In a two-country synthesis model we examine implications of a tariff reduction on the gains at extensive margin and the diversification of export basket through development of new varieties of a horizontally differentiated good. Through its resource allocation effect, the tariff reduction raises the number of varieties produced and exported by the Home country if the differentiated good is relatively labour intensive. Though, the worsening of terms of trade (TOT) consequent upon the tariff reduction has a dampening effect, overall, the number of varieties increases. The number of varieties produced and exported by the Foreign country, on the other hand, falls which makes the overall variety gains or gains at the extensive margin due to tariff reduction ambiguous.

*Keywords:* Trade liberalization, Export Diversification, Product Variety, Gains at Extensive Margin, Intra-Industry Trade

*JEL Classifications:* D50, F12, F13

1. **INTRODUCTION**

The paper examines whether a tariff reduction makes export basket of nations more diversified through the development of new product varieties by its resource allocation and terms of trade (TOT) effects. This is relevant in two contexts. First, the existing theoretical literature does not take into account the implications of TOT change on variety gains from trade though the empirical literature suggests that such changes may be important. Second, in the long run, diversified export basket means a higher rate of growth as more recent empirical analyses on trade and growth suggests.

The literature on trade and variety gains dates back to pioneering works of Krugman (1979, 1980) and Helpman on intra-industry trade, which show that opening up to trade will increase the number of varieties of a horizontally differentiated good all around. While love-of-variety preference pattern and homogeneous consumers were central to

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the analysis of Krugman (1979, 1980), goods as bundles of characteristics and heterogeneity of consumers were the building blocs of Helpman’s (1981) analysis. Subsequently, Arkolakis et. al. (2008) establish that with trade liberalization the total variety (domestic plus imported) can increase, decrease or remain constant in a model with firm-level increasing returns, differentiated goods, monopolistic competition, endogenous variety and free entry.

Based on these theoretical analyses of variety gains from trade (or gains at the extensive margin), Klenow and Rodriguez-Clare (1997) found that trade liberalization in Costa Rica during 1986 to 1992 led to an increase in the import varieties and the estimated welfare gains there from were 50% larger than the standard gains from trade. The study by Feenstra et al. (2005) also reveals that tariff reduction is important in expanding export variety both in Mexico and China. Own tariff cut is particularly important if the tariff of the importing country remains high and vice versa. Tariff reduction in both Mexico and the U.S. due to NAFTA also led to increased export variety of Mexico.

On the contrary, there is some evidence that trade liberalization leads to exit by domestic firms, thereby reducing the number of domestic varieties (Tybout, 2003). Similarly, in a cross country study for the period 1991-2000, Feenstra (2013) finds that the bilateral import tariff appears to discourage expansion in export variety. At the same time, some recent studies suggest that as trade liberalization can yield adverse terms of trade (TOT) effects, the Krugman (1979, 1980) model may overestimate the variety gains of (intra-industry) trade as it does not consider the terms of trade effect. Considering this, Ardelean (2006) estimated consumer’s love-of-variety as the elasticity of relative imports to extensive margin and found it to be 42% lower than that assumed in Krugman’s model.

All these empirical analyses bring out the importance of a change in TOT due to tariff reduction on the number of varieties exported and imported, one way or the other. But as mentioned above, the Krugman (1979, 1980) model does not consider the TOT effect and consequently may overestimate the variety gains from trade. The standard Heckscher-Ohlin-Samuelson (henceforth, HOS) model, on the other hand, though predicts a TOT deterioration when tariff rates are lowered, it cannot shed any light on implications of such TOT changes for variety gains since it does not consider different varieties of differentiated goods being traded. The purpose of the present paper is to fill in these gaps in these two main strands of general equilibrium trade models – variety effect in the HOS model and TOT effect in the Krugman model – by constructing a synthesis of these models and then analyzing implications of tariff reductions and TOT changes on the number of varieties being produced and exported.

Our theoretical exercise also has a dynamic or long run context. Recent empirical estimates reveal that export diversification leads to faster economic growth at cross-country level (Aditya and Acharyya, 2012, 2013; Hesse, 2008; Lederman and Maloney, 2007; Agosin, 2007). That is, what may matter for countries pursuing an export-led growth policy is what they export rather than how much they export.
Therefore, diversification of exports, which provides more stability and growth in export earnings, should be the primary focus of economic policy in the developing countries. In such a context, our extension of the HOS model in terms of different varieties of a good being traded enables us to address implications that trade liberalization policies in the present era of globalization may have for diversification of export basket of a country. This issue has not been explored adequately in the existing theoretical literature with the notable exception of Aditya and Acharyya (2015).

With the effects of TOT changes being the focal point of analysis, we consider a world economy with two countries, Home and Foreign, producing two goods with two factors of production, labour and capital. One of these goods is a homogeneous good, which is produced under constant returns to scale technology and perfectly competitive conditions. The other good is a horizontally differentiated manufacturing good with distinct set of varieties, which is produced in the two countries under increasing returns to technology and monopolistically competitive conditions. Whereas the Home country is assumed to be a net importer of the homogeneous good and thus protects its domestic sector by an ad-valorem tariff, unrestricted intra-industry trade takes place in the horizontally-differentiated manufacturing good between the two countries. Countries share the same technologies for the two goods and have homothetic and identical preferences as in the HOS and Krugman models. In the HOS tradition, countries differ only in respect of their endowments of the two factors of production.

As we show, the nature of factor abundance of the two countries and factor intensity ranking of the homogeneous and differentiated goods together determine the number of varieties of the differentiated good that the two countries produces. In this sense, our theoretical model can be seen more as an extension of HOS model in which the export sector produces a horizontally differentiated good under increasing returns to scale and monopolistic competition, than as an extension of the Krugman (1979)-model, to study the TOT effect of tariff reduction on consequent diversification of exports, variety gains and the growth implications.

In such a framework, a reduction of tariff on import of the homogeneous good by the Home country affects the number of varieties of the horizontally differentiated good in two ways. First is through a reallocation of resources across the two sectors consequent upon the tariff reduction since both the sectors draw resources from the same pool. Implication of this resource allocation effect on the number of varieties and gains at the extensive margin has not been explored in the existing theories mentioned above except by Feenstra et al. (2005) who argue that tariff reduction may boost export variety by

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1 The current empirical literature also emphasizes on the quality content of export baskets of the countries for faster export led growth. China’s growth driven largely by high technology exports and India’s growth caused primarily by ITeS are glaring examples of the importance of commodity composition of export baskets (Rodrik, 2006).

2 Aditya and Acharyya (2015) considered countries producing a differentiated good and a set of Ricardian continuum of goods to study diversification of export basket following trade liberalization. But, the implications of TOT changes consequent upon trade liberalization was not analyzed.
moving resources into the exporting sectors. Second, the reduction in the rate of tariff brings about a TOT change, which in turn changes the factor prices and leads to further reallocation of resources across the homogeneous and the differentiated sectors. The number of varieties produced thus changes further, as suggested by the empirical literature mentioned above but not addressed by the existing theoretical literatures.

More precisely, we show that if the varieties exported by the Home country are relatively labour intensive, the TOT worsens though less than proportionately to the rate of tariff reduction (like the usual case in the standard HOS model) under some fairly general conditions, which mitigates the initial increase in the number of domestic varieties to some extent. Thus, the increase in the number of domestic varieties at the new equilibrium is less than if there had been no TOT effect (such as in case of a small country). This supports the result of Ardelean (2006) that the variety gains in the Krugman model may be an overestimation. We also show that there are asymmetric changes in the number of varieties produced by the two countries and consequently asymmetric changes in diversity of their respective export baskets when the TOT worsens. This means that the growth experiences of the two countries in the long run may be different as well. However, with the number of foreign varieties decreasing in this case, the total number of varieties consumed in the two countries may go up or down and, therefore, the variety gains (or gains at the extensive margin) may not be realized, similar to what has been shown by Arkolakis et al. (2008). But, in contrast to their analysis, the forces at work behind the alternative possibilities in this paper are more in the HOS tradition. These twin implications of our result reveal a trade-off between growth and gains at the extensive margin.

One of the policy implications that emerges from our analysis is that the policy target of a more diversified export basket can be achieved by the Home country through a tariff reduction if the differentiated good is relatively labour intensive, though to a lesser extent than if there had not been any TOT change (i.e., if it had been a small country). Otherwise, raising the tariff rate will be the appropriate policy, regardless of whether the Home country is large or small.

The rest of the paper is organized as follows. In Section 2.1 we set out the analytical structure of our synthesis model and describe the equilibrium under tariff restricted trade. Section 2.2 examines the impact of a tariff reduction on the number of varieties produced in the two countries. In Section 2.3 we examine whether there are gains at the extensive margin. Section 3 discusses relevance of the factor intensity conditions underlying our results and other robustness issues and verifies the results in a numerical exercise. Finally, Section 4 concludes the paper.

