

SEASONAL FLUCTUATIONS AND ECONOMIC GROWTH

HERNANDO ZULETA*

Universidad de los Andes, Colombia

The industrial revolution and the subsequent industrialization of the economies occurred first in temperate regions. We argue that this and the associated positive correlation between absolute latitude and GDP per capita are due to the fact that countries located far from the equator suffered more profound seasonal fluctuations in climate, namely stronger and longer winters. We propose a growth model of biased innovations that accounts for these facts and show that countries located in temperate regions were more likely to create or adopt capital-intensive modes of production.

The intuition behind this result is that savings are used to smooth consumption; therefore, in places where output fluctuations are more profound, savings are bigger. Because the incentives to innovate depend on the relative supply factors, economies where savings are bigger are more likely to create or adopt capital-intensive technologies.

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1. INTRODUCTION

The industrial revolution occurred first in temperate regions and, since then, the world has experienced a massive absolute divergence in the distribution of incomes across countries. Indeed, today there is a positive correlation between absolute latitude and GDP per capita. Moreover, only three tropical economies (Hong Kong, Singapore, and Taiwan) are classified as high-income by the World Bank, while all countries within regions zoned as temperate had either middle or high-income economies.

We argue that climatic conditions affect incentives to save in primitive economies in such a way that economies located in temperate regions are more likely to become capital abundant. In primitive economies the main source of output fluctuations is climatic and these are bigger in places located far away from the equator. Indeed, both the harvest per year and the diversification of crops are higher in tropical countries (see

* All errors are those of the author.

Chang, 1997). During the frost days in winter there is no harvest, transportation is difficult, people need more energy and, in general, surviving demands much more work. The response of people to these natural forces is saving during the good days to make bad days better. The stronger the winter the higher the level of savings needed to survive during the frost days. Finally, economies where the savings are systematically bigger are more likely to both, create and adopt capital intensive technologies. Once an economy begins to use capital intensive technologies a process of capital accumulation and capital improvement starts. Capital abundance generates the incentives for capital-using innovations and capital-using technologies generate the incentives for capital accumulation.

Many endogenous growth models predict poverty traps, that is, depending on the initial abundance of capital economies can present long run growth or be trapped in a steady state with no economic growth.¹ In particular, according to the literature of biased technological change the incentives to innovate depend on the relative supply of factors: in capital abundant economies individuals have incentives to make capital-using innovations while in economies where capital is scarce there are no incentives for this type of innovation.² These innovations increase the elasticity of output with respect to capital as well as the incentives to save, so a virtuous circle derives capital abundant economies to long-run growth. Models of factor saving innovations generally predict that both the elasticity of output with respect to capital and the capital income share must be higher in richer economies. However, in general these models do not try to explain differences in initial conditions. Among these lines, Peretto and Seater (2006) present a model where the saving rate is exogenous and prove that differences in this variable can explain long run income differences. In other words, Peretto and Seater (2006) find that the key variable beneath the initial conditions is the saving rate. We contribute to this literature, explaining why, in primitive economies, the saving rate is correlated with the strength and the duration of seasonal fluctuations. Our model is consistent with the results of Bloom, Canning and Sevilla (2003), who found that the probability of being in a high-income-level equilibrium is higher in temperate countries. Seasonal fluctuations have been economically important in the past, when people were more dependent on nature. Communities with higher savings rates were more likely to adopt capital-using

¹ For a complete review of the topic of poverty traps see Azariadys and Stachurski (2005).

² According to the literature of biased technological change the incentives to innovate depend on the relative supply of factors: in capital abundant economies individuals have incentives to make capital-using innovations while in economies where capital is scarce there are no incentives for this type of innovation (Zeira, 1998 and 2006; Acemoglu, 2002; Boldrin and Levine, 2002; Zuleta, 2008; Peretto and Seater, 2006 among others). These innovations increase the elasticity of output with respect to capital as well as the incentives to save, so a virtuous circle derives capital abundant economies to long-run growth. Models of factor saving innovations generally predict that both the elasticity of output with respect to capital and the capital income share must be higher in richer economies.

innovations and such innovations had permanent effects in the economic growth path because of the relation between capital abundance and innovations described above. Nowadays, living infrastructure and new technologies of production have reduced the need of savings to smooth consumption but the effect of seasons on savings and GDP can help to explain the relation between geography and income in the present.

Summarizing, since in primitive economies savings depended on seasonal fluctuations, communities located in temperate regions were more likely to adopt capital- using technologies and, as stated before, after the adoption of a capital intensive technology a virtuous circle derived the economy to long-run growth. Agents can respond to seasonal changes in output in different ways: (i) using financial institutions that help people smooth consumption and allocate resources to productive projects, (ii) increasing trade with economies where the timing of seasons is different, or (iii) inventing or adopting machines that allow people to produce goods using less quantities of other factors, namely, land and raw labor. These three activities have positive effects on economic growth, and can be undertaken simultaneously and, in many times, are complementary. In addition, any of these activities can be interpreted as capital-using innovations. However, we want to stress the role of machines for two reasons: first, the industrial revolution was characterized by new ways of production that made use of machines and reduced the need of land; second, high machinery investment has generated rapid economic growth over the 19th century in Canada, Germany, Italy, Japan, the United Kingdom and the United States, and a similar association holds since World War II for a broader sample of nations (De Long, 1992). Thus, the role of machines seems to be important to explain both, the beginning of the industrial era and the subsequent economic growth of industrialized countries.

We refer to capital goods and capital-using innovations as goods and technologies used in the production process. Durable goods and technological advances that are not used to produce marketable goods are considered as consumption goods. This distinction is important because long before the industrial revolution communities located in tropical regions developed techniques and built facilities which increased their welfare but were not used to produce new goods.

Similarly, we assume that any capital intensive technology can be adopted paying a cost and agents decide whether or not to pay the cost. Therefore, capital-using modes of production are only adopted where agents have economic incentives to pay for them. Innovations like the ones that led to the industrial revolution had been made much earlier and, in many times, outside Europe. We argue that these innovations were not used for production before because a minimum level of capital was needed in order to make profitable the introduction of the new techniques. Similarly, the reason why some economies used the innovations for productive purposes and others did not was the difference in factor abundance. If this hypothesis is correct then the economies where industrialization occurred first had higher capital labor ratios. We do not have a direct measure of capital stocks before the industrial revolution. However, according to Maddison (2003) GDP per capita was higher in the countries where industrialization

occurred first: UK, Belgium and Netherlands (see Table 1). Therefore, under reasonable assumptions, we can infer that the capital-labor ratio was higher in UK, Belgium and Netherlands than in the rest of Europe.

Table 1. GDP Per Capita in Western Europe

	1700	1820
Netherlands	2.130	1.838
United Kingdom	1.250	1.706
Belgium	1.144	1.319
Italy	1.100	1.117
Denmark	1.039	1.274
Austria	993	1.218
Sweden	977	1.198
France	910	1.135
Germany	910	1.077

Source: Maddison (2003).

