

**CORRUPTION IN PUBLIC PROCUREMENT AUCTIONS:  
POSITIVE EQUILIBRIUM ANALYSIS, INCENTIVE MECHANISM  
DESIGN, AND EMPIRICAL STUDY**

MARIA M. WIHARDJA \*

*University of Indonesia*

We study how poor quality of institution, such as corruption in public procurement auction, could hurt welfare. We show how competition effect could improve the cost-efficiency but not the quality of a public procurement auction with corruption. In fact, no incentive mechanism can be efficient in this auction if qualities are non-contractible. An empirical study suggests that increasing the number of bidders does increase the percentage cost efficiency albeit at a decreasing rate and decreases the percentage cost efficiency after it reaches a certain number of bidders.

*Keywords:* Auctions, Procurement, Corruption

*JEL classification:* D44, H57, D73

## 1. INTRODUCTION

In this paper, we provide a positive equilibrium analysis for a two-dimensional public procurement auction with corruption, when the type space is two-dimensional in cost and quality, independently distributed. First, we show the case without corruption in an  $N$ -bidder auction. We discuss the efficiency of this auction, comparative static analysis, and the convergence results as the number of bidders increase. This paper contributes to the literatures on auctions and corruption by introducing incompleteness of information in auctions with two-dimensional-type bidders and extending a two-bidder auction into an  $N$ -bidder auction. With corruption, we analyze the

\* I would like to express my gratitude to Pedro Dal Bó, Enrico Spolaore, Andrés Carvajal, Ani Guerdjikova, David Easley, Peter Cramton, Iwan J. Azis, Christopher Cotton, Dino Gerardi, Walter Isard, Nancy Brooks, Marcela Gonzales-Rivas, Tom Pepinsky, Françoise Vermeylen, everybody who helped in conducting the empirical study (CSIS, BAPPENAS, DPU in Jakarta), participants at TWIPS (Cornell University, February 4, 2008), AEM Discussion Group (Cornell University, March 26, 2008), RS Discussion Group (Cornell University, April 11, 2008), RES Fourth Ph.D. Dissertation (London, January 17, 2009).

equilibrium bidding and bribing strategies in an incomplete information setting with a finite type space and an infinite strategy space.

The second section of this paper discusses the incentive mechanism design for an unobservable and non-contractible quality. We discuss the impossibility result and “constrained efficiency” (Dasgupta and Maskin, 2000; Jehiel and Moldovanu, 2001; Maskin, 1992) on this multidimensional-type-bidder model. The third section of this paper shows empirical evidence of the effect of number of bidders on the percentage cost efficiency of the auctions. We took 1,404 Semi Electronic public procurement auctions from Indonesia’s Department of Public Work in 2006, and we showed a quadratic relationship between the number of bidders and the percentage cost efficiency, which suggests that increasing the number of bidders increases the percentage cost efficiency at a decreasing rate and it starts to decrease the percentage cost efficiency after some number of bidders. The gap between the theoretical predictions and the empirical study may be due to factors, other than the competition effect, such as inefficiency of selection process with a very large number of bidders, that are not captured in the theoretical model.

## 2. LITERATURE REVIEW

Literature on corruption in first-price, sealed-bid public procurement auctions that is closest to our model is by Burguet and Che (2004). In our model, we study the  $N$ -bidder case while in their model there are only two bidders. Also, we study the incompleteness of information model, while in their model information is complete. Unlike the multidimensional type of the bidders in cost and quality in this paper, most of the literatures on auctions and corruption only study auctions in the presence of corruption with one-dimensional-type bidders (see Burguet and Perry, 2007; Lengwiler and Wolfstetter, 2005; Arozamena and Weinschelbaum, 2005; Menezes and Monteiro, 2006; Esō and Schummer, 2004).

Asker and Cantillon (2008) analyze the equilibrium of a scoring auction without corruption when the type space is multidimensional. In our paper, we also use the concept of “pseudotype” as it is used in Asker and Cantillon for auctions without the presence of corruption. This study gives a result in the case of a multidimensional type space that parallels the equilibrium result in Che (1993). The general existence of equilibrium result for an incomplete information game is given in Athey (2001, 1997). We will use the definition of efficiency by Dasgupta and Maskin (2000). Dasgupta and Maskin (2000), Jehiel and Moldovanu (2001), and Maskin (1992) introduce what is called “constrained efficiency” in the case of multidimensional-common values problem. We show in this paper that with non-contractible quality an auction is constrained efficient in which cost is minimized but quality is chosen randomly.

### 3. THE MODEL WITHOUT CORRUPTION

Each bidder  $i$  has a type that is private information:  $(q_i, c_i^0) \in \mathfrak{R}_+^2$ , where  $q_i$  is the quality and  $c_i^0$  is the initial cost. We first assume that  $F(q_i) = F(c_i^0) = \text{Uniform } [0,1]$  and later, generalize. In this model, we will specifically focus on what is called the scoring auction. In this auction, each bidder is evaluated by a score that is a combination of the quality and the bid. An example of the score in a procurement auction is a linear combination of the quality and the bid. In this model, the central planner uses a scoring rule we denote by  $s_i = q_i - c_i$ , where  $c_i$  is the bid. Let  $u_i = (c_i - c_i^0)p(s_i \geq s_{-i})$ . The strategy of each bidder is a mapping from his or her type in a two-dimensional real number to his or her bid in a one-dimensional real number:  $\mathfrak{R}_+^2 \rightarrow \mathfrak{R}_+$ . We denote  $F(v_i)$  as the cumulative distribution function of  $v_i$ , where  $v_i$  is the quality minus the cost, and  $\tilde{s}_i$  as a deviation from the scoring rule at the equilibrium,  $s_i$ .

Proposition 1 gives an explicit equilibrium bidding function in the case of a two-dimensional auction with two symmetric bidders and two-dimensional type of cost and quality. We use a substitution method in our proof. We could also use the method of a change variable used by Che (1993) that produces the same result.

*Proposition 1.* Without corruption, the equilibrium bidding strategy is given by:

$$c(v_i, c_i^0) = c_i^0 + \frac{\int_{v_i}^{v_i} F(\delta) d\delta}{F(v_i)},$$

where  $v_i = q_i - c_i^0$ .

Proofs of Propositions 1 to 8 and of Corollaries 1 to 3 are available upon request.

*Corollary 1.* The auction is efficient:

$$\frac{ds_i}{dv_i} = 1 - \frac{F(v_i)^2 - p(v_i) \int_{-1}^{(v_i)} F(\delta_i) d\delta_i}{F(v_i)^2} > 0.$$

*Example 1.*  $F(c_i^0) = F(q_i) = \text{Uniform } [0,1]$ .