2. THE ANALYTICAL FRAMEWORK OF THE SYNTHESIS MODEL

We consider a world economy consisting of two countries, Home and Foreign, producing a homogeneous good $Z$ and a differentiated good $X$. The homogeneous
good $Z$ is produced under perfectly competitive conditions using fixed units of labour and capital per unit of output. Good $X$ with different varieties are produced under increasing returns to scale (IRS) technology and monopolistically competitive conditions. The number of endogenously determined varieties of good $X$ produced by the Home and Foreign countries, denoted by $n$ and $n^*$ respectively, are distinctly different from each other due to IRS. Thus, intra-industry trade arises in the horizontally-differentiated manufacturing good between the two countries. On the other hand, the Home country is assumed to be a net importer of the homogeneous good $Z$, and it protects its domestic import-competing sector by an ad valorem tariff, $t \in (0,1)$ Production technologies for each good are identical and demand conditions are identical and homothetic in the two countries. Preference for the differentiated good displays love-of-variety.

By fixed coefficient production technology, one unit of good $Z$ requires $a_{KZ}$ units of capital and $a_{LZ}$ units of labour. Each variety of good $X$ requires $p$ units of capital regardless of its output level and $\beta$ units of labour per unit of output. This production technology is similar to that of Krugman (1979) except for the fact that instead of labour being used as both fixed and variable factors of production in his specification, we consider capital as the fixed factor of production. Of course, our two-factor model allows us to differentiate between the fixed and the variable factors in production of good $X$. The idea is that a fixed amount of capital ($\rho$), which can alternatively be interpreted as human capital (or skilled labour), is required to develop a new blueprint or make a variation in specification or design of an existing variety sufficient enough to warrant it as a different or newer variety. Once such a new blueprint or design is developed, or a variation on the existing one is achieved, production of such a new variety requires only labour and the total requirement of labour increases proportionately with the output level produced.

Endowment of labor and capital are exogenously given. The return to capital ($r$) is fully flexible. But the money wage is assumed to be fixed institutionally ($\bar{w}$). As we will see later, this assumption is needed to close the model since along with output of good $Z$, the number of varieties of good $X$ (or the number of firms in X-sector) is also endogenously determined. The rigid wage assumption is also not at odd with reality. Official minimum wage rates are prevalent in many countries across the globe. Not only the developing and less developed countries, the OECD countries also apply some kind of wage floor. Apart from five countries (South Sudan, Taiwan, Northern Cyprus, Hong

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3 By the same IRS property, $n$ and $n^*$ also denote the number of firms producing good $X$ in the Home and in the Foreign country respectively.

4 Helpman (1993) considered a separate research & development (R&D) sector where a new product is developed by fixed units of labour. The R&D technology, however, displayed learning by doing in the sense that units of labour required to develop a product falls with increase in the knowledge capital or the number of varieties already developed. In our specification, like in Krugman (1979), newer varieties are developed in-house with no learning-by-doing effect so the fixed (capital) cost can be regarded as the cost of in-house R&D.

5 The ILO encourages its member States to adopt minimum wage as part of its Decent Work Agenda to reduce poverty and provide social protection for vulnerable employees as pointed out by the Global Wage Report (2012-2013). ILO standards further recommend that minimum wages should be set by authorities after
Given the above set of assumptions, the following set of conditions specifies the domestic market equilibrium conditions in the Home country. First, perfect competition in the homogeneous good sector implies that producers earn zero profit at the long run equilibrium:

\[(1 + t) P^w = \alpha_{Lz} \bar{w} + \alpha_{Kz} r, \quad (1)\]

where, \( P^w \) is the world price of the homogeneous good.

Second, free entry and monopolistic competition in the X-sector means the equality between price \( (P_{xt}) \) and average cost, which ensures zero profit from producing each variety,

\[P_{xt} = \beta \bar{w} + \frac{\alpha r}{x_i}, \quad (2)\]

and the equality between marginal revenue and marginal cost, which reflects the monopoly power that each firm has over the variety it produces:

\[P_{xt} = \left( \frac{\varepsilon_i}{\varepsilon_i - 1} \right) \beta \bar{w}, \quad (3)\]

where \( \varepsilon_i \) is the price elasticity of demand for the \( i \)th variety faced by the \( i \)th firm.

Finally, there are the factor market equilibrium conditions:

\[\bar{L} = \alpha_{Lz} + n \beta X, \quad (4)\]

\[\bar{K} = \alpha_{Kz} Z + \rho n, \quad (5)\]

where \( \bar{L} \) and \( \bar{K} \) are endowments of labour and capital respectively, \( Z \) is the output level of good \( Z \), and using the symmetric equilibrium condition for the number of varieties of good \( X \), \( \sum x_i = nx \).

Given \( P^w \) and \( \bar{w} \), the above set of five equations determine the five variables: \( r \), \( x \), \( P_x \), \( Z \) and \( n \). It is now clear that we need to fix the money wage to close the model.\(^6\)

Consultation with social partners, and that a balanced approach should be adopted which takes into account the needs of workers and their families as well as economic factors, including levels of productivity, the requirements of economic development and the need to maintain a high level of employment. Similar in spirit the European Commission has suggested its Member States to establish “decent and sustainable wages” because “setting minimum wages at appropriate levels can help prevent growing in-work poverty and is an important factor in ensuring decent job quality” (European Commission 2012a, p. 9). However, debates continue regarding the level at which minimum wages should be set.

\(^6\) In fact, to close the model we need the return to either of the two factors be exogenously given. We choose the money wage as it fits well with what we observe in many countries as mentioned earlier.
The dichotomy property of the model in the HOS tradition is to be noted. Given the fixed money wage, the prices of different varieties and the rate of return to capital are determined solely and uniquely by the world price of good $Z$, independent of the factor market equilibrium conditions. The zero profit condition for good $Z$ determines the rate of return to capital, and then the two equilibrium conditions for good $X$ determine the price and output of each variety. Two comments are warranted at this point. First, the price of each variety of good $X$ is determined by mark-up over the (constant) wage cost. The mark up itself varies with the output of each variety being produced since the elasticity of demand $\varepsilon$ varies with the level of quantity demanded (and hence output level produced at equilibrium) for a large class of demand functions. However, to keep things simple we will assume constant price elasticity of demand so that price of each variety will be a constant mark-up over the wage cost and, therefore, will be invariant with respect to changes in the tariff rate. Thus, variations in the TOT will be brought about only by the variations in $P^W_i$. We will discuss the implications of variable price elasticity and consequent variability in prices of different varieties later. Second, since all firms in sector $X$ share the same technology and face the same factor prices, so $x_i = x_j$ and consequently $P_{x_i} = P_{x_j}$ for $\forall i \neq j$. Thus, we have a symmetric equilibrium as in Krugman (1979).

Once the output of each variety is determined by (1)-(3), the full employment conditions (4) and (5) together determine the output level of good $Z$ and the number of varieties of good $X$ (and the number of firms producing good $X$). Note that, despite money wage rigidity, labour is fully employed in both the countries because excess supply of labour can be absorbed through expansion of the number of varieties and number of firms. Of course, the nature of adjustment, that is, whether $n$ rises and $Z$ falls or the reverse depends on the factor intensity assumption.

The equilibrium conditions in the foreign country can be described by analogous conditions on the assumption that production technologies, demand for each variety and the rigid wages are identical in the two countries:

$$P^W_i = \alpha_{lZ} \bar{w} + \alpha_{kZ} r^*, \quad (6)$$

where, $r^*$ is the rate of return to capital in the foreign country, which differs from that in the Home country because of the tariff rate.

$$P_{x_i} = \frac{\varepsilon_i}{\varepsilon_{i-1}} \beta \bar{w}, \quad (7)$$

$$P_{x_i} = \beta \bar{w} + \frac{\beta r^*}{x_i}, \quad (8)$$

The model displays the HOS properties so that the nature of adjustments in case of excess supply of labour (or capital) are analogous to output magnification effect of an increase in the endowment of labour (or capital) in a standard HOS model.
The assumptions of identical technology and identical demand conditions as in HOS and Krugman (1979) model along with the same rigid money wage in the two countries imply that prices of all varieties are the same across these countries. This rules out any scope of arbitrage in different varieties of good $X$, and the intra-industry trade in these varieties arises solely due to economies of scale (which makes all varieties distinctly different from each other) and the love-of-variety preferences as in Krugman (1979). However, the two countries differ in their factor endowments, and accordingly the volume (or the number of varieties exported) and the value of exports of differentiated varieties differ: $n \neq n^*$. Figure 1 below illustrates the asymmetric equilibrium under the assumption that the Foreign country is relatively capital rich ($K^* > \bar{K}$ and $\bar{L} = \bar{L}')$ and the differentiated good is relatively labour intensive in the sense that the ratio of labour requirement per variety to labour requirement per unit of output of good $Z$ is larger than ratio of the capital requirement per variety to capital requirement per unit of output of good $Z$: $\beta x / \alpha_{12} > \rho / \alpha_{KZ}$. The solid lines represent the full employment lines for labour and capital in the Home country, whereas the broken lines represent those in the Foreign country. Note that the slope of the Foreign country’s labour constraint is flatter than that of the Home country even though $\bar{L} = \bar{L}'$. This is because, for any given $P^*_Y$, a tariff on import of $Z$ by the Home country makes the domestic price and correspondingly the rate of return to capital there higher than those in the Foreign country. As evident from the zero-profit conditions in the differentiated good sector, the output of each Home variety ($x$) will be larger than the output of each Foreign variety ($x^*$). Thus, if the tariff rate is not too high, the Home country will produce a larger number of varieties than the foreign country ($n > n^*$). For homothetic and identical demand for the homogeneous good and the differentiated goods across countries, this, in turn, means that the differentiated good sector $X$ is larger in size (and value) in the Home country than that in the Foreign country (see appendix).