According to the Unified Theory of Growth, human history is divided into three growth regimes, namely, a “Malthusian Regime”, a “Post-Malthusian Regime”, and a “Modern Growth Regime” (see Galor and Weil, 2000). The first two regimes are separated by an acceleration of technological progress while the latter two are separated by a demographic transition, driven by the micro fundamentals of utility maximizing fertility behavior.^{3,4} The demographic transition, on its turn, was the result of human

³ Galor and Moav (2002) highlight the long-run consequences of the constant interplay between Darwinian and Malthusian forces. Their model contains four essential ingredients: (i) a Malthusian element that explains the persistence of low per capita incomes over most of human history; (ii) a Darwinian evolutionary element, that highlights the slowly changing genetic composition of the population over thousands of years; (iii) an element relating the evolution of human species to human capital accumulation and technological progress; and finally, (iv) the acceleration of technological progress generates a demographic transition during the second phase of the Industrial Revolution.

⁴ Gregory Clark provides an explanation of human history based on Darwinian natural selection (see Clark and Hamilton, 2006 and Clark, 2007). According to Clark, in England, during the six hundred year period prior to the Industrial Revolution, the richest families had twice as many surviving children as the poorest. Because this was the Malthusian era of stagnation in per capita incomes, “the superabundant children of the rich had to, on average, move down the social hierarchy.” As a result of this downward social mobility, the survival of the richest demographic mechanism propagated and embedded within the English population the bourgeois virtues of thrift, hard work, education and patience.

capital accumulation (Issa, 2005).

In terms of the Unified Theory of Growth, this paper can explain why, during the Malthusian Regime societies located in temperate regions accumulated more capital than societies located in the tropics and, for this reason, helps to explain the acceleration of technological progress that separates the Malthusian Regime and the Post-Malthusian Regime. For the sake of simplicity, we assume constant and homogenous population so we cannot say anything about the demographic transition.

The model we present cannot account for exact timing of the industrial revolution and cannot explain why it occurred first in Great Britain. However, our story is consistent with some sensible explanations. Voth and Voigtländer (2006) explain why the industrial revolution occurred in Great Britain using a probabilistic two sector model. They argue that weather-induced shocks to agricultural productivity were the ultimate causes of the industrialization. As it will become apparent, in our model an exogenous shock to agricultural productivity generates an increase in savings and may trigger the industrial revolution. Joel Mokyr traces the intellectual roots of the Industrial Revolution to important changes in the method and culture involved with the creation and dissemination of new knowledge (see Mokyr, 2005a; 2005b and 2007). The scientific method that evolved in Western Europe before the industrial revolution was crucial for the emergence of new ways of production. Open science and verification, rewarded by fame and recognition became part of the “Industrial Enlightenment”. The idea that human progress was desirable created a climate supportive of new technological developments. In the same way, the enlightenment facilitated institutional change that lowered tolerance of rent-seeking activities and predation by the state.⁵ In terms of our model, the new scientific discoveries and inventions are the introduction of a new set of technologies and the decline in population is an increase in the capital labor ratio. As it will become apparent, both events can trigger the adoption of capital intensive technologies. Therefore, the idea of seasons and growth can be perfectly consistent with these broader explanations.

⁵ Other scholars identify the ultimate causes of the industrial revolution in the Crusades which re-established traffic between the East and West after having been suspended for several centuries. Along with trade, new scientific discoveries and inventions made their way from east to west and the Arabic advances including the development of algebra, optics, and refinement of engineering arrived to Europe. The new knowledge coming from the east contributed to the development of usable techniques of production that impulsed the Industrial Revolution (Sabato, 1991). Additionally, the Crusades exposed European people to new germs and viruses coming from eastern countries. As a result, new plagues with destructive social effects devastated European cities. Now, the decline in population increased the amount of food available per capita, raised nutritional status and population growth. Population growth, on its turn, fostered urbanization, knowledge creation and the expansion of the market. “The cycle of population growth, capital accumulation, market expansion, crisis, followed by population growth again could proceed until by the eighteenth century the European societies were sufficiently advanced to break out of the Malthusian trap.” (J. Komlos, 1989)

Finally, four pieces of evidence motivate this work:

(i) During the process of industrialization, the shift of labor away from agriculture and the increase in the size of the firms led to a decline in the proportion of self-employed (see Prados de la Escosura and Roses, 2003). Therefore, we should observe an increase in the labor income share during this process. However, the share of labor decreased between 1856 and 1913 for the UK, the Netherlands and the US: in the UK the wages and salaries as a percentage of national income fell from 50 to 48 (see Mathews, Feinstein and Odlig-Smee, 1982). In the Netherlands wages and salaries fell from 45 to 38 percent of national income. In the US labor share as a percentage of national income fell from 66 to 62. These facts imply that, as the proportion of self-employed declined, another process was driving up the capital income share. We claim that the force behind the decline in labor shares was a process of capital-using innovations. In a related work, regarding the British industrial revolution, Allen (2005) states that between 1800 and 1840, GDP per worker rose 37%, real wages stagnated, and the profit rate doubled. In summary, the share of profits in national income expanded at the expense of labor and land.

(ii) High rates of economic growth generally coincide with high levels of savings (see Aizenman, Pinto and Radziwill, 2004 and Hausmann, Pritchett, and Rodrick, 2005).

(iii) Previous empirical works in economic growth find that both the GDP per capita (Theil and Galvez, 1995; Irwin and Tervio, 2002) and the growth rate of GDP per capita (Sala-i-Martin 1997) depend positively on the absolute latitude.

(iv) Clearly, seasonality is not fully determined by latitude. Continental climate is characterized by important annual variation in temperature due to the lack of significant bodies of water nearby. For example, seasonality is bigger in Russia or Mongolia than England although they have similar latitudes. Therefore, in order to test the hypothesis other proxies for seasonality should be considered. Masters and McMillan (2001) provide a rigorous empirical analysis considering variables directly related to the strength and the length of the winter. They find that the frequency of frost in winter, after frost-free summer, is a key variable to explain differences in growth paths among countries.

The rest of the paper is organized as follows. In the second section, we explain how our theory fits into the debate “geography versus institutions”. In the third section, we present the general model and its results. In the fourth section we provide an example assuming an explicit production function. In the fifth section we provide some empirical evidence. In the fifth section we present an exploratory exercise in order to test if the correlation between seasonality and savings is supported by the data. Finally, some concluding remarks are provided.