For  $F(q_i) = F(c_i^0) = \text{Uniform } [0,1]$ ,  $F(v_i)$  is given by:

$$F(v_i) = \begin{cases} \frac{(v_i+1)^2}{2}, & -1 \leq v_i \leq 0 \\ 1 - \frac{(1-v_i)^2}{2}, & 0 < v_i \leq 1 \end{cases}.$$

The bidding function  $c(v_i, c_i^0)$  without corruption is given by:

$$c(v_i, c_i^0) = c_i^0 + \frac{\int_{-1}^{v_i} \frac{(\delta+1)^2}{2} d\delta}{\left(\frac{(v_i+1)^2}{2}\right)}, \text{ for } -1 \leq v_i \leq 0.$$

$$c(v_i, c_i^0) = c_i^0 + \frac{\int_{-1}^0 \frac{(\delta+1)^2}{2} d\delta + \int_0^{v_i} \left(1 - \frac{(1-\delta)^2}{2}\right) d\delta}{\left(1 - \frac{(1-v_i)^2}{2}\right)}, \text{ for } 0 < v_i \leq 1.$$

$$c(v_i, c_i^0) = c_i^0 + \frac{\frac{1}{6}v_i^3 + \frac{1}{2}v_i^2 + \frac{1}{2}v_i + \frac{1}{6}}{\left(\frac{(v_i+1)^2}{2}\right)}, \text{ for } -1 \leq v_i \leq 0.$$

$$c(v_i, c_i^0) = c_i^0 + \frac{\frac{1}{6} + \left(\frac{1}{2}v_i + \frac{1}{2}v_i^2 - \frac{1}{6}v_i^3\right)}{\left(1 - \frac{(1-v_i)^2}{2}\right)}, \text{ for } 0 < v_i \leq 1.$$

Note for  $v_i = -1$ , the second term of  $c(v_i, c_i^0)$  requires an application of the L'Hôpital's rule twice:

$$\lim_{v_i \rightarrow -1} \frac{\frac{d}{dv_i} \left( \frac{1}{6}v_i^3 + \frac{1}{2}v_i^2 + \frac{1}{2}v_i + \frac{1}{6} \right)}{\frac{d}{dv_i} \left( \frac{(v_i+1)^2}{2} \right)} = \frac{\frac{1}{2}v_i^2 + v_i + \frac{1}{2}}{(v_i+1)} = \frac{0}{0}.$$

$$\lim_{v_i \rightarrow -1} \frac{\frac{d}{dv_i} \left( \frac{1}{2}v_i^2 + v_i + \frac{1}{2} \right)}{\frac{d}{dv_i} (v_i+1)} = \frac{v_i+1}{1} = 0.$$

*Proposition 2.* Without corruption, the equilibrium bid increases in quality in the game with  $F(q_i) = F(c_i^0) = \text{Uniform } [0,1]$ .

Proposition 2 states that without corruption, in the game with uniform  $[0,1]$  distributions the equilibrium bid increases with the quality.

*Proposition 3.* Without corruption, the bid increases with the bidder's cost.

*Proposition 4.* Without corruption, the probability of the bidder winning depends positively on the bidder's quality.

*Proposition 5.* Without corruption, the probability of the bidder winning depends negatively on the bidder's cost.

Proposition 1, 3, 4, 5 and Corollary 1 generalize to any Probability Distribution Functions of  $q_i$  and  $c_i^0$ :  $F(q_i)$ ,  $F(c_i^0)$ .

*Corollary 2.* (N-Symmetric Player Game) Without corruption, the equilibrium bidding strategy in the  $N$ -symmetric player game is given by:

$$c(v_i, c_i^0) = c_i^0 + \frac{\int_{v_i}^{v_i} F(\delta)^{N-1} d\delta}{F(v_i)^{N-1}},$$

where  $v_i = q_i - c_i^0$ .

*Corollary 3.* Without corruption, the equilibrium bidding strategy in the  $N$ -symmetric player game decreases in  $N$  and converges to the true cost.

*Example 2.* For  $F(q_i) = F(c_i^0) = \text{Uniform } [0,1]$ , the bid in an  $N$ -player game is given by:

$$c(v_i, c_i^0) = c_i^0 + \frac{\int_{-1}^{v_i} \left[ \frac{(\delta+1)^2}{2} \right]^{N-1} d\delta}{\left( \frac{(v_i+1)}{2} \right)^{N-1}}, \text{ for } -1 \leq v_i \leq 0.$$

$$c(v_i, c_i^0) = c_i^0 + \frac{\int_{-1}^0 \left[ \frac{(\delta+1)^2}{2} \right]^{N-1} d\delta + \int_0^{v_i} \left[ \left( 1 - \frac{(1-\delta)^2}{2} \right) \right]^{N-1} d\delta}{\left( 1 - \frac{(1-v_i)^2}{2} \right)^{N-1}}, \text{ for } 0 < v_i \leq 1.$$

**Table 1.** Example 2: Bids Weakly Decrease in  $N$ 

$N/(c_i^0, q_i)$	(0,0)	(0,1)	(1,0)	(1,1)
2	0.333	1.167	1	1.333
4	0.143	0.650	1	1.143
5	0.111	0.576	1	1.111
10	0.053	0.401	1	1.053
20	0.026	0.282	1	1.025
50	0.010	0.177	1	1.010

*Notes:* That at  $v_i = -1 (c_i^0 = 0, q_i = 1)$ , the bid remains at 1. At  $v_i = -1$ , the second term of the  $c_i$  function is zero after we apply L'Hopital's rule. Moreover, we notice that as  $N$  increases the bids converge downwards to the true cost.

#### 4. THE MODEL WITH CORRUPTION

There are  $N+2$  players in this game: the central planner, the public official, and the  $N$  bidders. The central planner is a non-strategic, virtual, dummy, or passive player and chooses the winning bidder based on reported quality bids and cost bids. The central planner may represent a government whose utility reflects the social welfare that is the quality minus the cost of the project. We also assume that the public official is a non-strategic player whose moves are determined by nature and whose decision-making process is based on pair-wise comparison as explained below. The strategies of the non-strategic central planner and the non-strategic public official are consistent with the dominant strategies of a strategic central planner and a strategic public official.

The main purpose of treating the central planner and the public officials as non-strategic players is because it simplifies the  $N+2$ -player game into an  $N$ -player game enabling us to focus on the strategic behaviors of the bidder. Treating the central planner and the public official as non-strategic players does not change how the central planner and the public official move because a strategic central planner and a strategic public official have dominant strategies, and we assume that non-strategic central planner and the non-strategic public official will play these dominant strategies. The only difference between this model and the model with a strategic public official and a strategic central planner is that in the case of a tie in which a strategic central planner and a strategic public official are indifferent among a set of actions, bidders may have subjective beliefs about what actions are chosen by the central planner and the public official that do not have to coincide with the actual moves.<sup>1</sup> This is called the subjective belief model.

<sup>1</sup> We may think a horse race as an example of the subjective belief model in which bettors have subjective beliefs about which horse is going to win.

In the case of discrete payoffs, which is often the case in real life, tie-breaking rule is not trivial. We introduce the subjective belief model to tackle the tie-breaking issue. In this subjective belief model, bidders have subjective beliefs about what is going to happen in the case of a tie, i.e., who is going to win. Since the beliefs of the bidders are subjective, they can be different. Moreover, these beliefs do not have to coincide with the actual strategies of the central planner and the public official, who are assumed to be non-strategic players or “dummy” players. In other words, the beliefs of the bidders are non-equilibrium beliefs.