To put it alternatively, the Home country, which is shown as relatively labour rich in Figure 1 ($\bar{L}/\bar{K} > \bar{L}'/\bar{K}'$), will be exporting larger number of differentiated varieties of good $X$: $n > n^*$. Thus, even though within this sector intra-industry trade is driven by economies of scale and love-of-variety preference, the number of varieties traded will be determined by the endowment asymmetry. All these are obvious because our synthesis model is HOS in character at the sector level. On the other hand, by the resource constraint a lower output of good $Z$ will be produced at home than abroad. Homothetic and identical taste then implies the foreign country will be exporting good $Z$, which we have presumed.

\[ \bar{L}' = \alpha_{12} Z^* + n^* \beta x^* \]  \hspace{1cm} (9)
\[ \bar{K}' = \alpha_{KZ} Z^* + n^* \rho \]  \hspace{1cm} (10)

\(^8\) Tariff also leads to differences in rates of returns to capital in the two countries and thus in output levels of each varieties at equilibrium (see equations (2) and (8)).
However, similar asymmetric intra-industry equilibrium can be obtained if the Foreign country is relatively labour (capital) rich and good $X$ is relatively capital intensive in the sense that $\beta x/\alpha_{LZ} < \rho/\alpha_{KZ}$. This means, our presumption of good $Z$ being exported by foreign country requires it to be relatively capital rich when good $Z$ is relatively capital intensive and relatively labour rich when good $Z$ is relatively labour rich. At this point, however, we do not make any specific assumptions regarding factor intensity of goods and factor abundance of countries except for that these ranking along with the initial tariff rate and other parametric values be such that $n > n^*$ and that both the intra and the inter-industry trade coexists, with the Home (Foreign) country importing (exporting) good $Z$.

To close the model we turn to other two requirements. First of all, the world market for good $Z$ must clear, and second overall trade must be balanced for both the countries. First one is important because it ensures that the quantity of good $Z$ demanded by $H$ (the volume of import of $Z$) is exactly what is supplied by $F$ (its volume of exports of $Z$). Otherwise, $H$’s import of $Z$ will not be realized. At the same time, the overall trade balance of $H$ and $F$ must be balanced, which follows from their (or their consumers’) respective budget constraints. Given identical prices and (rigid) wages, it is easy to check that the share of foreign varieties in total expenditure on horizontally differentiated goods by all home consumers equals $\alpha \equiv n^*/n + n^*$ and the share of home varieties in total expenditure on horizontally differentiated goods by all foreign consumers equals $\alpha' \equiv n/n + n^*$.

Homothetic tastes, on the other hand means

\[ \text{If } c \text{ denotes consumption of } i\text{-th variety by a representative Home consumer, then the total expenditure on all varieties consumed is } P(n + n^*)(L + K)c \text{ and on foreign varieties consumed in the Home country is } Pn^*(L + K)c. \]
constant fractions of national incomes are spent on good $Z$, and on all varieties of good $X$ taken together, and let $\gamma$ and $(1 - \gamma)$ denote such fractions, which are identical for both the countries reflecting their identical taste as well. Thus given the income shares spent on all varieties together and on good $Z$, the trade balance condition for the Home country (evaluated at the world price) can be written as,

$$\frac{n^*}{n^* + n}(1 - \gamma)I + \frac{P^w_Z}{\bar{TP}_Z^w} \left[ \frac{\gamma I}{TP_Z^w} \right] - Z = \frac{n}{n^* + n}(1 - \gamma)I^*, \text{ or}$$  \hspace{1cm} (11) \\
$$\alpha(1 - \gamma)I + \left[ \frac{\gamma I}{\bar{TP}_Z} - P^w_Z \right] = \alpha^*(1 - \gamma)I^*, \hspace{1cm} (11a)$$

where, $I = \bar{w}L + r\bar{K} + t(yI/T - P^w_Z Z)$ is the sum of aggregate factor income and tariff revenue and and $I^* = \bar{w}L^* + r^*\bar{K}^*$.

Foreign’s trade balance condition on the other hand is:

$$\frac{n^*}{n^* + n}(1 - \gamma)I + [P^w_Z^* - \gamma I^*] = \frac{n}{n^* + n}(1 - \gamma)I^*. \hspace{1cm} (12)$$

From the two trade balance conditions it is immediate that the Home country must have a surplus in its intra-industry trade in good $Z$ in order to finance its import of good $Z$, whereas the Foreign country can have a deficit in its intra-industry trade which it can finance by its earnings from export of good $Z$. In other words, in our model, intra-industry trade is unbalanced, and this is because of the one-way trade in the homogeneous good.

Combining (11) and (12) we get,

$$\frac{n^*}{n^* + n}(1 - \gamma)I + \frac{P^w_Z}{TP_Z^w} \left[ \frac{\gamma I}{TP_Z^w} \right] - Z = \frac{n^*}{n^* + n}(1 - \gamma)I^*$$

$$= \frac{n^*}{n^* + n}(1 - \gamma)I + [P^w_Z^* - \gamma I^*]$$

$$\Rightarrow P^w_Z \left[ \frac{\gamma I}{TP_Z^w} - Z \right] = [P^w_Z^* - \gamma I^*] \hspace{1cm} (13)$$

which is the world market clearing condition for good $Z$.

In other words, when at the equilibrium, the overall trade balance conditions for both the countries are satisfied, the world market for good $Z$ clears as well. The value of $P^w_Z$, which satisfies (13), and hence the overall trade balance conditions (11) and (12), is thus the equilibrium value of the world price of good $Z$. Condition (13) thus closes the model.

2.1. Diversification Effect of a Tariff Reduction

In the above set up, we now examine the gains from tariff reduction (or trade
liberalization) at the extensive margin. We begin with implications for product diversification in terms of larger varieties in this sub-section.

A reduction of tariff on import of the $Z$-good by the Home country in the above setup affects the number of varieties produced and exported by it through the resource allocation effect in two ways. First is by lowering the domestic price of the $Z$-good relative to the differentiated manufacturing good at the initial world prices, which we call the direct resource reallocation effect of tariff reduction on the number of varieties produced and exported by the Home country. Second is the induced effect of tariff as it changes the world price of the $Z$-good, and hence induces further change in the domestic relative price and consequent resource allocation, given that the prices of different varieties of the manufacturing good are constant mark-up over the fixed labour costs per unit. We call this as the induced resource reallocation effect or TOT effect on the number of varieties produced and exported by the Home country. On the other hand, in the Foreign country there will only be induced resource allocation effect due to the change in TOT.

The direct effect of tariff reduction can be worked out as follows. At the initial world price of the $Z$-good and corresponding rate of return to capital, a tariff reduction causes production of good $Z$ unprofitable as the domestic tariff-inclusive price declines below the average (and marginal) cost. With the money wage being fixed, capital takes the hit and its rate of return falls through competitive forces as a consequence. This is evident from the zero-profit condition (1). The consequent lower capital cost in the differentiated manufacturing sector makes production of these varieties more profitable, which attracts new firms. Since free-entry leads to average-cost pricing so there is a downward pressure on price of each variety. But for institutionally fixed wage, a price decline would mean a loss for firms. With capital requirement being fixed for each variety being produced, they adjust to the competitive pressure on prices by lowering the output levels produced per variety of the horizontally differentiated export good proportionately to the fall in the rate of return to capital, and thereby cutting down the variable (labour) cost of production. All these are evident from the average-cost pricing and profit maximizing conditions in the $X$-sector:

$$\hat{x} = \hat{r} < 0. \quad (14)$$

However, whether the number of firms and varieties in the manufacturing $X$-sector will increase or not depends on the relative factor intensity of the two goods, $X$ and $Z$. The actual entry of firms and newer varieties are possible only when additional capital along with additional labour is made available. The output contraction by incumbent firms in the $X$-sector releases some labour but not any capital as its requirement per variety is fixed. But, capital being essential in developing newer varieties, availability of only labour due to scale contraction for each existing variety is not sufficient to attract new firms and produce newer varieties. On the other hand, without availability of capital, the production of good $Z$ cannot expand to absorb these excess workers either. So
either the production of good Z must contract to release some capital required for newer varieties to be developed, or some of the incumbent firms (and existing varieties) in the X-sector must exit so that capital released thereof can enable the production of good Z to expand. Now if good Z is relatively capital intensive, then a fall in its output would release capital sufficient for new entrants to design newer varieties and absorb labour released from the sector Z as well as by incumbents in the modern sector. Hence, at equilibrium, firms and varieties in the differentiated good sector will increase. But if good Z is relatively labour-intensive, it releases less capital and more labour required by the potential entrant in sector X. In such a case full employment is maintained only through an expansion of production of good Z, which in turn necessitates exit of some firms (and varieties) in sector X. That is, the contraction of scale of production of each variety and consequent release of labour due to a tariff reduction raises the number of varieties of good X (and correspondingly lowers the output of good Z) if it is relatively labour intensive. This is analogous to the output magnification effect of an increase in labour supply in a standard HOS model by which the production of relatively labour intensive good expands and that of the other good contracts.