2. GEOGRAPHY AND INSTITUTIONS

Several authors, following the classic work of Max Weber (1905), argue that differences in economic performance can be explained by religion (Barro and MacCleary, 2003). They explain the different performance of former English colonies compared to former Spanish or French colonies by the institutional bequest. However, the relation between geography and GDP holds even controlling for religion and language. Additionally, the superiority of British institutions is usually based on the records of the USA and Canada, but Barbados; Belize, Guyana and Jamaica did not perform well in economic terms and they were British colonies located closer to the equator. In the same vein, Hall and Jones (1999) argue that institutions (instead of climate or location) play a fundamental role in the economic performance of different countries. Along this lines, Acemoglu, Johnson and Robinson (2001 and 2002) state that the disease environment in former European colonies determined the number of European settlements and their subsequent institutional development. Therefore, countries where the environment was friendlier to European settlers benefited from better institutions and, for this reason, have had better economic performance. In the same line, Easterly and Levine (2003) claim that geographical variables affect country income through institutions. Finally, Rodrik, Subramanian and Trevi (2004) confirm that controlling for institutions, geography does not affect GDP.

In principle, the story of seasons and savings can be consistent with the institutional approach: the seasonality of output generates incentives to save and accumulate capital, strengthening the incentives for capital owners to vote, impose or ask for an institutional arrangement that protects their property rights. In countries where property rights are effectively protected the incentives to accumulate capital are bigger. Therefore, economic fluctuations and savings are determining capital abundance and capital abundance is the main cause of both capital intensive technologies and property rights protection. In this case, the result that geography affects GDP only through institutions can be hiding the fact that capital abundance and capital intensive technologies are determining both economic growth and institutions.

Along the institutional line, Engerman and Sokoloff (2002) study countries in the Americas and claim that the differences in economic performance can be explained by the differences in factor abundance. Their idea consists of 3 arguments: (i) differences in factor endowments determine differences in income distribution; (ii) income distribution affects the choice of institutions; and (iii) democratic institutions stimulate economic growth.

The idea of Engerman and Sokoloff (2002) is the observational equivalent to the predictions of a Biased Technological Change Model in the sense that factor abundance determines economic growth. According to this view, factor abundance affects the choice of technology, i.e., in labor abundant economies the optimal technologies are more labor intensive than in labor scarce economies. On their turn, technologies affect the marginal productivity of capital and the incentives to save. Therefore, independently

of the institutional framework, in labor abundant economies the incentives to save are weaker because the predominant production factor is labor, not capital.

Self and Grabowski (2006) relate institutional development with agricultural productivity. Specifically, the state revenue depends on a productive agricultural sector so if agricultural productivity is high then there is no need for a predatory state so political and social improvements are likely to occur. On the other side, if agricultural productivity is low then few sources of income exists and the chances for institutional development are small. Again, this story is the observational equivalent to the predictions of a Biased Technological Change Mode where more agricultural productivity increases the amount of savings so the economy becomes more capital abundant, the technologies more capital intensive and the economy grows faster.

As stated before, the story of seasons, savings and biased technological change complements the institutional approach. In essence, we find three reasons to believe that the effect of seasonality and biased innovations is important in its own right to explain economic development:

(i) There exists a positive correlation between GDP and both absolute latitude and frost frequency.

(ii) Huber, Rueschemeyer and Stephens (1993) point out that in Europe democracy was a result of economic development. To illustrate this point they indicated that in 1870, only one European country, Switzerland, was a democracy. In contrast, by 1920, almost all Western European countries were fully democratic.

(iii) If institutions are determined by GDP, as suggested by Huber *et al.* (1993), and Glaeser, La Porta, Lopez-de-Silanes and Shleifer (2005), the result that geography affects GDP only through institutions can be reflecting a positive correlation between current and past income level.⁶

The institutional view tells a story of why the industrial revolution spread to certain regions faster than others, but not why it happened in Europe in the first place. The traditional geographical view can explain why countries where commerce and production are obstructed by natural conditions have, in general, low relative levels of development, but this approach cannot account for the correlation between latitude and GDP. Finally, Diamond (1997) explains why industrialization happened on the Eurasian continent, but is not equally convincing on the issue of why South Asia did not industrialize before Europe. In this sense, our work complements the previous approaches and adds a missing link among their findings.

⁶ Other limitations of the “only institutions matter” approach have been pointed out by a number of scholars. Indeed, this model implies that good institutions should generate higher growth rates during any period. However, Przeworski (2004) finds that during the period 1950-1999 for countries that were colonies as of 1945 “the rate of growth of total output does not depend on institutions”. Similarly, Hausmann, Pritchett and Rodrick (2004) show that growth accelerations tend to be correlated with increases in savings and most growth accelerations are not preceded or accompanied by major changes in institutional arrangements.

3. THE MODEL

We formalize our argument with a model of endogenous growth with biased technological change. The model uses the notion of capital-using innovations to explain how economies switch from a storage technology to a technology of capital accumulation. We start by assuming a primitive economy where output is produced with not-reproducible factors (land and labor) and a storage technology. We also assume that output does not have a trend but fluctuates seasonally. In particular, we assume that output per worker can take two values, A_l (low) and A_h (high). Since output behaves cyclically, a storage technology is used to smooth consumption. Under such circumstances, savings (storage) are bigger in places where changes in output are bigger. Besides the primitive technology, there exists a set of capital intensive technologies differentiated by their capital intensity, that is, by the elasticity of output with respect to capital. These technologies are costly and the cost is increasing in the capital intensity. Under this setting, a minimum amount of savings is needed for the adoption of capital intensive technologies to be profitable. Therefore, economies where seasonal fluctuations are stronger are more likely to adopt capital intensive modes of production. Finally, once an economy is using capital intensive technologies a process of capital accumulation and technological improvement drives the economy to sustained growth.

We first present the case where only a primitive technology, without capital accumulation, is available and explain why a relation between absolute latitude and savings is likely to appear. After that, we consider the possibility of creating or adopting new technologies that use capital and show that economies where savings are bigger are more likely to adopt new technologies.

3.1. Storage and Technology

We assume identical agents; each agent devotes a constant amount of time to work, $L=1$ and there is no population growth. We also assume that the production function is linear in not reproducible factors and that agents can make use of a storage technology. Therefore, the budget constraint for each agent is given by, $c_t \leq A_t + \eta\zeta_t - \zeta_{t+1}$, where ζ is the amount of stored goods, η is the proportion of stored products that can be consumed after one period ($\eta \leq 1$) and A is the output per worker.

Note that in this model savings (s) are completely allocated to storage, namely, $s_t = \zeta_t$. We also assume that output behaves seasonally in such a way that if current output is low ($A_t = A_l$) then future output is high ($A_{t+1} = A_h$).