The non-equilibrium beliefs are justified by assuming that the non-bidders whose strategies constitute the bidders' subjective beliefs are non-strategic players or “dummy” players. This is motivated by the uncertainty of the bidders about the non-bidders' strategies given the weak institutions that are very common in developing countries. Central planner's decisions and public official's actions are often best described as accidental rather than strategic, which can hardly form any common objective beliefs among observers. Hence, this is what motivates the adoption of subjective belief model and the adoption of non-equilibrium behaviors for the non-bidders by excluding them from the game.<sup>2</sup>

The actual tie-breaking rule of the central planner and the public official is unknown. However, the bidders have subjective beliefs of what it might be. Uncertainty of the bidders about the moves of the non-bidders is often experienced in real life particularly in a public procurement setting where the central planner or the public official act seemingly erratically, probably because of political, social, monetary and other biases, or where there is a lack of evidence, observation, or experience to form a common objective belief a priori. Even when there are a few observations, it may not be possible for the bidders to guess the probability of a mixed strategy played by the central planner and the public official.

We start this section by stating an example with a strategic public official and a strategic central planner as a benchmark to the model with a non-strategic public official and a non-strategic central planner. In both models, the equilibrium bids and bribes do show similar strategic behaviors of the bidders with respect to their types.

#### 4.1. Bidder

Each bidder  $i$  has a type denoted by  $t_i$  that is two-dimensional in  $q_i$  and  $c_i^0$ , where  $q_i$  is the quality and  $c_i^0$  is the initial cost. The strategy of bidder  $i$  is denoted by  $s_i$  that is a two-tuple strategy in  $c_i$  and  $f_i$ , where  $c_i$  is the bid and  $f_i$  is the bribe. There are  $N$  player that is  $I = \{1, \dots, N\}$ , and  $i$  denotes an individual player. Denote  $q_i^m$

<sup>2</sup> Non-equilibrium belief model has been adopted by Costa-Gomes and Zauner (2003), other non-equilibrium model is the level- $k$  model that has been used by Crawford and Iriberri (2007).

as the post-manipulated quality that is the quality that the public official reports to the central planner, and  $s_i^m$  as the post-manipulated scoring rule that is equal to the post-manipulated quality minus the bid. The two-bidder model can be represented in the following game as follows:

$$\Gamma(N=2) = \{I = \{1,2\}, \{s_i = (c_i, f_i)\}_{\forall i \in \{1,2\}}, \{u_i(c, f, q^m | (c_i^0, q_i))\}_{\forall i \in \{1,2\}}, \\ \{t_i = (c_i^0, q_i)\}_{\forall i \in \{1,2\}}, \{(F(c_i^0), F(q_i))\}_{\forall i \in \{1,2\}}\},$$

where, bidders are symmetric;

$t_i \in T_i$ , where  $T_i$  is the type space of bidder  $i$ ;

$s_i : T_i \rightarrow \mathfrak{R}^2$ , where  $T$  is the type space of bidder  $i$ ;

$u(c, f, q^m | (c_i^0, q_i))$ :

$u(c, f, q^m | c_i^0, q_i) = (c_i - c_i^0 - f_i)(p(s_i^m \geq s_{-i}^m))$ ,

$p(s_i^m \geq s_{-i}^m) = p(q_i^m - c_i \geq q_{-i}^m - c_{-i} | f_i \geq f_{-i})p(f_i \geq f_{-i})$

$+ p(q_i^m - c_i \geq q_{-i}^m - c_{-i} | f_{-i} > f_i)p(f_{-i} > f_i), \quad \forall i = 1, 2.$

Note that bribes are contingent on the bidders winning and the quality report  $q^m$  is the strategy of the public official that is a function of bribes.

#### 4.2. Public Official

The utility function of the public official,  $A$ , is given by:  $u_A = f_1 p_1 + f_2 p_2$ , where  $p_i$  is the probability of bidder  $i$  winning. The strategy of the public official is a mapping from bribes offered to quality reports, that can be written by:  $q^m : \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$ .

#### 4.3. Central Planner

The utility function of the central planner,  $P$ , is given by:  $u_P = (q_1 - c_1)p_1 + (q_2 - c_2)p_2$ ,  $p_2 = 1 - p_1$ . The strategy of the central planner is to choose a winning bidder given the quality report and the cost bids by choosing a bidder with the highest score,  $s_i^m$ . The strategy of the central planner can be written as:  $p_1 : \mathfrak{R}^4 \rightarrow \mathfrak{R}$ . Note, that even though the central planner only gets the manipulated quality reports, the central planner cares about the true quality. Suppose in this model that the central planner is unaware of the bribery that takes place.



#### 4.4. Time Line

The sequence of the game is as follows. At time 0, nature chooses the types of bidders,  $\{c^0, q\}$ , and the types of bidders are privately observed by all bidders. The central planner does not observe the cost nor quality of the bidders. The central planner only observes the quality but not the cost. At time 1, the bid,  $c_i$ , is submitted to the central planner and (contingent) bribe,  $f_i$ , is offered to the public official simultaneously and privately by each bidder. At time 2, the public official observes  $q = \{q_1, q_2\}$  and reports  $q^m = \{q_1^m(f_1, f_2), q_2^m(f_1, f_2)\}$  that is not necessarily equal to  $q$  to the central planner without any information about  $c$ . At time 3, the central planner evaluates each bidder by a scoring rule:  $s_i^m = q_i^m - c_i$ , and chooses the winning bidder  $i$ ,  $i | s_i^m = \text{Max}\{s_1^m, s_2^m\}$ .

The immediate properties that can be derived from the utility functions of the bidders, public official, and central planner are the following.

*Lemma 1. (Type-Relevant Strategy)* Since the utility of the bidders is non-trivially dependent on  $c_i^0$  and is trivially dependent on  $q_i$ , then the only incompleteness of the payoff-relevant information is  $c_i^0$  and the equilibrium strategy will be only non-trivially dependent on  $c_i^0$ .

Proofs of Lemmas 1 to 3 are available upon request.

Lemma 1 says that since the utility of the bidder is only  $c_i^0$ -dependent, i.e., the incompleteness of payoff-relevant information comes only from one element of the bidder's type,  $c_i^0$ , and  $q_i$  does not affect the utility of the bidder, then the equilibrium strategy of the bidders will also be only dependent on  $c_i^0$  alone.

*Proposition 6. (Efficiency-the Impossibility Result)* An auction with the two-dimensional bid and two-dimensional type space with corruption cannot be efficient.

## 5. SUBJECTIVE BELIEFS MODEL

In this section, we simplify the model by assuming the following two conditions: (1) the public official is a non-strategic or dummy player whose moves are determined by nature and are consistent with the dominant strategy of a strategic public official; the central planner is a non-strategic or dummy player whose moves are determined by nature and are consistent with the dominant strategy of a strategic player; (2) in an

$N$ -player game, the nonstrategic public official moves according to a pair-wise comparison instead of a group-wise comparison as we will explain further below. Since both the public official and the central planner are dummy players, we allow the beliefs of the bidders on how the public official and the central planner move in the case of indifferences over possible actions to be subjective. In other words, the bidders do not have to correctly predict the moves of the public official and the central planner at the equilibrium. The actual moves of the public official and the central planner may therefore be inconsistent with the bidders' subjective beliefs. Condition one therefore allows us to assume that bidders' beliefs about the move of the public official and the central planner in the case of a tie of bribes and scores in which the public official and the central planner are indifferent over actions to take to be *subjective* beliefs instead of the objective beliefs. Subjective beliefs are common knowledge. We consider two examples where bidders behave differently under uncertainty. One is an example in which both bidders make decisions based on the best possible scenario over a set of probabilities. In this example, each bidder of the two or  $N$  bidders believes that in the case of a tie of bribe and scores, he/she will be favored by the public official and the central planner with probability one. Second is an example of the opposite case in which bidders make decisions over a set of actions based on the worst possible scenario they can possibly get over a set of probabilities.<sup>3</sup> In this example, each bidder believes that in the case of a tie of bribes and scores, the other bidder will be favored by the public official and the central planner with probability one.<sup>4</sup> We take these two extreme cases to make sure that solutions exist at the extreme cases, and if they do then solutions are also likely to exist in the intermediate cases.