This seemingly paradoxical outcome that the import-competing sector expands (when \(|\lambda| > 0\)) even when import-tariff is lowered arises because of the existing distortions. There are two types of distortions existing in this economy. First is distortion due to the economies of scale and consequent violation of marginal-cost-pricing in the differentiated good sector, which is endogenous in nature. The other is the policy induced distortion in the homogeneous good sector due to imposition of tariff there. Whereas the economies of scale induces specialization and reallocation of resources towards the differentiated good sector, tariff induces resource reallocation towards the homogeneous good sector. Since economies of scale arises due to the fixed factor capital, so when the differentiated good is relatively labour-intensive or the homogeneous good is relatively capital-intensive, the economies of scale effect was relatively weaker and the economy "over"-specialized in good-Z under tariff. Then a tariff reduction corrects this over specialization by contracting the size of the sector Z and expanding the number of firms and varieties in the modern sector. But when the horizontally differentiated sector is relatively capital-intensive, the economies of scale effect and consequent distortion is much larger. Thus, despite the tariff on imports of Z, the economy was over-specialized in sector X. A tariff reduction, in such a case acts like a second-best policy to correct this sub-optimal specialization pattern by expanding the size of the import-competing sector.

The direct effect of tariff reduction as explained above is illustrated in Figure 2, under the assumption that good Z is relatively more capital intensive in the sense defined earlier. As at the initial world price of good Z, the reduction in import tariff lowers the output per variety of the manufacturing good, the labour constraint rotates up (as shown by the broken line) and the equilibrium shifts from \(E_0\) to \(E_1\). Output of good Z falls from \(Z_0\) to \(Z_1\), and with the rise in the net availability of factors of production
the number of varieties of good \( X \) increases from \( n_0 \) to \( n_1 \). It is easy to check that if good \( Z \) is relatively labour intensive (in which case the labour constraint line in Figure 2 would have been steeper than the capital constraint line), the reduction of import tariff lowers the number of varieties of good \( X \). In rest of our analysis, we will assume that good \( X \) is relatively labour intensive in the sense defined earlier so that a tariff reduction leads to greater diversification in the differentiated good sector by its direct resource reallocation effect. The implication of good \( X \) being relatively capital intensive will be discussed later.

These initial adjustments in production levels and the number of varieties being produced and exported as explained above will bring in a change in the world price of the \( Z \)-good through corresponding changes in trade volumes. This change in the world price will induce not only subsequent changes in the number of varieties being produced and exported by the Home country but also similar changes in the Foreign country. The equilibrium change in the number of varieties produced and exported by the Home country thus depends on the direction of the change in the world price of good \( Z \).

Algebraically, as shown in the appendix, the equilibrium changes in the output levels of the \( Z \)-good and the number of varieties of the \( X \)-good in the Home country due to the direct and induced (or the TOT) effects taken together can be traced out as,

\[
\hat{n} = \frac{\lambda_X \zeta \left( \bar{T} + \bar{p}_Z^w \right)}{\theta_{KZ}^Z}, \tag{15}
\]

\[
Z = -\frac{\lambda_X \zeta \left( \bar{T} + \bar{p}_Z^w \right)}{\theta_{KZ}^Z}, \tag{16}
\]

where, \( [\lambda] = \lambda_{LZ} \lambda_{KZ} - \lambda_{LX} \lambda_{KZ} \) is the employment share matrix, \( \theta_{KZ} = \alpha_{KZ} r / p_w^Z \) is the capital cost share in unit production in the traditional sector, and \( \bar{T} = d (1 + t) / (1 + t) \) denotes the proportional change in ad-valorem tariff rate.

\[\text{Figure 2. Impact of Tariff Reduction}\]
On the other hand, in the Foreign country, changes in the output levels of the $Z$-good and the number of varieties of the $X$-good are brought about by the change in the world price of good $Z$ (or the TOT):

\[
\hat{n}^* = \frac{\lambda_{KZ}^L \lambda_{KZ}^L \lambda_{KZ}^L \hat{p}_Z^W}{|\lambda|' \theta_{KZ}^*,} \quad (17)
\]

\[
\hat{z}^* = -\frac{\lambda_{KZ}^L \lambda_{KZ}^L \lambda_{KZ}^L \hat{p}_Z^W}{|\lambda|' \theta_{KZ}^*}, \quad (18)
\]

where $|\lambda|' = \lambda_{IZ}^L \lambda_{IZ}^L \lambda_{IZ}^L$ is the employment share matrix, $\theta_{KZ}^* = \alpha_{KZ} r / \rho_{ZW}$ is the capital cost share in unit production in the traditional sector.

Note that, good $Z$ is relatively capital intensive means $|\lambda| < 0$, so that a tariff reduction ($\hat{T} < 0$) at the initial world price of good $Z$, raises the number of varieties produced in the Home country by the magnitude obtained by putting $\hat{p}_Z^W = 0$ in (15): $\hat{n} = \hat{T} (\lambda_{KZ}^L \lambda_{KZ}^L / |\lambda|' \theta_{KZ}^*)$. If tariff reduction raises the world price of good $Z$, i.e., $\hat{p}_Z^W > 0$, the number of varieties of good $X$ produced and exported in the Home country falls as a consequence for reasons just opposite to the one explained above. That is, the induced or the TOT effect of a tariff is adverse on the number of varieties. But as long as such world price rise is less than proportionate to the rate of tariff reduction so that the domestic price of good $Z$ still falls at the new equilibrium (i.e., $\hat{T} + \hat{p}_Z^W < 0$), as is usually the case in a standard Heckscher-Ohlin-Samuelson model, the subsequent fall in the number of varieties only dampens the initial increase in the number of varieties due to the direct effect. Referring back to Figure 2, the labour constraint rotates downward from its new position though not all the way back to its initial position, as shown by the broken line. But $n^*$ falls in this case (see (15)).

But if the world price falls, then the TOT effect reinforces the initial effect and the increase in the number of varieties gets magnified. Moreover, the number of varieties produced and exported by the Foreign country rises as well. This has some far reaching implications for gains at the extensive margin as we will see later.

What appears from the above discussion and changes in the number of varieties produced in the two countries as specified in (15) and (17), the direction and magnitude of the change in the world price is important. The change in the world price of good $Z$ can be worked out from the world market clearing condition stated in (13). As shown in the appendix, by total differentiation of this condition and substitution of values for changes in the rate of return to capital, output levels of good $Z$ and in the number of varieties, the change in the world price of good $Z$ equals,

\[
\hat{p}_Z^W = -\frac{\left[\phi - (\phi - 1) \lambda_{IZ}^L \lambda_{IZ}^L \lambda_{IZ}^L \frac{\theta_{KZ}^*}{\theta_{KZ}^*} + \phi \right]}{\left[(\phi - 1) + \phi \frac{\theta_{KZ}^*}{\theta_{KZ}^*} \right]^{1 - \frac{\lambda_{IZ}^L \lambda_{IZ}^L \lambda_{IZ}^L}{\theta_{KZ}^*} + \phi \left[1 - \frac{\lambda_{IZ}^L \lambda_{IZ}^L \lambda_{IZ}^L}{\theta_{KZ}^*} \right] + \phi \frac{\theta_{KZ}^*}{\theta_{KZ}^*} \theta_{KZ}^*}} \hat{p}, \quad (19)
\]

where $\phi \equiv \frac{\gamma_1 / T}{(\gamma_1 / T) - \hat{p}_Z^W} > 1$, $\phi^* \equiv p_Z^W Z^* / \hat{p}_Z^W Z^* - \gamma 1^* > 1$, $\lambda_{iz}$ is the employment
share of factor $i$ in sector $j$, $\theta_K = \theta_k/(1 - \tau)$, $\theta_K \equiv r\bar{R}/l$ is the share of capital in home national income, $\theta_K^* = \theta_k^*/(1 - \gamma)$, $\theta_K^* \equiv r\bar{R}^*/l$ is the share of capital in foreign national income, $\theta_Z = p^*_Z/(1 - \gamma)l = \theta_z/(1 - \gamma)$, $\theta_Z = p^*_Z Z/l$ is the share of sector $Z$ in national income of the Home country, $\psi = (\gamma l/T) - (1 + t)p^*_Z/(1 - \gamma)l$, and $T = 1 + t$.