The problem of the representative consumer is the standard one,

$$\begin{aligned}
& \text{Max}_{\{c_t\}} \sum_{t=0}^{\infty} \log(c_t) \beta^t, \\
& \text{s.t. } c_t \leq A_t + \eta \zeta_t - \zeta_{t+1}, \\
& \zeta_t \geq 0.
\end{aligned} \tag{1}$$

From where,

$$c_{t+1} = \begin{cases} c_t \eta \beta & \text{if } A_{t+1} + \eta \zeta_t \geq c_t \eta \beta \\ A_{t+1} + \eta \zeta_t & \text{if } A_{t+1} + \eta \zeta_t < c_t \eta \beta. \end{cases} \tag{2}$$

Therefore, the most desirable consumption path is given by $\frac{c_{t+1}}{c_t} = \eta \beta$. However, the fluctuations of output and the fact that storage cannot be negative constitute an impediment to achieve this path. For this reason, in some periods the consumer chooses to have zero or negative savings.⁷

Formally, savings in the long-run equilibrium can be characterized as follows (the proof is presented in the Appendix):

$$\zeta_t = \begin{cases} \frac{\eta \beta A_t - A_{t+1}}{(1 + \beta) \eta} & \text{if } \eta \beta A_t > A_{t+1} \\ 0 & \text{if } \eta \beta A_t \leq A_{t+1}. \end{cases} \tag{3}$$

Thus, the average size of assets per capita depends on the difference between A_t and A_h . Moreover:

(i) If $\frac{1}{\eta \beta} \geq \frac{A_h}{A_t}$ then $\zeta_t = 0 \forall t$, that is, in economies where fluctuations in output are small the optimal amount of savings is zero.

(ii) If $\frac{1}{\eta \beta} < \frac{A_h}{A_t}$ then savings are an increasing function of $(\eta \beta A_t - A_{t+1})$. In other words, savings are higher in places where changes in output are higher.

(iii) If $\frac{A_h}{A_t}$ is constant and $\frac{A_h}{A_t} > \frac{1}{\eta \beta}$ then savings are an increasing function of average output.

(iv) Holding the rest constant, savings are an increasing function of the

⁷ Note that individuals store goods only for the one period because it is optimal. They could store for more than one period but they choose not to do it.

productivity of the storage technology.

In primitive economies the main source of fluctuations is climatic and such fluctuations are bigger in places located far away from the equator. Therefore, results (i) and (ii) imply that in places located far away from the equator savings are likely to be higher. Result (iii) implies that savings are higher in places where land productivity is higher. Finally, result (iv) implies that savings are higher in places where the storage technology is better.

Another feature of the model is that, holding the rest constant, the consumers utility is higher in places where fluctuations are smaller (because $\eta \leq 1$). This means that in primitive economies, welfare was likely to be higher in places close to the equator.⁸ Recall that consumption goods include food, leisure, housing and public goods. Therefore, the fact that some countries located in tropical zones enjoyed this type of goods before any European country is consistent with our story.

3.2. Innovations

Now consider that savings (s) may be devoted to storage (ζ), to create or adopt new technologies (α) and to accumulate capital (K) that can be used in the new technologies ($F(K, \alpha)$).

Here, we assume that better technologies make capital goods more productive, that is, if $\alpha_1 > \alpha_0$ then $F(K, \alpha_1) > F(K, \alpha_0)$, but they are also more costly. The cost of a technology α is given by $g(\alpha)$ where $g'(\alpha) > 0$. Therefore, the cost of the technology increases as the technology becomes more capital intensive. This assumption may be justified in two ways. On the one hand, since Jones (1995) diminishing returns have been a standard assumption in growth models. On the other hand, relaxing this assumption does not affect qualitatively the main predictions of the model.

We also refer to α as the capital intensity of the technology because an increase in α augments the return on capital and, for this reason, stimulates capital accumulation. Similarly, we refer to increases in α as capital-using innovations. In this setting, the difference between production and consumption can be stored or exchanged for capital goods. In the latter case, savings can be used to increase the number of capital goods of a given quality or to improve the quality of a given number of capital goods. Therefore, $s_t = \zeta_t + \Delta K_t + g(\alpha)$, where $\Delta K_t = K_{t+1} - K_t$.

The new production function, $F(K, \alpha)$, has the following properties:

⁸ A possible extension of this model, including migrations, may help to explain why the first civilizations appeared in regions that were not temperate (Southern China, India, Mesopotamia, Peru, Guatemala and Mexico).

- If $K \geq 0$ then the production function is increasing and concave in the stock of capital ($F_K(\cdot) > 0$, $F_{KK}(\cdot) < 0$) for any technology $\alpha \in [0,1)$ and linear in K for $\alpha = 1$.
- It is increasing in the technology $F_\alpha(\cdot) > 0$ for any $K \geq 0$.
- Technology and capital are essential ($F(0,\alpha) = 0$ and $F(K,0) = 0$).
- Technology and capital are complements $F_{K\alpha}(\cdot) > 0$.
- The stock of capital depreciates at a rate δ .

As in the previous section, the consumer has to choose savings and consumption. In a period of high output the consumer saves part of it. Savings can be stored and consumed during the next period or invested in a technology of capital accumulation. In the second case, savings have to be divided between capital accumulation and technology improvement.

The problem of the representative consumer is the following:

$$\begin{aligned} & \text{Max}_{\{c_t\}} \sum_{t=0}^{\infty} \log(c_t) \beta^t, \\ & \text{s.t. } c_t \leq A_t + F(K_t, \alpha_t) + (\eta \zeta_{t-1} - \zeta_t) + (K_{t+1} - (1 - \delta)K_t) - g(\alpha_{t+1}), \\ & \zeta_t \geq 0, \quad K_t \geq 0, \quad \alpha_t \geq 0, \quad \alpha_0 = 0. \end{aligned} \quad (4)$$

From the first order conditions it follows that the growth rate of consumption is given by

$$\frac{c_{t+1}}{c_t} = \beta \max \{ \eta, [(1 - \delta) + F_K(K_{t+1}, \alpha_{t+1})], [F_\alpha(K_{t+1}, \alpha_{t+1}) - g'(\alpha_{t+1})] \}. \quad (5)$$

Since there are three different ways to accumulate wealth, the consumer chooses the most productive one, namely,

(i) If $F_K(K_{t+1}, \alpha_{t+1}) < \delta + \eta - 1$ and $F_\alpha(K_{t+1}, \alpha_{t+1}) - g'(\alpha) < \eta$ only the storage technology is used ($K = 0$ and $\alpha = 0$).

(ii) If $F_K(K_{t+1}, \alpha_{t+1}) > \delta + \eta - 1$ and $F_\alpha(K_{t+1}, \alpha_{t+1}) - g'(\alpha) > \eta$, the storage technology is not used anymore and the economy accumulates capital ($\zeta = 0$; $K > 0$; $\alpha > 0$).

(iii) If $\frac{1}{\eta\beta} \geq \frac{A_h}{A_t}$ and $\frac{1}{((1 - \delta) + F_K(\cdot))\beta} \geq \frac{A_h}{A_t}$ then $\zeta_t = 0 \forall t$ and there are no incentives to save, $s = 0$.