Condition two allows us to simplify our calculation on the powers and combinations of the probabilities of the bidders winning in the case of an  $N$ -bidder auction. The difference between pair-wise and group-wise is essentially this: evaluating the probability of winning of a bidder  $i$  over bidder  $j$ , group-wise comparison takes into account the values (or parameters of interest) of all other bidders, while pair-wise comparison only takes into account the values (or parameters of interest) of bidder  $i$  and  $j$ .

The following Lemma will give a formal proof of how a strategic corrupt public official whose utility is only the bribe he/she receives will behave. This strategy of a strategic public official will underlie our assumption of how a non-strategic public

<sup>3</sup> This is what is also called the *Maxmin* decision rule (Gilboa and Schmeidler, 1989; Knight, 1921; Bewley, 1986).

<sup>4</sup> To axiomatize the different subjective beliefs of the bidders and represent it in a utility-functional form, we may use Chateauneuf, Eichberger, and Grant (2007) (CEG). These *neo-additive* beliefs can be represented by the multiple priors form. The notion of Nash Equilibrium in the model under uncertainty aversion is weaker since perfect consistency between beliefs and the actual plays fails. However, in our examples, since uncertainty is over the strategies of a fictitious player (i.e., the public official or the central planner) and not over the other strategic bidders' strategies, we do not need to check for these inconsistencies.

official is expected to move.

*Lemma 2.* (Dominant Strategy of A Strategic Public Official) Suppose a public official has a utility function as follows:  $u_A = \sum_{k \in I} f_k p_k$ , where  $f_i \in \mathfrak{R}_+^1$ ,  $0 \leq p_i \leq 1$ , and  $\sum_{i=1}^N p_i = 1$ . Then, the solution to the maximization problem of the public official,  $\max_{\{p_i\}_{i \in I}} u_A$ , s.t.  $f_i \in \mathfrak{R}_+^1$ ,  $0 \leq p_i \leq 1$ , and  $\sum_{i=1}^N p_i = 1$ , is given by the following:  $p_i[f_i - \max_{k \in I} f_k] = 0$ ,  $\forall i \in I$ .

Note that by “dominant strategy” here, we mean that regardless of what the bidders’ bribes are, a strategic public official will always play the above class of strategies. These strategies themselves among this class of strategies are not unique (in the case of a tie, the public official is indifferent among a set of mixed strategies). Hence, the above strategies are dominant only among classes of strategies but not among strategies.

Similarly, the strategy that underlies the move of a non-strategic central planner is the dominant strategy of a strategic central planner, which is given by the following:

*Lemma 3.* (Dominant Strategy of a Strategic<sup>5</sup> Central Planner) Suppose a central planner has a utility function as follows:  $u_p = (q_1 - c_1)p_1 + (q_2 - c_2)p_2$ . Then, the solution to the maximization problem of the central planner,  $\max_{p_1, p_2} u_p$ , s.t.  $0 \leq p_i, p_2 \leq 1$ , is given by the following:  $p_i[(q_i^m - c_i) - \max_{k \in \{1, 2\}} (q_k^m - c_k)] = 0$ ,  $\forall i \in I$ .

### 5.1. Positive Bidders

We are going to assume that there are two types of bidders whom we are going to call the “positive” bidders and the “negative” bidders. Positive bidders makes decisions based on the best possible scenario while negative bidders makes decisions based on the worst possible scenario. The set of subjective beliefs of the bidders in a two-player game can be illustrated as follows

$$r_i = \text{subjective belief of } i \text{ on } \{1 \text{ wins, } 2 \text{ loses}\} \text{ if } \{q_1^m - c_i = q_2^m - c_2\}, \quad r_i \in [0, 1],$$

$$o_i = \text{subjective belief of } i \text{ on } \{q_1^m = 1, q_2^m = 0\} \text{ if } \{f_1 = f_2\}, \quad o_i \in [0, 1].$$

Note that the probability space of the prior beliefs of the bidders are the same that is

<sup>5</sup> “Strategic” here means maximizing social welfare,  $q_i - c_i$ , without considering different types of auctions. The strategy space for a social planner is the probabilities of winning assigned to bidder 1 and bidder 2.

in the interval  $[0,1]$  for both  $r_i$  and  $o_i$ . In the first example of positive bidders, we assume the following.

*Condition 1.* Positive Bidders (2- player and  $N$ -player games)

1. The public official is a non-strategic player or a dummy player. The uncertainty over the public official's behavior is illustrated as follows:

$$q^m = \left( \begin{array}{l} \left( \begin{array}{l} (1,0), \text{ if } f_1 > f_2 \\ (0,1), \text{ if } f_1 < f_2 \end{array} \right) \\ (1,0) \text{ with probability } o_i, \text{ if } f_1 = f_2 \\ (0,1) \text{ with probability } (1-o_i), \text{ if } f_1 = f_2 \end{array} \right), \quad o_i \in [0,1], \quad i = 1,2.$$

The subjective probabilities of the bidders on the public official's behavior are as follows: bidder 1's subjective belief is  $o_1 = 1$  and bidder 2's subjective belief is  $o_2 = 0$ .

2. The central planner is a non-strategic player or dummy player. The uncertainty over the central planner's moves can be illustrated as follows:

$$\pi = \left( \begin{array}{l} \left( \begin{array}{l} (1,0), \text{ if } q_1^m - c_1 > q_2^m - c_2 \\ (0,1), \text{ if } q_1^m - c_1 < q_2^m - c_2 \end{array} \right) \\ (1,0) \text{ with probability } r_i, \text{ if } q_1^m - c_1 = q_2^m - c_2 \\ (0,1) \text{ with probability } (1-r_i), \text{ if } q_1^m - c_1 = q_2^m - c_2 \end{array} \right),$$

where  $r_i \in [0,1]$ ,  $i = 1,2$ ,  $\pi = (\pi_1, \pi_2)$  is the probability of bidder 1 and bidder 2 winning respectively. Bidder 1's subjective belief is  $r_1 = 1$  and bidder 2's subjective belief is  $r_2 = 0$ .

3. Probability of the bidders winning is based on pair-wise comparison and not group-wise comparison. The difference between a pair-wise comparison and a group-wise comparison can be illustrated by the following tables.

Let for example, bribes are  $f = (50,50,100)$ . Then, based on pair-wise comparison, the probabilities of the bidders winning given that bidder 1 and 2 offer the same bribe that is strictly smaller than bidder 3's bribe are given by the following table.