It is easy to check that $\hat{p}_{Z}^w$ rises if the following two (sufficient) conditions hold together:

\[ (\phi - 1) \frac{\lambda_{XZ} \lambda_{LZ}}{\lambda |\theta_K^*|} > \phi \frac{\theta_K^*}{\theta_K} \tag{20} \]

\[ -\frac{\phi \lambda_{1X} \lambda_{1K}}{\lambda |\theta_K^*|} > \phi \frac{\theta_K^*}{\theta_K} \tag{21} \]

A reduction of tariff, ceteris paribus, triggers several effects on the demand for and supply of the homogeneous good $Z$, at the initial $p_{Z}^w$, with different implications for the change in $p_{Z}^w$. First, is the output magnification effect, as mentioned above, due to excess supply of labour as the tariff reduction and consequent proportionate decline in the tariff-inclusive price at initial $p_{Z}^w$ lowers output of each existing varieties in the home country. If good $X$ is relatively labour intensive ($|\lambda| < 0$), such an excess supply of labour will lower the output of good $Z$, by the magnitude $\hat{Z} = \lambda_{XZ} \lambda_{LZ} / |\lambda| |\theta_K^*| \text{at the margin at initial } p_{Z}^w$. This adverse domestic supply effect, ceteris paribus, raises the excess demand for good-$Z$ in the home country and consequently its import demand. This effect thus serves to raise $\hat{p}_{Z}^w$. Second, the decline in the tariff-inclusive domestic price of good $Z$ increases the import demand proportionately given the assumption of constant fraction of (national) income being spent on this good. Thus, this price effect also puts an upward pressure on $p_{Z}^w$. In the above expression (19), the first two terms in the numerator capture respectively these price and supply effects of the tariff reduction on $p_{Z}^w := [\phi - (\phi - 1) \lambda_{XZ} \lambda_{LZ} / |\lambda| |\theta_K^*|]$ at the margin. Finally, the tariff reduction triggers an adverse income effect on the demand for and import of good $Z$, since it lowers national income by lowering the rate of return to capital given the fixed money wage and endowments of labour and capital, and also by lowering the tariff revenue at initial value of imports. This income effect thus lowers $p_{Z}^w$, and is captured by the term $-\phi \bar{\theta}_K / |\theta_K^*|$ at the margin. Condition (20) essentially means that the domestic supply effect is larger in magnitude than the income effect on the import demand for good $Z$. This is, however, a sufficient condition for a net upward push on $p_{Z}^w$.

There are also the secondary effect triggered by such an upward push on $p_{Z}^w$. Given (20), as tariff reduction starts raising $p_{Z}^w$, the domestic price starts rising which triggers similar secondary effects at the margin in the Home country. But now these effects work in the opposite direction of the initial effects, since the rise in $p_{Z}^w$ pulls up the tariff-inclusive domestic price in the Home country. On the other hand, the rise in $p_{Z}^w$ (which is also the domestic price of good $Z$ in the Foreign country) triggers supply, price and income effects in the Foreign country. Both the increase in output of good $Z$ and the price effect of demand for good $Z$ in the Foreign country raise its export supply,
whereas the rise in Foreign national income consequent upon an increase in the rate of return to capital there raises the local demand for good $Z$ and thus lowers the export supply. All these secondary effects in the Home and Foreign countries at the margin are shown by the terms in the denominator on the right hand side of (19), which together have a dampening effect on $P^W_Z$ if conditions (20) and (21) hold. That is, the rise in $P^W_Z$ is mitigated by these secondary effect so that at the new equilibrium $P^W_Z$ rises less than proportionate to the rate of tariff reduction and the domestic relative price falls:

$$\tilde{P}^W_Z + \tilde{T} = \frac{\phi^{*} - 1 + \frac{\theta^*_Z}{\theta^*_W} + \phi^{*} - 1 + \frac{\theta^*_Z}{\theta^*_W}}{\phi^{*} + \phi^{*} - 1 + \frac{\theta^*_Z}{\theta^*_W}} \tilde{T} < 0 \quad (22)$$

The sufficient conditions specified in (20) and (21) for $P^W_Z$ to rise, but less than proportionately, are not unusual to be satisfied. These conditions require that the supply effect of a reduction in tariff on good $Z$ must be larger than its income effect in each country. Both these effects are triggered by the change in the rate of return to capital – a fall in the Home country and a rise in the Foreign country – following a tariff reduction. But, as explained above, the domestic supply effect arises due to the output magnification effect, which is second-order large.

**Proposition 1**

Given conditions (20), (21) and that the differentiated good $X$ is relatively labour intensive, a tariff reduction raises the number of varieties produced by the Home country and thus makes its export basket more diversified, but lowers the number of foreign varieties and makes its export basket less diversified.

**Proof**

By (20), (21), $|\lambda| < 0$ and $|\lambda^*| < 0$, from (19) and (22) we have $\tilde{P}^W_Z > 0$ and $\tilde{T} + \tilde{P}^W_Z < 0$. Thus, given the expressions (15) and (17) for changes in the number of varieties in the two countries, it follows that $\tilde{n} > 0$ and $\tilde{n}^* < 0$. Q.E.D.

A reduction of tariff by the Home country changes the domestic prices of good $Z$ in the two countries – the tariff-inclusive price in the Home country ($TP^W_Z$) and the world price in the Foreign country ($P^W_Z$) – in opposite direction and accordingly leads to asymmetric changes in the number of varieties being produced.

A few comments are warranted at this point. First, the above results may hold even if the parametric values and initial conditions (such as the initial level of tariff) are such that (20) and (21) are not satisfied, because these are only sufficient conditions. Alternatively, conditions that the price effect on the demand for good $Z$ is stronger than the income effect induced by changes in domestic price in both the countries – $\theta^*_Z / \theta^*_W < 1$ and $\theta^*_Z / \theta^*_W < 1$ – can also ensure a (less than proportionate) rise in $P^W_Z$. 
Since we assume that good-Z is relatively capital intensive, it is quite possible that share of capital in its average cost of production is larger than the share of capital in national income, and thus these conditions being satisfied. Again, these are only sufficient conditions. Thus, it is unlikely that the TOT will improve ($\tilde{P}_Z^W < 0$), or that the tariff inclusive domestic price in the Home country will increase ($\tilde{T} + \tilde{P}_Z^W > 0$) when the TOT worsens ($\tilde{P}_Z^W > 0$).

Second, the increase in the number of Home varieties at the new equilibrium is less than the initial increase. This provides some theoretical support to the result of Ardelean (2006) that increase in variety in Krugman model is an overestimation. Third, to the extent to which more diversified export baskets augments output growth, as suggested by recent empirical estimates mentioned earlier, the asymmetric changes in the number of varieties produced and exported by the two countries means that they will have asymmetric growth experiences.

### 2.2. Gains at the Extensive Margin

The gains from extensive margin arises when the total number of varieties consumed by each country increases after the reduction of tariff ($\hat{n} + \hat{n}^* > 0$). But, as has been already discussed in the preceding section (Proposition 1), under the factor intensity assumption that the horizontally differentiated good is relatively labour intensive, the number of such varieties produced and exported by the Home country increases and those by the Foreign country falls. Thus, a priori we cannot say that there will be variety gains or gains at the extensive margin.

Adding the values for changes in the number of horizontally differentiated variety of good $X$ produced and exported by the Home and Foreign countries specified in (15) and (17) we get

$$\hat{n} + \hat{n}^* = c\tilde{T} + d\tilde{P}_Z^W, \quad (23)$$

where $c = \frac{\lambda_{KZ}\lambda_{LX}}{|\lambda|\theta_{KZ}}$ and $d = \frac{\lambda_{KZ}\lambda_{LX}}{|\lambda|\theta_{KZ}} + \frac{\lambda_{KZ}\lambda_{LX}}{|\lambda^*|\theta_{KZ}} = c + \frac{\lambda_{KZ}\lambda_{LX}}{|\lambda^*|\theta_{KZ}}$. Given the factor intensity assumption that the production of the differentiated varieties is relatively labour intensive in both the countries $|\lambda|$ and $|\lambda^*|$ are negative. Further, $|\lambda^*| < 0$ implies $|c| < |d|$.