(iv) If the economy accumulates capital, savings are devoted to new technologies and capital accumulation in such a way that the marginal productivity of capital must be equal to the marginal productivity of innovation, namely,

$$(1 - \delta) + F_K(K_{t+1}, \alpha_{t+1}) = F_\alpha(K_{t+1}, \alpha_{t+1}) - g'(\alpha_{t+1}). \quad (6)$$

Notice that $\alpha_0 = 0$, so initially only the primitive technology is used and there is no capital accumulation. Note also that when the new technology is used, savings are partially devoted to capital accumulation and partially to technological change. Any increase in α generates an increase in the marginal productivity of capital, and an increase in K generates an increase in productivity of technology. Therefore, when both K and α are growing, the production function can be convex in the amount of savings. If this is the case, there exists a minimum level of savings such that agents have incentives to use capital intensive technologies (in the next section we provide an example).

From results (i) and (ii) it follows that $s > 0$ is a necessary condition for new technologies to be adopted. The indivisibility of capital goods imposes a minimum level of savings needed to adopt the technology of capital accumulation. Moreover, when the function $F(K_t, \alpha_t)$ is convex in s , the minimum level-of-savings condition holds. Similarly, from (i), (ii), (iii) and (iv) it follows that when the accumulation technology is used there exists a positive relation between capital intensity (α) and stock of capital (K) (see proof in the Appendix).

Finally, when the capital intensive technology is used, both the number and the quality of capital goods grow, so the marginal productivity of capital does not necessarily decrease as the stock of capital grows.

From results (i) and (ii) it also follows that, the lower the cost of storage (higher η), the higher the savings needed to adopt a technology of capital accumulation. Therefore, the net effect of the productivity of the storage technology is ambiguous.

These results can be summarized by saying that when savings are low it is optimal for the individuals to use the primitive technology. If savings are high enough it is optimal to create or adopt new technologies and start accumulating capital. Now, since technology is embodied in capital goods which are costly and indivisible, communities with low levels of savings are not able to adopt capital intensive technologies. Finally, once a capital intensive technology ($\alpha > 0$) is adopted, a process of capital accumulation and technological progress starts. Depending on the functions $F(\cdot)$ and $g(\cdot)$ the process of capital accumulation can last forever or end with a finite stock of capital (see Boldrin and Levine, 2002; Zuleta, 2008 and Peretto and Seater, 2006). If there is long run growth then the differences in GDP per worker are likely to increase over time.

The main implications of the model are consistent with the empirical evidence. First, GDP per capita correlates with absolute latitude and with frost frequency. Second, the industrial revolution occurred first in countries located far away from the equator. Third, there was an important increase in agriculture productivity preceding industrial revolution. Fourth, the share of capital increased during the industrialization process in the UK. Fifth, both the share of reproducible factors and the elasticity of output with regard to reproducible factors are positively correlated with GDP per capita. Sixth,

growth accelerations are generally associated with high saving rates. The model also predicts that any increase in the capital-labor ratio generates incentives to increase the capital intensity of the technology. Therefore, holding the rest constant, a reduction in the size of the populations generates an increase in the capital intensity of the technology.

4. EXAMPLE

In this section we present an example where the production function of the capital intensive sector is a Cobb-Douglas, $Y = Bk^\alpha$. Thus, changes in α are the only way to have capital-using innovations. Savings can be stored and consumed during the next period or invested in technology of capital accumulation. In the latter case, savings have to be divided between capital accumulation and technological change. b is the fraction of savings devoted to storage, u the fraction of savings devoted to increase the number of capital and $1-b-u$ is the fraction of savings devoted to increase the quality of capital goods. Even if technologies were free, in capital scarce economies there are no incentives to adopt capital-using technologies.⁹ Figure 1 illustrates this fact: when the capital-labor ratio is smaller than one ($k < 1$), a capital-using innovation (increase in α) reduces output, so it is better to use the primitive technology. Now, for capital abundant economies ($k > 1$) it is better to adopt capital-using innovations because such innovations increase output.

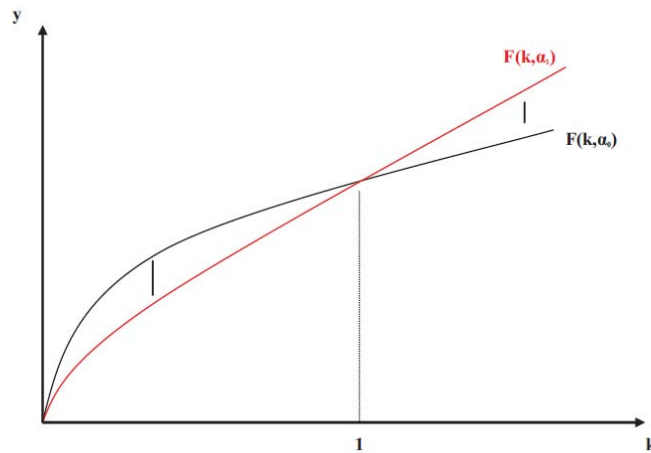


Figure 1.

⁹ This argument was first presented by Zuleta (2012).

The cost of increasing α is captured by the following function: $\alpha_{t+1} = \alpha_t + (1 - \alpha_t) [1 - e^{-[1-b_t-u_t]s_t}]$. Therefore, the capital intensity of the technology is a function of the technology used in the past and of the amount of savings devoted to increase the capital intensity of the technology.

For simplicity the depreciation rate is assumed to be zero and the storage technology is assumed to be efficient $\eta=1$. The results of the model do not depend on this assumption. Under this setting, the problem of the representative agent is the following:

$$\begin{aligned} & \text{Max}_{\{c_t\}} \sum_{t=0}^{\infty} \log(c_t) \beta^t, \\ & \text{s.t. } \zeta_t = \zeta_{t-1} + b_t s_t, \\ & k_{t+1} = k_t + u_t s_t, \\ & \alpha_{t+1} = \alpha_t + (1 - \alpha_t) [1 - e^{-[1-b_t-u_t]s_t}], \\ & \zeta_t \geq 0, \quad K_t \geq 0, \quad \alpha_t \geq 0, \quad \alpha_0 = 0. \end{aligned} \quad (7)$$

To find the solution we combine the first order conditions (complete derivation in the Appendix 6.3). The optimal growth rate of consumption is the following:

$$\frac{c_{t+1}}{c_t} = \beta \max \{1, [1 + \alpha_{t+1} B(k_{t+1})^{\alpha_{t+1}-1}], [1 + (1 - \alpha_{t+1}) B(k_{t+1})^{\alpha_{t+1}} \ln(k_{t+1})]\}. \quad (8)$$

At any t , the agent chooses the share of savings devoted to increase the number of capital goods u , to store b , and to increase the capital intensity of the technology $1-b-u$. If the marginal productivity of savings is higher in the capital intensive technology than in the storage technology there is no storage, $b=0$. In this case, savings are allocated in such a way that the marginal productivity of capital is equal to the marginal productivity of technology, that is,

$$\alpha_{t+1} B(k_{t+1})^{\alpha_{t+1}-1} = [(1 - \alpha_{t+1}) B(k_{t+1})^{\alpha_{t+1}} \ln(k_{t+1})]. \quad (9)$$

