

NON-LINEAR ADJUSTMENT PROCESS IN WON/DOLLAR AND WON/YEN REAL EXCHANGE RATES

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We examine the purchasing power parity (PPP) hypothesis in won/dollar and won/yen real exchange rates using a non-linear framework. Many empirical studies based on the linear framework have failed to find clear supporting evidence for the validity of PPP hypothesis. We test the PPP hypothesis using a two-stage procedure suggested by Engle and Granger (1987), and show that it fails to reject non-cointegration. Evaluating the linear model against the nonlinear STAR model, we find that linearity is clearly rejected, but ESTAR process is accepted. Moreover, the parameter estimates of the ESTAR model establish a certain pattern of random walk behavior for small deviations and of fast adjustment for large deviations, thus providing strong evidence for mean-reverting behavior in real won/dollar and won/yen exchange rates.

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JEL classification: C52, F31

1. INTRODUCTION

The development of time series techniques for non-stationary series enabled the use of unit root and cointegration procedures on the real exchange rate to test the validity of the PPP condition in the long run. Nevertheless, many studies using univariate unit root tests and cointegration analysis also obtain mixed results in testing for PPP. To overcome limited test power, some researchers have applied unit root tests to long-span data sets, while others have applied panel unit root and panel cointegration techniques.¹

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¹Sarno and Taylor (2002) show, on the basis of Monte Carlo experiments calibrated on typical results from the empirical real exchange rate literature, that the probability of rejecting (at the 5% significance level) the null hypothesis of a random walk with data from the early 1970s to the late 1980s, when in fact the real exchange rate is mean reverting, would only be somewhere between 5-7%. Sarno and Taylor (2002) also show that, even with the benefit of an additional 10 or 15 years or so of data which are now available, the power of the test increases only slightly to a maximum of around 11% on the most optimistic view of the speed of mean reversion (Taylor (2006, pp. 6-7)).

While these approaches find significant evidence of mean reversion of the real exchange rate, there still remains considerable skepticism on the results.

The economic implications of studies using long-span data sets are unclear since the data typically mix fixed and floating exchange rates. In addition, the data potentially contain serious structural breaks, and these studies have serious size biases. As data are available only over long spans for industrial countries, favorable results may be exaggerated by survivorship bias.²

On the other hand, as Taylor and Sarno (1998) noted, there is an important caveat in the interpretation of panel unit root tests. They argue that the null hypothesis in most of these panel unit root tests is a joint hypothesis that all of the real exchange rates under consideration are realizations of unit root processes. Rejecting the null hypothesis implies that at least one of the real exchange rates is stationary. However, it does not mean that all of the real exchange rates are stationary. Another problem with the studies that support long-run PPP is that the half-life of real exchange rate adjustment is generally estimated to be in the range of three to five years for industrialized economies.³ Such a slow speed of reversion to PPP is difficult to reconcile with the observed large short-term volatility of real exchange rates. Real shocks cannot fully account for the short-term volatility of real exchange rates, and nominal shocks can have strong effects only over a time frame in which nominal wages and prices are sticky. Hence such a high degree of persistence in the real exchange rate is something of a puzzle, as Rogoff first noted.⁴

Recently, the possibility of nonlinearity in the adjustment process has been examined as a potential source of downward bias in the estimates of the speed of real exchange rate adjustment. Most of the literature testing the PPP hypothesis has assumed linear specifications, where the rate of mean reversion is invariant to the distance from parity. However, if the adjustment behavior of the real exchange rate is nonlinear, employing a linear specification leads to a wrong conclusion. It may increase the likelihood of retaining the null hypothesis of a unit root even if the data suggest parity. Furthermore, by averaging across the varying speeds of adjustment, linear methods may bias downward the speed of adjustment.

There are three potential sources of non-linearity in real exchange rates adjustment.⁵ First, non-linearity may arise from friction due to transport costs, tariffs, non-tariff barriers and the sunk costs of international arbitrage. For example, transport costs make arbitraging deviations from the law of one price unprofitable, unless the deviation

² See Taylor (2006, pp. 6-7).

³ Half-life is the time that it takes for 50% of a shock to the real exchange rate to dissipate. The formula for a half-life is given as $\ln(0.5)/\ln(\hat{\rho})$, where $\hat{\rho}$ is the estimated value of ρ in the equation of $q_t = \alpha + \rho q_{t-1} + \varepsilon_t$ ($0 < \rho < 1$) (MacDonald (2007, p. 44)).

⁴ See Rogoff (1996, pp. 647-648).

⁵ See Taylor (2006, p. 9).

reaches a certain level, which leads to a discontinuity in the behavior of international arbitrage. A second potential source of non-linearity may arise from the interaction of heterogeneous agents in the foreign exchange market. If there is a wide range of opinions relative to the appropriate level of the nominal exchange rate in the market, it is difficult to predict the movement of exchange rates and the exchange rates are likely to follow random walks. However, as the degree of misalignment from PPP increases, a market consensus on the direction of the exchange rates forms. Then given relatively sticky prices of goods, one would expect to see the increase in the degree of mean reversion of the real exchange rates. A third potential source of non-linearity arises from the government intervention in the foreign exchange market. As the currency becomes misaligned in the foreign exchange market, traders will lose confidence in the market. Traders need a coordinating signal in order to reenter the market and to alleviate the misalignment. This coordinating signal is provided by government intervention when the degree of misalignment becomes large.