From (23) it then appears that a less than proportionate increase in $P_Z^W$ (or worsening of the TOT for the Home country) is not sufficient for an increase in the total number of varieties being traded and consumed in both the countries. The tariff reduction raises the total number of varieties ($\hat{n} + \hat{n}^* > 0$) only when it does not worsen the TOT too much in the following sense:

$$\tilde{P}_Z^W < -\frac{\lambda_{KZ}\lambda_{LX}}{|\lambda|\theta_{KZ}} \tilde{T} < -\tilde{T}. \quad (24)$$
Thus, we need that the TOT effect must not be large and accordingly more stringent conditions than the two sufficient conditions (20) and (21) that only ensure that the world price of good rises but less than proportionate to the fall in tariff ($\bar{p}_1^y < \bar{F}$). As shown in the appendix, if we assume that that the price effect on the demand for good is stronger than the income effect on it induced by a tariff reduction – $\theta_K/\theta_{KZ} < 1$ and $\theta_K^*/\theta_{KZ}^* < 1$– then a necessary (though may not be a sufficient) condition for a smaller TOT effect in the sense defined in (24) above, and consequently for variety gains to be realized ($\bar{n} + \bar{n}^* > 0$), is that at the initial equilibrium we must have:

$$\frac{\phi^*}{\phi-1} > \frac{\lambda_{KX}\lambda_{KZ}}{\lambda_{KZ}\lambda_{KX}}.$$  

(25)

But this is not guaranteed a priori. Since by definition, $\phi$ depends on the initial tariff, so we can say at most that the initial tariff rate matters. In sum, the aggregate number of varieties may increase or decrease as a consequence of tariff reduction, so that variety gains may or may not be realized.

As shown in the appendix, condition (25) can be reduced to

$$n^* > n.$$  

(25a)

To see the plausibility of this condition, recall the assumption of the same fixed wage in the two countries. Thus, if there had not been any tariff imposed by the Home country on imports of good from the Foreign country ($T = 1$), producers of good in both countries having identical technology would have chosen the same capital-output ratio so that $\alpha_{KZ}/\alpha_{KZ} = 1$. Since, an import tariff lowers the world price of good $Z$ but raises the tariff-inclusive price of good $Z$, so by the zero-profit conditions the rate of return to capital will be higher in the Home country and lower in the Foreign country for any positive tariff rate ($T > 1$). But due to the assumption of fixed coefficient production technology, still we have $\alpha_{KZ}/\alpha_{KZ} = 1$. Referring back to Figure 1, the resource constraint in each country would mean a consequent larger output of good $Z$ in the Home country and a smaller output of good $Z$, making it less likely that the Foreign country will be an exporter of good $Z$. In other words, our presumption that the Foreign country is an exporter of good $Z$ likely to be more consistent with $n > n^*$ at the initial equilibrium, in which case it is not possible for condition (25a) to be satisfied. In sum, it is less likely that a tariff reduction will lead to gains at the extensive margin.

10 Of course, given identical taste, the Foreign country can still have an excess supply of good $Z$ and therefore export it if its national income is smaller than that of the Home country. This in turn requires specific assumptions regarding sizes of the labour force and capital stocks in the two countries.
Proposition 2

The variety gains from trade liberalization in the sense that \( \hat{n} + \hat{n}^* > 0 \) may not be realized. Given conditions (20) and (21), and that the price effects are stronger than the income effects on the demand for good \( Z \) in both countries, the variety gains from trade liberalization may be realized if condition (25a) is satisfied.

Proof

See Appendix A.3. Q.E.D.

The above result is similar to what has been shown by Arkolakis et al (2008). But, in contrast to their analysis, the forces at work behind the alternative possibilities here are more in the HOS tradition. The differences in factor endowment and in fixed money wages in the two countries appear to be of critical importance.

3. FACTOR INTENSITY CONDITION, POLICY IMPLICATIONS AND ROBUSTNESS

The above results are derived under the assumption that the differentiated manufacturing good \( X \) is labour intensive relative to the homogeneous good \( Z \). In this section we briefly discuss its relevance and appropriateness, and policy implications thereof. Some robustness issues are also discussed.

3.1. Factor Intensity Condition and Policy Implications

Relatively larger labour intensity of the differentiated manufacturing good \( X \) may make sense once we think of this good being different varieties of apparel or leather manufacture. Such goods typically require more labour than capital. Moreover, the varieties considered here being horizontal in nature, that is being differentiated from each other only in terms of their superficial characters (such as design) rather than in terms of their intrinsic quality (such as texture in case of apparels and superior tanning in case of leather manufacture), do not require a very high amount of capital per product. That is, the varieties that we consider involve small fixed (capital) cost but significant variable (labour) cost. On the other hand, if we consider the homogeneous good \( Z \) as refined vegetable oil, heavy manufacturing like iron and steel, petroleum products and the like, then it may not be at odd to assume that this good is relatively capital intensive.

In situations where the assumed factor intensity condition does not hold, and the differentiated manufacturing good is relatively capital intensive requiring larger amounts of capital to develop blueprint or design of a differentiated variety, by its initial or direct effect a tariff reduction lowers the number of varieties in the Home country. This is
evident from (15) for $|\lambda| > 0$. This is because the output magnification effect triggered by availability of labour due to contraction of the scale of production of the initial number of varieties work in the opposite direction. Excess labour now expands the production of Z-good and the additional capital required for such an increase in the output requires fall in the number of varieties being produced in the Home country. As evident from (19) and (22), the TOT is again likely to rise (though conditions (20) and (21) are no longer relevant), but less than proportionately. Consequently, the induced and subsequent effects of a tariff reduction are not substantial enough to reverse its initial adverse effect of tariff on the number of varieties produced and exported by the Home country. The number of Foreign variety, on the other hand, rises now. The asymmetric result holds again but in the opposite direction.

The policy implication in such a scenario where good X is relatively capital intensive ($|\lambda| > 0$) is thus to raise the tariff rate instead of lowering it, if achieving larger number of varieties and making export basket more diversified is the policy target from a long run perspective as mentioned earlier. Thus,

**Proposition 3**

The policy target of a more diversified export basket can be achieved by the Home country through a tariff reduction if the differentiated good is relatively labour intensive. Otherwise, raising the tariff rate will be the appropriate policy.

**Proof**

Follows from Proposition 1 and the discussion above. \textbf{Q.E.D.}

However, under the same set of assumptions along with the differentiated good being relatively labour intensive, a tariff reduction is more likely to cause a smaller number of varieties being consumed at Home through domestic production and imports. This reveals a possible trade-off between growth objective and gains at the extensive margin. Similar trade off may exist when the differentiated good is relatively capital intensive, but now gains at the extensive margin can be realized along with a less diversified export basket and consequently a lower growth rate in the long run.

### 3.2. Robustness

We now examine how far our results derived above depend on the assumptions of constant price elasticity of demand for each variety.

For a large class of demand functions (including a linear demand function), the absolute value of price elasticity falls as more and more quantities are demanded and produced. Thus, we can write,
\[ \varepsilon = \varepsilon(x), \text{ and } \dot{\varepsilon} = -\mu \dot{x}, \]  
\[ \hat{\beta} = -\frac{1}{\varepsilon - 1} \dot{x}. \]  

where \( \mu > 0 \) is the percentage change in the absolute value of the price elasticity when quantity demanded and output per variety rises by one percent. On the other hand, it is easy to check from (3) that the price of each variety will decline with an increase in the value of the price elasticity:

\[ \hat{\beta} = -\frac{1}{\varepsilon - 1} \dot{x}. \]  

This is obvious because the more price elastic the demand function is, less will be the monopoly power of the firm over its produced variety and accordingly the mark-up over (constant) wage cost will be smaller. It is easy to check then that the decline in output per variety is less than proportionate to the decline in the rate of return to capital brought about by a tariff reduction:

\[ \dot{x} = \Omega \hat{x}, \]  

where \( \Omega = \theta_{xx}/(\theta_{ex} + \mu/1 - \varepsilon) < 1 \).

It immediately follows then that the initial effect of a tariff reduction will be smaller than when the price elasticity is constant. This is because, with the fall in scale contraction being lower, less amount of labour will be made available for expansion of the number of varieties \( n \) by the output magnification effect when good \( X \) is relatively labour intensive. Though, as shown in the appendix, the reduction of tariff will still raise \( P^w_z \) less than proportionately. Thus, the number of varieties does increase at the final equilibrium, but the magnitude of such increase will be smaller because of the smaller initial expansion, than when the price elasticity is constant. That is, Proposition 1 remains the same. There is only the magnitude difference.

### 3.3. Numerical Example

To exemplify the results discussed above in Propositions 1 and 2, consider the parametric configurations in Table 1. We consider values of labour and capital endowments such that the Home country is relatively labour intensive. The values of input coefficients, on the other hand, are taken in a manner so that good \( Z \) requires less labour per unit of output than the per unit of output of each variety of good \( x \) requires. But the capital requirement per unit of output of good \( Z \) is the same as the capital requirement for each variety. Thus, the values of \( \alpha_{lz}, \alpha_{kz}, \beta \) and \( \rho \) are chosen in consonance with the assumption that good \( Z \) is relatively capital intensive. Given these parametric configurations, three sets of values of price elasticity of demand for each variety \( \varepsilon \) and the rate of tariff are considered to find out the equilibrium value of \( P^w_z \), and number of varieties produced in the two countries. Variations in the rate of tariff and corresponding changes in the equilibrium value of \( P^w_z \), for any given value of \( \varepsilon \), verify
Proposition 1. For the relevant ranges of $P_Z^W$ for which both countries produce strictly positive number of varieties, we consider only the value at Walrasian-stable equilibrium wherever there are multiple equilibrium values of $P_Z^W$. The relative number of varieties produced by the two countries at the initial (stable) equilibrium, on the other hand, verifies whether condition (25a) for variety gains to be realized (see Proposition 2) is satisfied or not.