Note that $k > 1$ is a necessary condition for innovations to be profitable. Therefore, $s > 1$ is a necessary condition for the representative agent to adopt capital-using innovations and Equation (9) can be rewritten in the following way:

$$\alpha_{t+1} = \max \left(\alpha_t, \frac{k_t \ln(k_t)}{k_t \ln(k_t) + 1} \right). \quad (10)$$

So, in equilibrium α_t is a function of k_t , namely, $\alpha_t = \alpha(k_t)$, where $\alpha(k_t) = 0$ for any $k_t < 1$ and $\lim_{k \rightarrow \infty} \alpha(k) = 1$. Additionally, for any $k > 1$, the marginal

productivity of capital increases as the capital stock grows (proof in the Appendix). Therefore, for $k > 1$, the growth rate of the economy increases as the capital stock augments and converges to a finite number when the capital stock goes to infinity ($\lim_{k \rightarrow \infty} \frac{c_{t+1}}{c_t} = \beta(1+B)$). Finally, since $\eta = 1$ then for any $k > 1$ the technology of capital accumulation is preferred.

Note also that depending on the values of α_t , B and β the growth rate of consumption can be positive or negative. Indeed, from Equation (8) it follows that $\frac{c_{t+1}}{c_t} \geq 1$ implies $\alpha_{t+1}B(k_{t+1})^{\alpha_{t+1}-1} \geq \frac{1-\beta}{\beta}$.

Now, the marginal productivity of capital increases as the capital stock grows, so it is possible to define k_m as the capital stock such that the growth rate of consumption is equal to zero, namely,

$$\alpha(k_m)(k_m)^{\alpha(k_m)-1} = \frac{1-\beta}{B\beta}. \quad (11)$$

So $\frac{c_{t+1}}{c_t} < 1$ for any $k_{t+1} < k_m$ and $\frac{c_{t+1}}{c_t} > 1$ for any $k_{t+1} > k_m$.

Equation (11) indicates the levels of the state variables for which the discount rate is equal to the marginal productivity of savings. If under the initial conditions the state variables are high ($\alpha > \alpha(k_m)$ and $k > k_m$) then the optimal consumption growth rate is positive and, consequently, it is optimal to save and increase both the number and the quality of capital goods. If under the initial conditions the state variables are low ($\alpha < \alpha(k_m)$ and $k < k_m$) then the optimal consumption growth rate is negative and, consequently, it is optimal to consume part of the capital stock. Therefore, for $1 < k < k_m$, the new technology is preferred over the storage technology but there are no incentives to accumulate capital. In general, this implies that, under such circumstances, capital goods instead of consumption goods are used to smooth consumption. However, if capital is irreversible, that is, if capital goods cannot be consumed then the new technology is preferred over the storage technology only if $\alpha_t B(k_t)^{\alpha_t-1} > 1$. Now, given that $k < k_m$, for this condition to hold the discount factor must be unrealistically low,¹⁰ $\beta < 0.5$. Summarizing, if savings are low the storage technology is preferred over the capital accumulation technology and if savings are high agents prefer the capital intensive technology. In general, once the capital intensive technology is used, a process

¹⁰ Note that $k < k_m$ implies $\alpha_t B(k_t)^{\alpha_t-1} < \frac{1-\beta}{\beta}$. So, if $\alpha_t B(k_t)^{\alpha_t-1} < 1$ then $\beta < 0.5$.

of capital accumulation and technological improvement drives the economy to sustained growth.

Finally, note that once the capital intensive technology is adopted, the production function becomes

$$Y_t = A_t + Bk_t^{\alpha(k_t)}. \quad (12)$$

So there are two sources of seasonality, the natural sources, that is seasonal fluctuations in A , and fluctuations in the stock of capital k . However, if savings are high enough ($s > k_m$) the capital intensive technology is used and the optimal growth rate of consumption is positive for every t , so savings must be positive for every t and the stock of capital k grows every period. Now, capital growth is also seasonal because output is seasonal. However, as the economy accumulates capital the seasonal component of output (and savings) becomes less important.

Table 2. Growth Rates GDP per capita

	1700-1820	1820-1850	1850-1900	1900-1950	1950-2000
Austria	0.17%	1.02%	1.12%	0.50%	3.50%
Belgium	0.12%	1.13%	1.42%	0.77%	2.69%
Denmark	0.17%	1.10%	1.08%	1.68%	2.42%
Finland	0.17%	0.51%	1.22%	1.89%	3.10%
France	0.18%	1.14%	1.18%	1.22%	2.83%
Germany	0.14%	0.94%	1.49%	0.53%	3.23%
Italy	0.01%	0.63%	0.56%	1.36%	3.42%
Netherlands	-0.12%	0.85%	0.74%	1.13%	2.60%
Norway	0.09%	0.59%	1.36%	2.15%	3.11%
Sweden	0.17%	0.24%	1.38%	1.95%	2.27%
Switzerland	0.17%	1.04%	1.91%	1.74%	1.83%
U. K.	0.26%	1.05%	1.32%	0.87%	2.16%
Greece	0.16%	0.81%	1.01%	0.70%	3.76%
Portugal	0.10%	0.00%	0.69%	0.95%	3.85%
Spain	0.14%	0.23%	1.01%	0.41%	4.01%
Australia	0.22%	4.56%	1.43%	1.23%	2.16%
New Zealand	0.00%	3.57%	2.68%	1.36%	1.31%
Canada	0.62%	1.29%	1.58%	1.85%	2.27%
United States	0.73%	1.21%	1.65%	1.71%	2.20%

Source: Maddison (2003).

In summary, the output growth rate predicted by the model is zero for primitive economies but economies where savings are high have incentives to adopt capital intensive technologies. Once an economy begins to use capital intensive technologies, growth rates may become positive. Moreover, the rate of economic growth increases as the economy accumulates capital and converges to a finite number in the long run. This result is consistent with the evolution of the growth rate of GDP per capita for Western European Countries and for Western Offshoots (see Table 2): Until 1800 GDP per capita grows at a very small rate and, since then, the growth rate augments period by period with the only exception of 1900-1950 characterized by the two world wars.

5. EMPIRICAL EVIDENCE

We claim that in primitive economies the main source of fluctuations is the climate, and such fluctuations are bigger in places located far from the equator. Now, the strength of the winter can be captured by variables like Absolute Latitude, Proportion of land with more than N frost-day per month in winter or Average number of frost days per unit of land area. As we stated before, during frost days there are no harvests, transportation is difficult, people need more energy and, in general, surviving demands much more work, and, for this reason, people save part of the output produced during the rest of the year. Therefore, holding the rest constant, savings should be higher in places where the winter is stronger and longer.