**Table 2.** Pair-wise Comparison

$i-i$	1	2	3	$P_i$
1	1	$p_1(c_2 \geq c_1 - 1)$	$p_1(c_3 \geq c_1 + 1)$	$p_1(c_2 \geq c_1 - 1)p_1(c_3 \geq c_1 + 1)$
2	$p_2(c_1 \geq c_2 - 1)$	1	$p_2(c_3 \geq c_2 + 1)$	$p_2(c_1 \geq c_2 - 1)p_2(c_3 \geq c_2 + 1)$
3	$p_3(c_1 \geq c_3 - 1)$	$p_3(c_2 \geq c_3 - 1)$	1	$p_3(c_1 \geq c_3 - 1)p_3(c_2 \geq c_3 - 1)$

Each cell represents the probability of winning of a row-bidder against one other column-bidder given only the bribes of these two bidders (pair-wise comparison). The probability of bidder 1 winning, for example, is the product of the cells of row 1,  $p_1(c_2 \geq c_1 - 1)p_1(c_3 \geq c_1 + 1)$  Group-wise comparison table is given by the following table:

**Table 3.** Group-wise Comparison

$i-i$	1	2	3	$P_i$
1	1	$p_1(c_1 \leq c_2)$	$p_1(c_3 \geq c_1 + 1)$	$p_1(c_1 \leq c_2)p_1(c_3 \geq c_1 + 1)$
2	$p_2(c_2 \leq c_1)$	1	$p_2(c_3 \geq c_2 + 1)$	$p_2(c_2 \leq c_1)p_2(c_3 \geq c_2 + 1)$
3	$p_3(c_1 \geq c_3 - 1)$	$p_3(c_2 \geq c_3 - 1)$	1	$p_3(c_1 \geq c_3 - 1)p_3(c_2 \geq c_3 - 1)$

Note that the difference from the pair-wise comparison. Here, each cell represents the probability of winning of a row-bidder against one other column-bidder given the bribes of **all** of the bidders (group-wise comparison).

These two conditions essentially allow the utility function of the bidders to be written as follows:

1. Two-player game:

$$u(c, f, q^m | c_i^0, o_i, r_i) = (c_i - c_i^0 - f_i)(p(s_i^m \geq s_{-i}^m)),$$

$$p(s_i^m \geq s_{-i}^m) = p(c_{-i} \geq c_i - 1)p(f_i \geq f_{-i}) + p(c_{-i} \geq c_i + 1)p(f_{-i} > f_i), \quad \forall i = 1, 2.$$

2.  $N$ -player game:

$$\begin{aligned} u(c, f, q^m | c_i^0, o_i, r_i) &= (c_i - c_i^0 - f_i)(p(c_{-i} \geq c_i - 1)^{N-1} p(f_i \geq f_{-i})^{N-1} \\ &+ \sum_{j=1}^{N-2} \binom{N-1}{j} p(c_{-i} \geq c_i - 1)^{N-1-j} p(c_{-i} \geq c_i + 1)^j p(f_i \geq f_{-i})^{N-1-j} p(f_i < f_{-i})^j \\ &+ p(c_{-i} \geq c_i + 1)^{N-1} p(f_i < f_{-i})^{N-1}). \end{aligned}$$

Note that the utility of the bidders now depend on the subjective beliefs,  $(o_i, r_i)$ . Note that  $\binom{N-1}{j} = \frac{(N-1)!}{(N-1-j)!(j)!}$  is the formula for a combination without replacement when order is not important.

The general structure of the game with positive bidders in which Condition 1 above holds given Lemma 1 can be written as follows:

$$\Gamma(N) = \{I = \{1, 2, \dots, N\}, \{s_i = (c_i, f_i)\}_{\forall i \in \{1, 2, \dots, N\}}, \{u_i(c, f | c_i^0, o_i, r_i)\}_{\forall i \in \{1, 2, \dots, N\}}, \\ \{c_i^0\}_{\forall i \in \{1, 2, \dots, N\}}, \{F(c_i^0)\}_{\forall i \in \{1, 2, \dots, N\}}\},$$

$$u(c, f, q^m | c_i^0, o_i, r_i) = (c_i - c_i^0 - f_i)(p(c_{-i} \geq c_i - 1)^{N-1} p(f_i \geq f_{-i})^{N-1} \\ + \sum_{j=1}^{N-2} \binom{N-1}{j} p(c_{-i} \geq c_i - 1)^{N-1-j} p(c_{-i} \geq c_i + 1)^j p(f_i \geq f_{-i})^{N-1-j} p(f_i < f_{-i})^j \\ + p(c_{-i} \geq c_i + 1)^{N-1} p(f_i < f_{-i})^{N-1}).$$

*Example 3.* Discrete Type Space and Infinite Strategy Space with  $N$  players (Multiplicity of Equilibria)

Let:  $i = 1, 2, \dots, N$ , and  $\forall i \in I$ .

Type space:  $c_i^0 \in \{0, 1\}$ ,  $p(c_i^0) = \frac{1}{2}$ .

Strategy and Strategy space:  $(c_i, f_i) : \{0, 1\} \rightarrow \mathfrak{R}_+^0$ .

$u(c, f, q^m | c_i^0, o_i, r_i) = (c_i - c_i^0 - f_i)p_i$ , where

$$p_i = (p(c_{-i} \geq c_i - 1)^{N-1} p(f_i \geq f_{-i})^{N-1} + \sum_{j=1}^{N-2} \binom{N-1}{j} p(c_{-i} \geq c_i - 1)^{N-1-j} \\ p(c_{-i} \geq c_i + 1)^j p(f_i \geq f_{-i})^{N-1-j} p(f_i < f_{-i})^j + p(c_{-i} \geq c_i + 1)^{N-1} p(f_i < f_{-i})^{N-1}).$$

$$\Gamma = \{I = \{1, 2\}, \{s_i = (c_i, f_i) \in \mathfrak{R}_+^2\}_{\forall i \in \{1, 2\}}, \{u_i(c, f | c_i^0, o_i, r_i)\}_{\forall i \in \{1, 2\}}, \\ \{c_i^0\}_{\forall i \in \{1, 2\}}, \{F(c_i^0)\}_{\forall i \in \{1, 2\}}\}.$$

Equilibrium strategy profile:  $c(c_i^0 = 1) = c(c_i^0 = 0) - 1$ ,  $f(c_i^0 = 0) > f(c_i^0 = 1)$ .

There exists a multiplicity of equilibria. The following table shows the equilibrium bids and bribes that maximize the sum of the expected utilities of the bidders (Pareto Dominant equilibrium bids and bribes) as  $N$  increases (numbers are rounded up to 5 d.p.).

(Available upon request).

Example 3 shows that the equilibrium bid converges downward as  $N$  increases. This example predicts the size of the corruption and efficiency of an  $N$ -bidder auction. As  $N$  gets very large, the low-cost bidder offers a bribe of one and the high-cost bidder offers a bribe of zero. Hence, only for the high-cost bidder does the equilibrium bribe converges to zero, while for the low-cost bidder, the equilibrium bribe converges to a positive number. The low-cost bidder bids two and the high-cost bidder bids one. Hence, only for the high-cost bidder does the equilibrium bid converges to the true cost, while for the low-cost bidder, the equilibrium bid does not converge to the true cost. Note that in Example 3, the probabilities of the high- and low-cost bidders win are equal. We can also show that the higher the probability of  $c_i^0 = 0$  is, the higher the number of bidders,  $N$ , needed for the bids and the bribes to converge.