<table>
<thead>
<tr>
<th>Value of demand elasticity</th>
<th>Rate of Tariff</th>
<th>Range of $P_Z^W$ for which $n &gt; 0$ and $n^* &gt; 0$</th>
<th>Range of $P_Z^W$ for which $P_Z^W$ (if it exists)</th>
<th>Walrasian stable equilibrium value of $P_Z^W$</th>
<th>Possibility of Variety Gains after tariff cut in terms of initial condition (25a): $n - n^* &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon = 1.5$</td>
<td>$t = 0.5$</td>
<td>$P_Z^W \in (20,30)$</td>
<td>$P_Z^W &lt; 24$</td>
<td>Solution does not exists for parametric values defined above</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t = 1$</td>
<td>$P_Z^W \in (15,30)$</td>
<td>$P_Z^W &lt; 20$</td>
<td>$P_Z^W = 17.36$</td>
<td>Not likely</td>
</tr>
<tr>
<td></td>
<td>$t = 2$</td>
<td>$P_Z^W \in (10,30)$</td>
<td>$P_Z^W &lt; 15$</td>
<td>$P_Z^W = 12.94$</td>
<td>Not likely</td>
</tr>
<tr>
<td>$\varepsilon = 2.5$</td>
<td>$t = 0.5$</td>
<td>$P_Z^W \in (11,16.6)$</td>
<td>$P_Z^W &lt; 13.3$</td>
<td>$P_Z^W = 12.6$</td>
<td>Not likely</td>
</tr>
<tr>
<td></td>
<td>$t = 1$</td>
<td>$P_Z^W \in (8,3,16.6)$</td>
<td>$P_Z^W &lt; 10.1$</td>
<td>$P_Z^W = 9.11$</td>
<td>Not likely</td>
</tr>
<tr>
<td></td>
<td>$t = 2$</td>
<td>$P_Z^W \in (5,5,16.6)$</td>
<td>$P_Z^W &lt; 8.3$</td>
<td>$P_Z^W = 6.14$</td>
<td>Not likely</td>
</tr>
<tr>
<td>$\varepsilon = 3$</td>
<td>$t = 0.5$</td>
<td>$P_Z^W \in (10,15)$</td>
<td>$P_Z^W &lt; 12$</td>
<td>$P_Z^W = 10.98$</td>
<td>Not likely</td>
</tr>
<tr>
<td></td>
<td>$t = 1$</td>
<td>$P_Z^W \in (7,15)$</td>
<td>$P_Z^W &lt; 10$</td>
<td>$P_Z^W = 8.08$</td>
<td>Not likely</td>
</tr>
<tr>
<td></td>
<td>$t = 2$</td>
<td>$P_Z^W \in (5,15)$</td>
<td>$P_Z^W &lt; 7.5$</td>
<td>$P_Z^W = 5.42$</td>
<td>Not likely</td>
</tr>
</tbody>
</table>

Note: Solution values are obtained by using Scientific Workplace 5.5.

From the above numerical solutions, the following observations are in order. First, for all the three values of $\varepsilon$, lower rate of tariff means a less than proportionate higher equilibrium value of $P_Z^W$. That is, at least for the set of parametric configuration consistent with Home country being relatively labour rich and good $Z$ being relatively capital intensive, reductions in the rate of tariff mean $P_Z^W > 0$ and $\hat{T} + \hat{P}_Z^w < 0$ so that $n > 0$ and $n^* < 0$ as stated in Proposition 1. Second, for at least two values of the tariff $-t = 1$ and $t = 2$ at the initial equilibrium $n - n^* > 0$. That is, condition (25a) is not satisfied meaning that reduction of tariff rates from either of these initial rates would most likely lower the total number of varieties $n + n^*$, so that the variety gains from tariff reductions may not be realized (see Proposition 2). For $\varepsilon = 2.5$ and $\varepsilon = 3$, this result holds even for a very small initial rate of tariff $t = 0.5$. Third, the variability in $\varepsilon$ does not change the results qualitatively as stated in sub-section 3.2.
4. CONCLUSION

The paper examines the impact of trade liberalization on export diversification through its resource reallocation and TOT effects in a synthesis of HOS and the Krugman (1979, 1980) models by considering the variety effect in the HOS model and TOT effect in the Krugman model. Under the factor intensity assumption that the differentiated good is relatively labour intensive it is seen that the reduction of import tariff on the homogeneous good by the Home country increases its number of varieties and lowers those of the Foreign country. The TOT worsens for the Home country but less than proportionate to the rate of tariff reduction so that the increase in the number of domestic varieties is less than without the TOT effect (such as in case of a small country). This is supportive of the result of Ardelean (2006) that the variety gains in the Krugman model may be an overestimation. Overall, however, there will be gains at the extensive margin in terms of increased availability of total variety of the differentiated good.

On the other hand, when the differentiated good is relatively capital intensive, the policy target of a more diversified export basket can be achieved by the Home country by raising the tariff rate, irrespective of whether the Home country is large or small.

One possible extension of the paper would be to endogenize the pattern of trade. Instead of assuming the trade pattern ex ante we can begin with the autarchic equilibrium and study how the pattern of trade itself depends on the factor endowment of countries. Moreover, the numerical exercises done in sub-section 3.3 for verifications and exemplifications of results stated in Propositions 1 and 2 are contingent upon parametric configurations consistent with relative labour abundance of Home country and relative labour-intensity of the differentiated good. It might be interesting to see how the results change for alternative assumptions of factor abundance and factor intensity as indicated in Proposition 3. We put all these in our future research agenda.

APPENDIX

A.1. Number of Varieties and Size of Sector X

The total demand for \( n \) and \( n^* \) varieties in H-country are \( n(L + K)c \) and \( n^*(L + K)c \) respectively, where \( c \) is the unit of each variety consumed by each Home consumer. So the total expenditure on all varieties consumed is \( Pn(L + K)c + P^*n(L + K)c \) and the share of foreign varieties in the total expenditure on horizontally differentiated goods by all home consumers is

\[
\frac{P^*n'(L+K)c}{(Pn+P^*n')(L+K)c} = \frac{P'n'^*}{(Pn+P^*n'^*)} = \frac{\frac{\varepsilon}{\varepsilon_1}\tilde{w}n^*}{\frac{\varepsilon_1}{\varepsilon}\tilde{w}n + \frac{\varepsilon}{\varepsilon_2}\tilde{w}^*n^*} = \frac{\tilde{w}n^*}{\tilde{w}n + \tilde{w}^*n^*} = \frac{n^*}{n + n^*} \equiv \alpha,
\]
Similarly, the share of home varieties in total expenditure on horizontally differentiated goods by all foreign consumers is \( n/n + n' \equiv \alpha' = (1 - \alpha) \). Using these shares, we can write the aggregate value of demand for Home varieties or the size of its X-sector \( S_X \) in terms of value of production as,

\[
S_X = \frac{n}{n+n'}(1 - \gamma)I + \frac{n}{n+n'}(1 - \gamma)I' = \frac{n}{n+n'}(1 - \gamma)(I + I').
\]  \hspace{1cm} \text{(A.1)}

Similarly,

\[
S_X' = \frac{n'}{n+n'}(1 - \gamma)I + \frac{n'}{n+n'}(1 - \gamma)I' = \frac{n'}{n+n'}(1 - \gamma)(I + I').
\]  \hspace{1cm} \text{(A.2)}

It is immediate then that \( S_X > S_X' \) if \( n > n' \).

### A. 2. Effect of Tariff Reduction

Totally differentiating the zero-profit condition (1) in the traditional sector yields:

\[
TdP^w_z + P^w_z dT = \alpha_{KZ} dr \Rightarrow \hat{P}^w_z + \hat{T} = \theta_{KZ} \hat{\rho},
\]  \hspace{1cm} \text{(A.3)}

where \( \theta_{KZ} = \alpha_{KZ} r/P^w_z \) is the capital cost share in unit production in the traditional sector, thus \( \hat{\rho} = \hat{P}^w_z + \hat{T}/\theta_{KZ} \).