Three non-linear econometric models incorporate the idea that the speed of mean reversion may rise as the deviation from PPP increases: the threshold autoregressive (TAR) model, the exponential smooth transition autoregressive (ESTAR) model and the logistic smooth transition autoregressive (LSTAR) model. Studies using these non-linear models provide confirmation that real exchange rates are well characterized by non-linearly mean reverting processes.⁶

The primary objective of this paper is to examine whether won/dollar and won/yen real exchange rates are well characterized by the smooth transition autoregressive models. The remainder of the paper is organized as follows. Section 2 describes the procedures for tests and estimations of the nonlinear models as well as for the linear cointegration test of the PPP hypothesis. Section 3 reviews earlier studies based on the non-linear framework. Section 4 presents the empirical results and section 5 provides concluding remarks.

2. MODELLING NON-LINEAR MEAN REVERSION

2.1. Linear Cointegration Test

As a preliminary to the tests of linearity in adjustment process of real exchange rates, we need to establish the stationarity of the PPP deviations. Verifying stationarity consists of first regressing the nominal exchange rate and relative prices using the cointegration method, and then testing the linearity of adjustment in the residuals.⁷

⁶See MacDonald (2007, pp. 71-73).

⁷ Michael *et al.* (1997) use Engle-Granger cointegration method and Hasan (2006) uses Johansen cointegration method. Taylor *et al.* (2001) test directly on the linearity of adjustment in real exchange rates.

Following the approach of Michael *et al.* (1997), we use the two-stage procedure suggested by Engle-Granger (1987) to test the stationarity and linearity of the PPP deviations.

The first step in the Engle-Granger testing procedure for cointegration is to pretest the variables for their order of integration.⁸ For the PPP hypothesis, the variables are nominal exchange rate, domestic price, and foreign price. If the variables are integrated of same order, it is possible to proceed to test the cointegration of these variables. The augmented Dickey-Fuller (ADF) test can be used to infer the number of unit roots in each of the variables.

The second step is to estimate the long-run equilibrium relationship in the form of (1), where s is log of nominal exchange rate, p is log of domestic price, p^* is log of foreign price and y_t is an error:

$$s_t = \beta_0 + \beta_1 p_t + \beta_2 p_t^* + y_t. \quad (1)$$

If the variables are cointegrated, an ordinary least squares (OLS) regression yields a super-consistent estimator of the cointegration parameters β_0 , β_1 and β_2 . The PPP assumes a proportional relationship between nominal exchange rate, domestic price and foreign price, given by $\beta_1 = 1$ and $\beta_2 = -1$. But, in the presence of transport costs and measurement errors, the price ratio may not be unity. The only necessary condition for the weak form of PPP to hold is that the coefficients are correctly signed with $\beta_1 > 0$ and $\beta_2 < 0$. Imposing the condition that $\beta_1 = 1$ and $\beta_2 = -1$ a priori defines the strong form of the PPP.⁹

The final step is to test the stationarity of the residuals from (1). The residual series from (1), which are denoted by $\{\hat{y}_t\}$, are the estimated values of the deviations from the long-run relationship. If the variables are actually cointegrated, $\{\hat{y}_t\}$ series should be stationary. The augmented Dickey-Fuller (ADF) test can be used to test the stationarity of the residual $\{\hat{y}_t\}$ series. If we reject the null hypothesis $\gamma = 0$ in (2), we can conclude that the residual series are stationary and that the variables are cointegrated.

$$\Delta \hat{y}_t = \gamma \hat{y}_{t-1} + \sum_{i=1}^k \beta_i \Delta \hat{y}_{t-i} + \varepsilon_t. \quad (2)$$

The lag length k in the right side of (2) is selected such that the residuals of (2) appear to be white noise, that is, they do not exhibit serial correlation. It is notable that it

⁸See Charemza and Deadman (1997, pp. 127-129) and Enders (2004, pp. 335-339).

⁹See Hasan (2006, p. 147).

is not possible to use the critical values in the ordinary ADF tables to test the magnitude of γ . Instead, the specific critical values in the table for Engle-Granger cointegration should be used. This is because the residual $\{\hat{y}_t\}$ series are generated from a regression equation; the researcher does not know the actual error y_t , only the estimates of the error \hat{y}_t .

2.2. Non-linear Models

While cointegration implies mean-reverting behavior, the conventional framework assumes a linear process for y_t , which means that the adjustment process is continuous and has a constant speed. The presence of a non-linear process has implications for the conventional cointegration tests of PPP that it may bring low power of PPP test and slow speed of mean reversion.