**Table 4.** Example 3: Pareto Dominant Equilibrium Bids and Bribes

$N$	$c(c_i^0=0)$	$c(c_i^0=1)$	$f(c_i^0=0)$	$f(c_i^0=1)$	$u(c,f,q^m   c_i^0=0)$	$u(c,f,q^m   c_i^0=1)$
2	6	5	2	0	4	3
3	4.28571	3.28571	2	0	2.28571	1.28571
4	3.72973	2.72973	2	0	1.72973	0.72973
5	3.46286	2.46286	2	0	1.46286	0.45202
6	3.31114	2.31114	2	0	1.31114	0.31114
7	3.21651	2.21651	2	0	1.21651	0.21651
8	3.15405	2.15405	2	0	1.15405	0.15405
9	3.11125	2.11125	2	0	1.11125	0.11125
10	3.08118	2.08118	2	0	1.08118	0.08118
20	3.00428	2.00425	2	0	1.00425	0.00425
30	3.00024	2.00024	2	0	1.00024	0.00024
40	3.00001	2.00001	2	0	1.00010	0.00013
50	3	2	2	0	1	$P_i$
75	3	2	2	0	1	$P_i$
76	2	1	1	0	1	0
100	2	1	1	0	1	0
1000	2	1	1	0	1	0

## 5.2. Negative Bidders

Otherwise stated, the structure of the game is the same as of Example above (positive bidders). Let  $p(c_i^0 = 0) = p(c_i^0 = 1) = \frac{1}{2}$ . Given these conditions, the utility of the negative bidders can be written as follows:

$$u(c, f | c_i^0, o_i, r_i) = (c_i - c_i^0 - f_i)(p(c_{-i} > c_i - 1)p(f_i > f_{-i}) \\ + p(c_{-i} > c_i + 1)p(f_i \leq f_{-i}), \quad \forall i = 1, 2.$$

In words, each bidder believes that in the case of a tie of bribes, the other bidder will get the highest possible quality of one and he/she will get a quality of zero. In the case of a tie of scores, each bidder believes that the other bidder will win with probability one. Now, suppose we put an upper bound on the bidders' bid that is  $c_i \leq \max_{c_i^0 \in \{0,1\}} c_i^0 = 1$ . The following strategy is an equilibrium:  $c(c_i^0 = 0) \rightarrow 1$ ,  $c(c_i^0 = 1) = c(c_i^0 = 0) - 1$ ,  $f(c_i^0 = 0) \rightarrow 0.999999998$ ,  $f(c_i^0 = 1) \rightarrow 0.999999997$ . At the equilibrium,  $u(c_i^0 = 0) \rightarrow 0$   $\left( p(c_i^0 = 0) = \frac{1}{4} \right)$ , and  $u(c_i^0 = 1) = 0$   $(p(c_i^0 = 1) = 0)$ . This example suggests that negative bidders will be indifferent between entering the auction and not entering the auction. In other words, uncertainty over the moves of the public official and the central planner in the case of a tie of bribes and scores may deter bidders to enter the auction. Similar to Myerson's argument on subjective beliefs or inconsistency of beliefs among players (Myerson, 1991, p.251), in the model with a non-strategic public official, the bidders could enter the auction having an expected utility that is higher than it truly is (see the positive-bidders case above) or the bidders could opt not to enter the auction having a zero expected utility that is lower than it truly is (see the negative-bidders case).

### MAIN PROPOSITIONS

*Proposition 7.* In an  $N$ -positive-bidder auction game with corruption above in which the type space is discrete, the strategy space is infinite, the central planner and the public official are non-strategic players and bidders' beliefs on the moves of the central planner and the public official are subjective beliefs, there exist multiple equilibria with respect to the bidders' subjective beliefs of the following form:  $c(c_i^0 = 0) = c(c_i^0 = 1) + 1$ ,  $f(c_i^0 = 0) > f(c_i^0 = 1)$ ,  $\forall i = \{1, 2\}$ . At these equilibria:

$$\frac{\partial u(c, f | c_i^0, o_i, r_i)}{\partial c_i} \Big|_{c^*, f^*} = - \frac{\partial u(c, f | c_i^0, o_i, r_i)}{\partial f_i} \Big|_{c^*, f^*}, \quad \forall c_i^0 \in \{0, 1\}, \quad \forall i = \{1, 2\}.$$

*Proposition 8.* In a two-negative-bidder auction game with corruption above in which  $c_i \leq \max_{c_i^0 \in \{0,1\}} c_i^0 = 1$ , the type space is discrete, the strategy space  $(c_i, f_i)$  is in  $[0, 1] \times \mathfrak{R}_+^1$ , the central planner and the public official are non-strategic players and bidders' beliefs on the moves of the central planner and the public official are subjective beliefs, there exists an equilibrium in pure strategy with respect to the bidders' subjective



beliefs of the following form:  $c(c_i^0 = 0) = c(c_i^0 = 1) + 1$ ,  $f(c_i^0 = 0) > f(c_i^0 = 1)$ ,  $\forall i = \{1, 2\}$ .

At this equilibrium:

$$\frac{\partial u(c, f | c_i^0, o_i, r_i)}{\partial c_i} \Big|_{c^*, f^*} = - \frac{\partial u(c, f | c_i^0, o_i, r_i)}{\partial f_i} \Big|_{c^*, f^*}, \quad \forall c_i^0 = \{0, 1\}, \quad \forall i = \{1, 2\}.$$

One could extend the scope of this paper by taking bidders with different attitudes towards uncertainties, for example, both bidders believe that they win with probability one-half, one-half, or one bidder has an optimistic belief while the other has a pessimistic belief, and analyze how the existence of an equilibrium and an equilibrium strategy will change.

The more important issue here is not about a specific tie-breaking rule, but about the bidders' beliefs that are at the extreme opposites, i.e., both bidders think that they are going to win (lose) in the case of optimistic (pessimistic) beliefs. In the long run, both players will observe the same events and their beliefs will eventually converge. Further study, such as dynamic auction, is needed to resolve this issue.

#### *HOW MUCH EFFICIENCY IS LOST?*

Note that the probability of the bidder winning is not a function of the true quality of the bidders, and hence, the probability of the bidder winning in terms of quality is a perfect randomization. This is what is called "constrained efficiency" (Dasgupta and Maskin, 2000). In other words, what an auctioneer or a central planner can best hope for is cost minimization and but not full efficiency.

The distributions of type are not the same in the models with and without corruption, however, we can still predict that the presence of corruption increases bids by the following argument. In the model without a corruption, regardless of quality, the bid converges to the true cost that is 0 for  $c_i^0 = 0$  and 1 for  $c_i^0 = 1$ , while in the model with a corruption, the bid never converges to the true cost for  $c_i^0 = 0$  even though it does converge to the true cost for  $c_i^0 = 1$ . Notice, however, that the equilibrium bribe is positive only for  $c_i^0 = 0$ , while no bribe is offered by the high-cost bidder. Hence, this suggests the mark-up in the case of the low-cost bidder can be blamed partly on bribes.