Totally differentiating price equal to average cost condition (2) gives:

\[
dP_{x_i} = \frac{\rho}{x_i} dr - \frac{\rho r}{x_i^2} dx_i \Rightarrow \tilde{P}_{x_i} = \frac{\rho r}{p_{x_i x_i}} (\hat{\rho} - \tilde{x}_i).
\]

With constant demand elasticity, \( \tilde{P}_{x_i} = 0 \) from condition (3) which implies

\[
\tilde{x}_i = \hat{\rho}.
\]  \hspace{1cm} \text{(A.4)}

So from these two conditions:

\[
\tilde{x}_i = \hat{\rho} = \frac{\rho^w + \hat{T}}{\theta_{KZ}}
\]  \hspace{1cm} \text{(A.5)}

Total differentiation of the full employment condition of labour in Home country yields:

\[
\alpha_{KZ} dZ + \rho dn = 0 \Rightarrow \lambda_{KZ} \ddot{Z} + \lambda_{KX} \dot{n} = 0.
\]  \hspace{1cm} \text{(A.8)}

(A.7) and (A.8) can be written as matrix form in the following way:
Using Cramer’s rule the equilibrium changes in the number of varieties of the differentiated goods and the output of the homogeneous goods are given by:

\[
\hat{n} = \frac{\lambda_{KZ} \lambda_{LX}}{|\lambda| \theta_{kz}}, \quad (A.9)
\]

\[
\hat{Z} = \frac{\lambda_{KZ} \lambda_{LX}}{|\lambda| \theta_{kz}}, \quad (A.10)
\]

\[
\hat{n}^* = \frac{\lambda_{KZ} \lambda_{LX}}{|\lambda| \theta_{kz}}, \quad (A.11)
\]

\[
\hat{Z}^* = \frac{\lambda_{KZ} \lambda_{LX}}{|\lambda| \theta_{kz}}, \quad (A.12)
\]

Total differentiation of the world market clearing condition (13) in the text yields:

\[
\frac{\gamma}{T} dI - \frac{\gamma}{T^2} dT - (P_{2w} Z)(\bar{P}_{Z} + \bar{Z}) = (P_{2w} Z^*)(\bar{P}_{Z} + \bar{Z}^*) - \gamma dI
\]

\[
\Rightarrow \frac{\gamma}{T} [\bar{I} + \bar{T}] - (P_{2w} Z)(\bar{P}_{Z} + \bar{Z}) = (P_{2w} Z^*)(\bar{P}_{Z} + \bar{Z}^*) - \gamma I^* \bar{I}
\]

\[
\Rightarrow [\frac{\gamma}{T} - P_{2w} Z] \left[ \left( \frac{\gamma}{T} - P_{2w} Z \right) \left( \bar{I} - \bar{T} \right) - \left( \frac{P_{2w} Z}{T - P_{2w} Z} \right) \left( \bar{P}_{Z} + \bar{Z} \right) \right]
\]

\[
=[P_{2w} Z^* - \gamma I^*] \left( \frac{P_{2w} Z^*}{P_{2w} Z^* - \gamma I^*} \left( \bar{P}_{Z} + \bar{Z}^* \right) - \frac{\gamma I^*}{P_{2w} Z^* - \gamma I^*} \right)
\]

\[
= \phi [\bar{I} - \bar{T}] - (\phi - 1) (P_{2w} Z^* + \bar{Z}) = \left[ \phi^* \left( \bar{P}_{Z} + \bar{Z}^* \right) - (\phi^* - 1) \right].
\]

Now, from the definitions of I and I* in the text, it is easy to check that,

\[
I = \frac{\theta_k}{(1 - \varphi)} \bar{r} - \frac{\theta_p}{(1 - \varphi)} \left( \bar{Z} + \bar{P}_{2w} \right) + \psi T, \quad I = \theta_k^* \bar{r}^*,
\]

where \( \theta_k \equiv rR/I \) and \( \psi = (\gamma l/T) - (1 + t)P_{2w} Z/(1 - \gamma r)I \). Substitution of these values yield,

\[
\left[ \phi \left( \bar{r} + (\psi - 1) \bar{T} \right) - (\phi - 1) + t \phi \bar{r} \right] \left( \bar{P}_{Z} + \bar{Z} \right)
\]

\[
= \left[ \phi^* \left( \bar{P}_{Z} + \bar{Z}^* \right) - (\phi^* - 1) \theta_k^* \bar{r}^* \right]
\]

\[
= \phi \left[ \theta_k \left( T - P_{2w} \right) + (\psi - 1) \bar{T} \right] - (\phi - 1) + t \phi \bar{r} \left[ \frac{\lambda_{LZ} \lambda_{KX}}{|\lambda| \theta_{kz}} \left( T - P_{2w} + P_{2w} \right) \right]
\]
\[
\hat{\beta}_Z = -\frac{\left[\phi^{+} (\phi - 1) \lambda_{KX}^2 \phi K + \phi K \theta K - (\phi^+ - 1) \frac{\theta K}{\theta K} \hat{\beta}_Z \right]}{\left[\left((\phi-1)+i\theta_{\beta}\phi\right)\left[1-\frac{\lambda_{KX}^2}{|\lambda|^{\theta_{KZ}}}-\phi\frac{\theta K}{\theta K} - (\phi^+ - 1) \frac{\theta K}{\theta K}\right]\right]^\top}.
\]

(A.13)

A.3. Variety Gains

Adding (A.9) and (A.11) we get,
\[
\hat{n} + \hat{n}^* = \frac{\lambda_{KZ} \lambda_{XX}(\hat{\pi} + \hat{\pi}^y)}{|\lambda| \theta_{KZ}} + \frac{\lambda_{KZ} \lambda_{XY} \theta Y}{|\lambda| \theta_{KZ}},
\]

which upon substitution of values from (22) in the text and (A.13) above boils down to,
\[
\hat{n} + \hat{n}^* = \frac{\lambda_{KZ} \lambda_{XX}(\hat{\pi} + \hat{\pi}^y)}{|\lambda| \theta_{KZ}} + \frac{\lambda_{KZ} \lambda_{XY} \theta Y}{|\lambda| \theta_{KZ}},
\]

\[
= \frac{\lambda_{KZ} \lambda_{XX}}{|\lambda| \theta_{KZ}} \left(\phi^+ - 1\right) + \frac{\theta K^2}{\theta K} + \phi^+ \left\{\left(\frac{\lambda_{KZ} \lambda_{XX}}{|\lambda| \theta_{KZ}} - \frac{\theta K}{\theta K}\right)\right\} \frac{\theta K^2}{\theta K} \hat{\beta}_Z.
\]

(A.14)

Since the denominator is positive by (20) and (21), so for \(\hat{n} + \hat{n}^* > 0\) we require the numerator to be negative. If we assume that the price effect on the demand for good \(Z\) is stronger than the income effect on it induced by a tariff reduction \(-\theta_{KZ} < 1\) and \(\theta K^2 / \theta K^2 < 1\) then the first term in the numerator is negative whereas the third term is positive. In that case, a necessary (though may not be a sufficient) condition for the value in the numerator being positive and consequently variety gains to be positive \((\hat{n} + \hat{n}^* > 0)\) is that at the initial equilibrium we must have:
which is condition (25) in the text.

Now by definition, we can rewrite the above condition as,

$$\frac{\frac{n_a^*Z^*}{\frac{n^*a_{KZ}^*}{n^*a_{KZ}^*}}}{\frac{1}{\gamma l}} = \frac{n_a^*Z^*}{\frac{n^*a_{KZ}^*}{n^*a_{KZ}^*}}$$

By the initial trade balance condition $\gamma I - TP_Z^W Z/T = Z^W Z^* - \gamma I^*$ this boils down to,

$$\frac{\frac{n_a^*Z^*}{\frac{n^*a_{KZ}^*}{n^*a_{KZ}^*}}}{\frac{1}{\gamma l}} > \frac{n_a^*Z^*}{\frac{n^*a_{KZ}^*}{n^*a_{KZ}^*}} \frac{\frac{n^*a_{KZ}^*}{n^*a_{KZ}^*}}{\frac{n^*a_{KZ}^*}{n^*a_{KZ}^*}} = 1,$$

which is condition (25a) in the text.

**A.4. Variable Elasticity of Demand**

Given (26) and (27), from the zero profit condition we get

$$\hat{\beta} = \theta_{KX}(\hat{\sigma} - \hat{\xi}) \Rightarrow \frac{\theta_{KX}(\hat{\sigma} - \hat{\xi})}{\hat{\sigma} - \hat{\xi}} = \theta_{KX}(\hat{\sigma} - \hat{\xi}) \Rightarrow \hat{\sigma} = \frac{\theta_{KX}}{\theta_{KX} + \frac{\theta_{KX}}{\hat{\sigma} - \hat{\xi}}}. \quad (A.15)$$

Using this and proceeding as before, the change in the price of good $Z$ can be obtained as,

$$\hat{p}_Z^W = -\frac{[\phi - \alpha(\phi - 1)\Lambda_{KX}^p \phi_{KZ}^p + \phi]}{[(\phi - 1) + \alpha_{KZ}^p \phi_{KZ}^p] + \phi \left[1 - \alpha_{KZ}^p \phi_{KZ}^p \right] + \phi \left[1 - \alpha_{KZ}^p \phi_{KZ}^p \right] - \phi \left[1 - \alpha_{KZ}^p \phi_{KZ}^p \right] - \phi \left[1 - \alpha_{KZ}^p \phi_{KZ}^p \right]} \hat{\sigma}. \quad (A.16)$$

Though now we require more stringent conditions than those specified in (20) and (21) in the text, the alternative sufficient condition that the price effect be stronger than the income effect still ensures $P_Z^W$ to rise, but less than proportionately.
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