Masters and McMillan (2001) find that a key variable explaining the different economic performance between tropical and temperate countries is Frost Frequency. According to them, this relation is explained by the effect of ground frost on productivity. As we stated before, the arguments of Masters and McMillan (2001) are captured in our model: if ground frost implies higher productivity, holding the rest constant, it also implies higher savings.¹¹

Here we consider these variables but instead of using growth as the dependent variable we use the savings rate. Our claim is that in regions where seasons are stronger agents are likely to have higher savings and for this reason, are more likely to adopt capital intensive technologies. The adoption of such technologies, on their turn, increases the return on savings (investment).

A complete test of the model would require a historical data set containing savings rates for a broad sample of countries. Unfortunately this information is not available. However, we can test the hypothesis that the savings rate in 1960 depend on seasonal

¹¹ Previous empirical works in economic growth find that both the GDP per capita and the growth rate of GDP per capita depend positively on proxies of seasonality like the absolute latitude and frequency of frost in winter (Sala-i-Martin 1997; Theil and Galvez, 1995; Irwin and Tervio, 2002; Masters and McMillan, 2001). However, there are no tests for the link between seasonality and savings.

fluctuations. Indeed, today part of the output depends on seasonal fluctuations and some expenditures fluctuate with the seasons (heating, cleaning streets and roads, etc.)

As we stated above the lack of information does not allow us to test the effect that seasonal fluctuations had on savings before the industrial revolution or during the industrialization process. However, we can use more recent data to see if there exists a positive relation. If such a relation exists in the 20th century, when the share of the output depending of climatic conditions is smaller, it should also have been present in the previous centuries.

The savings rate can be associated with variables like productivity and discount factors, among others. For this reason we use GDP per capita as a control variable. If controlling for GDP the proxy variables for seasonal fluctuations positively affect savings in 1960 then we can claim that the effect of seasons on savings is important.

Table 3. Dependent Variable: Saving Rate 1960

	(1)	(2)	(3)	(4)
Constant	-0.814 (0.000)	-0.566 (0.000)	-0.749 (0.000)	-0.691 (0.000)
Average Number of Frost Days per Unit of Land Area		0.0012 (0.000)		0.0066 (0.005)
Absolute Latitude	0.0042 (0.000)		0.0024 (0.023)	
Ratio of Population within 100Km of Ice-free Coast		0.154 (0.000)	0.116 (0.000)	0.144 (0-000)
Elevation		-5.48exp(-05) (0.023)		
Log GDPpc 1960	0.118 (0.000)	0.089 (0.000)	0.106 (0.000)	0.101 (0.000)
Percentage of Catholics			-0.091 (0.049)	-0.082 (0.012)
Social Infrastructure			0.192 (0.036)	0.159 (0.019)
Adjusted R-Square	0.712	0.76	0.78	0.79
Observations	101	99	80	79

Note: p-values in parenthesis.

Sources: Barro-Lee data set: GDP, absolute latitude, percentage of Catholics. Hall and Jones (1999): Social infrastructure. Masters and MacMillan (2001): Proportion of land with more than 3-frost days per month in winter and average number of frost days per unit of land area.

Table 3 presents the results. In column 1 the independent variables are absolute latitude, geographical variables and GDP per capita (we exclude non-significant variables). In column 2 we include Average number of frost days per unit of land area instead of absolute latitude. In column 3 we repeat the exercise of column 1 controlling for institutional variables and excluding variables with coefficient not significantly different from zero. Finally, in column 4 we repeat the exercise of column 2 controlling for institutional variables and excluding variables with coefficients not significantly different from zero.

The effect of the proxies for seasonal fluctuations on savings is positive and significantly different from zero.

6. CONCLUSIONS

We have studied a simple general equilibrium model of capital accumulation. Assuming that primitive economies start with a production function that uses only non-reproducible factors and a storage technology, we find that savings are bigger in economies where changes in output are greater. In economies with high savings, individuals have incentives to make capital-using innovations. In other words, these economies are more likely to create or adopt technologies that use capital more intensively. Finally, in primitive economies the main source of fluctuations is the climate and such fluctuations are bigger in places located far away from the equator. Therefore, economies where seasonal changes in output are higher are more likely to make capital-using and labor saving innovations. Once the process of capital accumulation begins, there is a positive relation between technology and capital. Thus, as long as economies accumulate capital the differences in GDP are not likely to be reduced.

The model contributes to the literature on biased innovations, explaining why, in primitive economies, the saving rate is correlated with the strength and the duration of seasonal fluctuations.

Finally, the main implications of the model are consistent with the empirical evidence related with the industrial revolution, the subsequent industrialization of some economies and the economic stagnation of others.

APPENDIX

A.1. Savings*A.1.1. Existence*

In this section we prove that

$$\zeta_t = \begin{cases} \frac{\eta\beta A_t - A_{t+1}}{(1+\beta)\eta} & \text{if } \eta\beta A_t > A_{t+1} \\ 0 & \text{if } \eta\beta A_t \leq A_{t+1}. \end{cases}$$

is a long run equilibrium.

Consider the case where $\zeta_{t-1} = 0$. A rational consumer decides the consumption level according to the following rule:

- A. If $A_t = A_l$ then $\zeta_t = 0$ and $c_t = A_l$.
- B. If $A_t = A_h$ and if $\frac{A_h}{A_l} < \frac{1}{\eta\beta}$ then $\zeta_t = 0$ and $c_t = A_h$.
- C. If $A_t = A_h$ and if $\frac{A_h}{A_l} > \frac{1}{\eta\beta}$ then $\zeta_t > 0$ and $c_t < A_h$. Moreover, since

the optimal consumption growth rate is given by $\frac{c_{t+1}}{c_t} = \eta\beta$ then savings are given by

$$\zeta_t = \frac{\eta\beta A_h - A_l}{(1+\beta)\eta} \text{ and consumption by } c_t = \frac{\eta A_h + A_l}{(1+\beta)\eta}.$$

Consider the case where $\zeta_{t-1} = \frac{\eta\beta A_h - A_l}{(1+\beta)\eta}$. A rational consumer decides the consumption level according to the following rule:

If $A_t = A_l$ then $\zeta_t = 0$ and $c_t = \frac{\beta}{1+\beta}(A_l + \eta A_h)$. Now, from literal c we know that if $\zeta_{t-1} = 0$ then $c_{t+1} = \frac{\eta A_h + A_l}{(1+\beta)\eta}$. Note also that $\frac{\eta A_h + A_l}{(1+\beta)\eta} > \frac{\beta}{1+\beta}(A_l + \eta A_h)$. Therefore, $c_{t+1} > c_t$ and $c_{t+1} > \eta\beta c_t$. So, the current consumption c_t level is lower than the most desirable. However, $\zeta_t \geq 0$ so the best choice is to consume the entire output plus ζ_{t-1} .

From 1 and 2 it follows that

- A. If $\eta\beta A_t > A_{t-1}$ and $\zeta_{t-1} = 0$ then $\zeta_t = \frac{\eta\beta A_t - A_{t-1}}{(1+\beta)\eta}$.
- B. If $\eta\beta A_t < A_{t-1}$ and $\zeta_{t-1} = \frac{\eta\beta A_t - A_{t-1}}{(1+\beta)\eta}$ then $\zeta_t = 0$.