## 6. INCENTIVE MECHANISM DESIGNS: A GLIMPSE

Our attempt in this paper is to design an efficient incentive mechanism design in a procurement auction model when bidders' private information are multidimensional. We restrict to unobservable and non-contractible quality. Non-contractible quality means that either qualities are ex-post non-verifiable or ex-post verifiable but there is no legal

enforcement to punish corrupt bidders and/or public officials. In some cases, this assumption is more realistic than earlier literatures that assume contractible qualities for a couple of reasons. One is that qualities of the bidders are most of the time not directly observable and ex-post are not perfectly verifiable. Second, we may also assume that the quality is not contractible even though it is ex-post verifiable because, in the worst possible case, legal enforcement is not effective and hence, corrupt bidders and/or public officials are never caught and bidders are not deterred to bribe the public officials.

### MODEL AND EXAMPLE

We restrict the number of player to two, however, the results in this section still holds for  $N$  players. An individual player is denoted by  $i=1,2$ . The mechanism designer denoted by  $P$  is also the procurer and the social planner.<sup>6</sup> The social choice function is denoted by  $f(\theta)$  that is a function of the players' types,  $\theta$ . Denote  $y_i(\theta)$  as the decision function for player  $i$ , where  $y_i(\theta)=1$  if the procured good/service goes to player  $i$ ,  $y_i(\theta)=0$  otherwise. The monetary transfer to player  $i$  is denoted by  $t_i(\theta)$ . The function  $\bar{y}_i(\theta_i)$  denotes the marginal decision function for player  $i$  given  $\theta_i$  and all agents  $j \neq i$  reveal their types truthfully. Similarly, the function  $\bar{t}_i(\theta_i)$  denoted the marginal transfer payment function to player  $i$  given  $\theta_i$  and agents  $j \neq i$  reveal their types truthfully. The mechanism  $\Gamma$  implements that the social choice function  $f(\theta)$  is there is an equilibrium strategy profile  $s^*$  such that  $g(s^*(\theta))=f(\theta)$ , where  $g(s(\theta))$  is the outcome function given  $s(\theta)$ . The strategy set of each player  $i$  is denoted by  $S_i$ .

Otherwise stated, let the social choice function be:

$$f(\theta)=[y_1(\theta), y_2(\theta); t_1(\theta), t_2(\theta)]: \Theta_1 \times \Theta_2 \rightarrow X,$$

$$\{y_i(\theta) : y_i(\theta)(v_i - \max(v_1, v_2)) = 0\}_{i=1,2}, v_i = q_i - c_i^0, \left( \sum_{i=1,2} t_i \right) = -t_p,$$

where  $X$  = the set of alternatives,  $\theta_i = (q_i, c_i^0)$ ,  $\theta = (\theta_1, \theta_2)$ ,  $\theta \in \Theta_1 \times \Theta_2$ ,  $\Theta_i =$  common knowledge,  $F(\theta_i) = \text{Uniform}[0,1]^2$ ,  $\forall \theta_i \in \Theta_i$ .

Define a direct mechanism:  $\Gamma = (S_1, S_2, g(s))$ ,  $S_i \in \Theta_i$ ,  $s \in S = S_1 \times S_2$  and  $\Gamma$

<sup>6</sup> Without a loss of generality, we can also assume that the mechanism designer who is also the procurer and the social planner is a player. The set-up of the model will be slightly different although it will not change the theoretical results.

implements  $f(\theta)$  if:  $g(s^* = \theta) = f(\theta)$ , or  $u_i(f(\theta), \theta_i) \geq u_i(f(\tilde{\theta}_i, \theta_{-i}), \theta_i)$ ,  $\forall \tilde{\theta}_i \in \Theta_i$ ,  $\forall i = 1, 2$ .

*Example 4. Two-Dimensional Private Information*

Let  $u_p = \sum_{i=1,2} (y_i(\hat{\theta})q_i - t_i)$ ;  $u(\hat{\theta}|\theta_i) = -y_i(\hat{\theta})c_i^0 + t_i$ ;  $Max.U_s = \sum_{i=1,2} y_i(\hat{\theta})v_i$ , where  $U_s = [u_p + \sum_{i=1,2} u(\hat{\theta}|\theta_i)]$ .

*Claim 1.* There does not exist an efficient mechanism in the above example.

### MAIN THEOREMS

In fact, we can put the claim above more formally by the following Theorem.

*Theorem 1.* Define a Social Choice Function,  $f(\theta) = (t(\theta), y(\theta))$ , whose objective function is  $y_i(\theta)((q_i - c_i^0) \max_{k=1,2} (q_k - c_k^0)) = 0$ ,  $\sum_i y_i(\theta) = 1$ ,  $y_i(\theta) \in [0,1]$ ,  $\forall i = 1, 2$ . The utilities of the procurer and the bidders are the following:

$u_p = \sum_{i=1,2} (y_i(\hat{\theta})q_i - t_i(\hat{\theta}))$ ;  $u(\hat{\theta}|\theta_i) = -y_i(\hat{\theta})c_i^0 + t_i(\hat{\theta})$ ;  $U_s = \sum_{i=1,2} y_i(\hat{\theta})(q_i - c_i^0)$ , then there cannot be an efficient mechanism that implements  $f(\theta)$ .

Proofs are available upon request.

This theorem comes directly from the Jehiel and Moldovanu's impossibility result theorem (Jehiel and Moldovanu, 2001), which is also proven in Dasgupta and Maskin (2000) and Maskin (1992), that states that there is no efficient mechanism when there is at least one player (the bidders) whose private information (their qualities) affects the choice of the most efficient bidder but does not directly affect the owner(s) of that information (the bidders).

## 7. EMPIRICAL STUDY: INEFFICIENCY WITH HIGH NUMBERS OF BIDDERS

We examine a case study on Land Management and Policy Development Project from BAPPENAS (Indonesia's National Development Planning Agency) in 2006 and we perform empirical study on 1,404 auctions conducted by Indonesia's Department of Public Work in 2006.<sup>7</sup> In the theoretical section above, it is argued that increasing the number of bidders increases competition, i.e., bidders bid less (and they bribe less). Hence, the appropriate empirical approach is to test whether the percentage cost efficiency of the auction (which means how much lower bidders bid relative to the original budget allocated to the auction) is positively correlated to the number of bidders.

<sup>7</sup> This section is taken from Wihardja (2007).

In the empirical study, we run a fixed-effects regression to test this.

Auctions' subjects, i.e., the pools of bidders, are not identical. The fixed-effect regression is conducted by controlling for all variables that are not identical, except for the numbers of bidders, that might give rise to different pools of bidders. The controlling variables in this fixed-effect model include the state department that conducts the auction, the province where the auction is conducted, the type of auction (or the object being auctioned), the initial budget of the auction (or the value of the auction), and the method used to conduct the auction. The regression is given by the following equation:

$$\begin{aligned} \ln(\% \text{ Cost Efficiency}) = & a + b(n - \bar{n}) + c(n - \bar{n})^2 \\ & + \sum_{i=1}^7 I(\text{Satkal}) + \sum_{i=1}^{13} I(\text{Prov}) + \sum_{i=1}^8 I(\text{Category}) \\ & + \sum_{i=1}^{10} I(\text{Value}) + \sum_{i=1}^2 I(\text{Metode}), \end{aligned}$$

where,

$I(.)$ : Indicator variable,

$\ln(\% \text{ Cost Efficiency})$ :

Natural Log of the percentage cost efficiency =  $\ln(((\text{Initial Budget-Contract Price})/(\text{Initial Budget})) \times 100\%)$ ,

$(n - \bar{n})$ : Centered Number of Bidders = Number of bidders - Mean of number of bidders,

$(n - \bar{n})^2$ : Centered Number of Bidders Squared = (Number of bidders - Mean of number of bidders)<sup>2</sup>,

Satkal: Department, Prov: Province, Category: Type of auctions, Value: Initial Budget, Metode: Method of the Auctions.

