselves in terms of their algebraic formulations and points of emphasis, they have in common the following inflexible features: all the models assume no capital-labor substitution in production;\(^6\) they

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\(^6\) Chenery and Strout (1966) justify the assumption of a fixed capital-output function 'as a matter of convenience' since production function estimates are 'inconclusive because of the limited data available'. However, at the present stage, this excuse seems no longer tenable.

assume implicitly or explicitly no substitutability between domestic and foreign capital; further, all these models assume no substitutability between domestic and foreign consumption goods—thus ignoring the role of relative prices. All of the above assumptions seem to be quite far from being realistic. It would be highly desirable to include these possibilities in the planning models focusing on the foreign resource implications of growth.

The second aspect that needs to be emphasized is that none of above-mentioned aggregative models can in any way handle or allow for structural change. It may be pointed out that the implicit assumption of a macro-model is that the structural composition of outputs remains the same throughout the period. This assumption may be valid for a developed country, but not quite so in the context of a LDC—because the very purpose of development is to bring about structural change. Therefore, we contend that the two-gap type aggregative models, though illuminating as a first approximation, are rather ill-suited by their very nature to tackle the problem of development.

Next, turning to the multi-sector models, one can observe that they are rich in empirical content, and eschew the problems of aggregation bias as discernible in the aggregative models of Section II. For instance, export and import figures in aggregative models, as it is often argued, are too simplistic to be of use for policy decisions. Multisector programming models are clearly at an advantage here. By disaggregating exports and imports by sectors, these models are more helpful not only in illuminating the structural

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7 Ronald Findlay has indeed rightly pointed out that the two-gap type analysis 'depends critically on the impossibility of substituting domestic for imported inputs' in the production function.

8 Again, Findlay has emphasized the role of relative change between domestic and foreign prices in the determination of consumption import requirements. See Robinson and De Melo for a model which emphasizes product differentiation between domestic and foreign goods, recognizes the importance of relative price changes and includes the possibility of 'two-way' trade.

9 Even McKinnon (1966), in his 'Rejoinder', admitted that the dual gap analysis implied an extreme form of lack of flexibilities and substitutabilities in resources; but, he, however, believed that they were inherent features of developing countries. As international evidence is accumulated, the belief he expressed there does not seem to be vindicated by facts.

10 Commenting on the fact that two-gap type models ignore structural change, Micksell rightly complains: "Economists have become so wedded to growth models involving continuous functions that they are unable to see in terms of structural changes which must occur if the economy is to develop." (pp. 93-94)

11 In a similar vein, L. Taylor (1975) puts it: "Trying to alter many of these 'fixed parameters (of the two-gap type models) is at the heart of planning in a developing economy experiencing structural change." (p. 41).
relationships, but also in providing policy guidances. In other words, the multi-sector models, through the disaggregation of exports and imports by sectors, can often bring into the open some of the options and their consequences that face the policy maker, but which, by the very nature of aggregation, remain hidden in the context of aggregative two-gap models. However, this is not to say that multi-sector programming models incorporating two-gap features are without difficulties; indeed far from it. The criticism relating to the lack of substitutabilities carries over verbatim to these models as well.

The interactions between aid and economic development can be handled in the multi-sector optimizing models only indirectly--through sensitivity analysis. Here one examines the impact of an increased inflow of external resource on the objective function--the increment in the value of the objective function due to an extra inflow of foreign resource thus measuring the shadow price of it. This shadow price approach to the problem is no doubt insightful--since it reflects in a general way the real resource trade-offs in the economy, but is not without difficulties. 12

In conclusion, we can say that while the aggregate gap models are useful to indicate the limits to growth, they conceal the available growth options and give very little guidance to specific policies. In particular, these models ignore the options of substitution in consumption and production, and the resulting structural change. On the other hand, the programming models, which can incorporate substitution only indirectly through trade, are a step forward over the aggregative two-gap models, but in no way near the ideal--they still ignore direct substitutabilities in consumption and production. We contend that the interactions between growth and aid can be handled in a more illuminating fashion--and of course more directly--if we posit the problem in the context of a computable general equilibrium model. Such a model can remedy most of the shortcomings inherent in the past modeling efforts in this area and also accommodate a wide range of important features of reality--like non-linearities of functions and different types of substitution possibilities. Such general equilibrium models were recently undertaken within a two-sector framework of the Chilean economy by Taylor (1973) and multi-sector framework of the Bangladesh economy by Quibria.

12 The problems that shadow prices run into the context of linear optimizing models are by now well known and we do not discuss them here. However, for a good summary discussion, see Taylor (1975) or Westphal.
References


Robinson S. and J. DeMelo, "A Planning Model Featuring Trade,