Therefore,

$$\zeta_t = \begin{cases} \frac{\eta\beta A_t - A_{t+1}}{(1+\beta)\eta} & \text{if } \eta\beta A_t > A_{t+1} \\ 0 & \text{if } \eta\beta A_t \leq A_{t+1}. \end{cases}$$

is a long run equilibrium.

A.1.2. Uniqueness

In this section we prove that the long run equilibrium is unique.

Suppose there exists a long run equilibrium where $\zeta_t > 0$ for every t . In this case the solution of the optimization problem is interior for every t , namely, $\frac{c_{t+1}}{c_t} = \eta\beta$ for every t . But note that $\eta\beta < 1$ so $\lim_{T \rightarrow \infty} \frac{c_T}{c_t} = 0$ which cannot be optimal. Therefore, an equilibrium where $\zeta_t > 0$ for every t does not exist. Finally, from the proof of existence it follows that, starting from $\zeta_{t-1} = 0$, if $\eta\beta A_t > A_t$ then the only possible equilibrium is $\zeta_t = \frac{\eta\beta A_t - A_{t+1}}{(1+\beta)\eta}$ and if $\eta\beta A_t < A_t$ then $\zeta_t = 0$.

A.2. Technology and Capital

From Equation (6),

$$(1-\delta) + F_K(K_{t+1}, \alpha_{t+1}) - F_\alpha(K_{t+1}, \alpha_{t+1}) + g'(\alpha_{t+1}) = 0.$$

Applying the implicit function theorem we prove that $\frac{\partial \alpha}{\partial K} > 0$.

Define $G(\alpha, K) = (1-\delta) + F_K(K, \alpha) - F_\alpha(K, \alpha) + g'(\alpha)$.

So, $G_\alpha(\alpha, K) = F_{K\alpha}(K, \alpha) - F_{\alpha\alpha}(K, \alpha) + g''(\alpha)$ and $G_K(\alpha, K) = F_{KK}(K, \alpha) - F_{\alpha K}(K, \alpha)$.

Notice that $g''(\alpha) > F_{\alpha\alpha}(K, \alpha)$, $F_{KK}(K, \alpha)$ and $F_{K\alpha}(K, \alpha) > 0$. Therefore, $G_\alpha(\alpha, K) > 0$ and $G_K(\alpha, K) < 0$. From where

$$\frac{\partial \alpha}{\partial K} = -\frac{G_\alpha(\alpha, K)}{G_K(\alpha, K)} > 0.$$

A.3. The Cobb-Douglas Example

A.3.1. The Problem

The problem can be written as,

$$\begin{aligned} & \text{Max}_{\{c_t\}} \sum_{t=0}^{\infty} \log(y_t - s_t) \beta^t, \\ & \text{s.t. } s_t = \zeta_{t+1} - \zeta_t + k_{t+1} - k_t + \log(1 - \alpha_t) - \log(1 - \alpha_{t-1}), \\ & \zeta_t \geq 0, \quad K_t \geq 0, \quad \alpha_t \geq 0, \quad \alpha_0 = 0. \end{aligned}$$

This is equivalent to

$$\begin{aligned} & \text{Max}_{\{c_t\}} \sum_{t=0}^{\infty} \log(y_t - [\zeta_{t+1} - \zeta_t + k_{t+1} - k_t + \log(1 - \alpha_t) - \log(1 - \alpha_{t-1})]) \beta^t, \\ & \text{s.t. } \zeta_t \geq 0, \quad K_t \geq 0, \quad \alpha_t \geq 0, \quad \alpha_0 = 0. \end{aligned}$$

The first order conditions are

$$\begin{aligned} & -\frac{1}{c_t} \beta^t + \frac{1}{c_{t+1}} \beta^{t+1} + \chi_1 = 0, \\ & -\frac{1}{c_t} \beta^t + \frac{1}{c_{t+1}} \beta^{t+1} + (1 + \alpha_{t+1} B(k_{t+1})^{\alpha_{t+1}-1}) + \chi_2 = 0, \\ & -\frac{1}{1 - \alpha_{t+1}} \frac{1}{c_t} \beta^t + \frac{1}{c_{t+1}} \beta^{t+1} \left(\frac{1}{1 - \alpha_{t+1}} + B(k_{t+1})^{\alpha_{t+1}} \ln(k_{t+1}) \right) + \chi_3 = 0, \end{aligned}$$

where χ_1 , χ_2 , and χ_3 are the multipliers of the restrictions $\zeta_t \geq 0$, $K_t \geq 0$, $\alpha_t \geq 0$.

From the FOC it follows that:

- (I) If $\chi_1 = 0$ then $\chi_2 \neq 0$.
- (II) $\chi_2 \neq 0$ iff $\chi_3 \neq 0$.
- (III) If $\chi_2 = \chi_3 = 0$ then $\chi_1 \neq 0$ and

$$\alpha_{t+1} (k_{t+1})^{\alpha_{t+1}-1} = (1 - \alpha_{t+1}) (k_{t+1})^{\alpha_{t+1}} \ln(k_{t+1}).$$

Rearranging,

$$\alpha_{t+1} = \frac{k \ln(k)}{1 + k \ln(k)}.$$

Therefore, $\Delta\alpha = \frac{\ln k + 1}{(1 + k \ln(k))^2} \Delta k$.

Using Equation (14) again,

$$\Delta\alpha = (1 - \alpha)^2 (k \ln(k) + 1) \Delta k, \text{ and } \frac{\Delta\alpha}{\alpha} = (1 - \alpha) \left(\frac{\ln(k) + 1}{\ln(k)} \right) \frac{\Delta k}{k}.$$

A.3.2. The Marginal Productivity of Capital

Consider the marginal productivity of capital and take logarithms

$$\ln(\alpha B k^{\alpha-1}) = \ln B + \ln \alpha - (1 - \alpha) \ln k.$$

Therefore the growth rate of the marginal productivity of capital is given by,

$$\frac{\Delta MPK}{MPK} = \frac{\Delta\alpha}{\alpha} - (1 - \alpha) \frac{\Delta k}{k} + \Delta\alpha \ln k.$$

Combining with Equation (15),

$$\frac{\Delta MPK}{MPK} = (1 - \alpha) \left(\frac{1}{\ln(k)} \right) \frac{\Delta k}{k} + \Delta\alpha \ln k.$$

Therefore, $\frac{\Delta MPK}{MPK} > 0$.

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Mailing Address: Hernando Zuleta, Centro de Estudios Sobre Desarrollo Económico (CEDE), Facultad de Economía, Universidad de los Andes, Cra 1 N° 18A- 12 Bogotá, Colombia. E-mail: hernando.zuleta@gmail.com.

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