The following table is the result of the regression (Available upon request for the complete regression result):

**Table 5.** Regression Coefficients

Regression	Coefficient	t-value	$P >  t $
$b$	0.0098179	5.74	0
$c$	-0.0000882	-6.93	0
Number of Observations	1332		
$F(46,1285)$	11.95		
$Prob > F$	0		
R-Squared	0.2996		
Root MSE	0.8217		

We transform the dependent variable into the natural logarithmic form to normalize the residuals and we center the independent variables in order to avoid multicollinearity between the number of bidders and the squared number of bidders. We drop one observation that has a very high residual (an outlier). We drop observations that use Direct Selections/Auctions<sup>8</sup> and Direct Appointment methods. Direct Selections/Auctions methods are used for low values projects and hence observations that fall into Direct Selections/Auctions also fall into one of the dummy variables for “Value” (or initial budget). In order to avoid multicollinearity, we drop these observations. Direct Appointments directly appoints one bidder and hence, it could not capture the effect of the number of bidders on the percentage cost efficiency.

The regression result suggests the concave relationship between the number of bidders and the natural log of the percentage cost efficiency. Both the linear and the quadratic coefficients are significant. The optimum point of the concave graph is at 149 bidders. In other words, percentage cost efficiency starts to *decline* as the number of bidders increases at 149 bidders. The mean of the number of bidders in this study is 37.97166. Although the theoretical model above captures the competition effect as the number of bidders increases, this empirical study suggests that increasing the number of bidders can have a negative effect for high numbers of bidders above 149 bidders. The factors that could negatively affect the percentage cost efficiency of the auctions with high numbers of bidders may include an inefficient selection process with too many bidders that may not be directly related to the competition effect. A theoretical model that could capture this effect and explaining the discrepancy between the theoretical predictions and the empirical study will be useful for future study.

## 8. CONCLUSION

The main contribution of this paper is to show that the equilibrium bids and bribes in the presence of corruption with a discrete type space and an infinite strategy space decrease as the number of bidders increases. The analysis on the incentive mechanism design shows a disappointing result that when quality is not fully verifiable and quality is not contractible, there is no mechanism that is efficient. The second-best option is to minimize cost but to randomize the choice of quality. The empirical study on Semi E-Procurement from Indonesia’s Department of Public Work in 2006 suggests that increasing the number of bidders starts to give a negative effect of the percentage cost efficiency at a high number of bidders. A theoretical model that could capture the discrepancy between the theoretical model in this paper and the empirical study is needed.

<sup>8</sup> Public Selection and Direct Selection methods are used for consultant services while Public Auction and Direct Auction methods are used for construction and goods services.

## REFERENCES

- Arozamena, L., and F. Weinschelbaum (2005), "The Effect of Corruption on Bidding Behavior in First-Price Auctions," SSRN Working Paper, *mimeo*.
- Asker, J., and E. Cantillon (2008), "Properties of Scoring Auction," *RAND Journal of Economics*, 39(1), RAND Corporation, 69-85.
- Athey, S. (2001), "Single Crossing Properties And The Existence Of Pure Strategy Equilibria In Games Of Incomplete Information," *Econometrica*, 69(4), 861-889.
- \_\_\_\_\_ (1997), "Single Crossing Properties And The Existence Of Pure Strategy Equilibria In Games Of Incomplete Information," MIT Working Paper, 97/11.
- Bewley, T. (1986), "Knightian Decision Theory: Part I," Yale University, New Haven, CT.
- Bliss, C., and R.D. Tella (1997), "Does Competition Kill Corruption?" *Journal of Political Economy*, 105(5), 1001-1023.
- Burguet, R., and Y.K. Che (2004), "Competitive Procurement with Corruption," *RAND Journal of Economics*, 35(1), 50-68.
- Burguet, R., and M.K. Perry (2007), "Bribery and Favoritism by Auctioneer in Sealed-Bid Auctions," *The B.E. Journal of Theoretical Economics*, 7(1).
- Celentani, M., and J. Ganuza (2002), "Corruption and Competition In Procurement," *European Economic Review*, 46, 1273-1303.
- Chateauneuf, A., J. Eichberger, and S. Grant (2007), "Choice under Uncertainty with the Best and Worst in Mind: Neo-additive Capacities," *Journal of Economic Theory*, 137, 538-567.
- Che, Y.K. (1993), "Design Competition Through Multidimensional Auction," *RAND Journal of Economics*, 23, 668-680.
- Costa-Gomes, M.A., and K.G. Zauner (2003), "Learning, Non-equilibrium Beliefs, and Non-pecuniary Payoffs in an Experimental Game," *Economic Theory*, 22(2).
- Crawford, V.P., and N. Iriberry (2007), "Level-k Auctions: Can Non-equilibrium Model of Strategic Thinking Explain the Winner's Curse and Overbidding in Private-Value Auctions," *Econometrica*, 75(6).
- Dasgupta, P., and E. Maskin (2000), "Efficient Auctions," *Quarterly Journal of Economics*, 115(2), 341-388.
- Eső, P., and J. Schummer (2004), "Bribing and Signalling in Second Price Auctions," *Games and Economic Behavior*, 47, 299-324.
- Gilboa, I., and D. Schmeidler (1989), "Maximin Expected Utility With A Unique Prior," *Journal of Mathematical Economics*, 18, 141-153.
- Jehiel, P., and B. Moldovanu (2001), "Efficient Design with Interdependent Valuations," *Econometrica*, 69(5), 1237.
- Knight, F. (1921), *Risk, Uncertainty, and Profit*, Boston, Mass.
- Lengwiler, Y., and E. Wolfstetter (2005), "Bid Rigging, An Analysis of Corruption In Auctions," *mimeo*.
- Maskin, E. (1992), "Auctions and Privatization," in H.Siebert, ed., *Privatization*,

- (Institut für Weltwirtschaften der Universität Kiel, 1992), 115-136.
- Menezes, F.M., and P.K. Monteiro (2006), "Corruption and Auctions," *Journal of Mathematical Economics*, 42(1), 97-108.
- Myerson, R.B. (1991), *Game Theory. Analysis of Conflict*, Harvard University Press, Cambridge, Massachusetts, London, England.
- Wihardja, M.M. (2007), "Competition and Corruption in Public Procurement Auction: Practical Application of the E-Procurement in Indonesia," Cornell University, *mimeo*.

*Mailing Address: Centre of Strategic and International Studies, Jakarta/ University of Indonesia, Indonesia. Tel:62-81-2122-0313. E-mail: mariamonicawihardja@gmail.com.*

*Received June 23, 2009, Revised January 8, 2010, Accepted February 5, 2010.*