The non-linear adjustment process can be characterized in terms of the threshold autoregressive (TAR) model. In the TAR model, within the band no adjustment takes place so that deviations from PPP may exhibit unit root behavior, while outside the band the process is mean-reverting. A TAR model is appropriate for an explicit band such as the exchange rate mechanism of the European Monetary System. However, the application of the TAR model is problematic in the analytical structure where transaction costs vary with trading goods and trading partners, and heterogeneous economic agents do not act simultaneously in aggregates.¹⁰ Another model for non-linear adjustment process is the smooth transition autoregressive (STAR) model. Here, adjustments take place in every period, but the speed of adjustment varies with the extent of the deviation from parity. In contrast with the TAR model, regime changes occur gradually rather than abruptly. The more fully developed statistical modeling procedures make the STAR model more attractive.¹¹

The STAR model is classified into the exponential smooth transition autoregressive (ESTAR) model and the logistic smooth transition autoregressive (LSTAR) model, based on the shape of the transition function. While the ESTAR model assumes symmetric behavior for the adjustment process, the LSTAR model assumes asymmetric behavior depending on whether the real exchange rate is above or below the equilibrium. The LSTAR model might be inappropriate for modeling real exchange rate movement, because it is hard to suggest economic reasons why the speed of adjustment of real exchange rate should depend on whether it is overvalued or undervalued, especially when a goods arbitrage is believed to be the ultimate driving forces toward the long-run

¹⁰ See Hasan (2006, p. 153).

¹¹ See Michael *et al.* (1997, p. 865).

equilibrium.¹² The transition function of the ESTAR model is described in (3), and the transition function of the LSTAR model is described in (4).

$$F(y_{t-d}, \gamma, \mu) = 1 - \exp(-\gamma(y_{t-d} - \mu)^2), \quad \gamma > 0. \quad (3)$$

$$G(y_{t-d}, \gamma, \mu) = [1 + \exp(-\gamma(y_{t-d} - \mu))]^{-1}, \quad \gamma > 0. \quad (4)$$

The deviations from PPP can be described by the ESTAR model.

$$y_t = k + \sum_{i=1}^p \pi_i y_{t-i} + (k^* + \sum_{i=1}^p \pi_i^* y_{t-i}) F(y_{t-d}, \gamma, \mu) + \varepsilon_t. \quad (5)$$

In (5), d is a delay parameter, p is the lag length, $\{y_t\}$ is a stationary and ergodic process, $\varepsilon_t \sim iid(0, \sigma^2)$ and y_{t-d} is a benchmark time period where the regime has changed. The parameter γ is assumed to be positive and signifies the speed of the transition process between two extreme regimes.

The transition function of the ESTAR model is U-shaped around the equilibrium level μ . The middle regime corresponds to $y_{t-d} = \mu$ when $F = 0$ and (5) becomes a linear AR(p) model as (6). The outer regime corresponds to $y_{t-d} = \pm\infty$ when $F = 1$ and (5) becomes a different linear AR(p) model as (7). As γ approaches zero, F becomes 0 and the model becomes a linear AR(p) model as (6). And as γ approaches infinity, F becomes 1 and the model becomes a different linear AR(p) model as (7).

$$y_t = k + \sum_{i=1}^p \pi_i y_{t-i} + \varepsilon_t. \quad (6)$$

$$y_t = k + k^* + \sum_{i=1}^p (\pi_i + \pi_i^*) y_{t-i} + \varepsilon_t. \quad (7)$$

It is convenient to reparameterize the ESTAR model.

$$\Delta y_t = k + \lambda y_{t-1} + \sum_{i=1}^{p-1} \pi_i \Delta y_{t-i} + (k^* + \lambda^* y_{t-1} + \sum_{i=1}^{p-1} \pi_i^* \Delta y_{t-i}) F(y_{t-d}, \gamma, \mu) + \varepsilon_t. \quad (8)$$

In this form, the crucial parameters are λ and λ^* . The non-linear adjustment process of real exchange rate assumes that the larger the deviation from PPP, the

¹²See Taylor *et al.* (2001, p. 1021).

stronger the tendency to move back to equilibrium. This implies that while $\lambda \geq 0$ is possible, we must have $\lambda^* < 0$ and $\lambda + \lambda^* < 0$. That is, for small deviations, y_t may follow a unit root or even explosive behavior, but for large deviations the process is mean-reverting.

This analysis has implications for the conventional cointegration test of PPP, which is based on the linear ADF regression. The failure to find cointegration on the basis of an ADF model does not necessarily invalidate long-run PPP. If the true process for y_t is given by the non-linear model (8), then the parameter λ' in (2) will be between λ and $\lambda + \lambda^*$. Hence, the null hypothesis $H_0 : \lambda' = 0$ (no linear cointegration) may not be rejected against the stationary alternative $H_1 : \lambda' < 0$, even though the true non-linear process is globally stable ($\lambda + \lambda^* < 0$).¹³

2.3. Estimation of STAR Model

If the real exchange rate is assumed to follow a non-linear adjustment process, the first hypothesis we need to test is that of linearity. Terasvirta (1994) outlines the linearity tests against LSTAR or ESTAR and also suggest a decision rule for choosing between the two models. If the delay parameter d is fixed, the linearity test against ESTAR consists of estimating the artificial regression (9) by OLS and testing the null hypothesis $H_0 : \pi_{2i} = \pi_{3i} = 0$ ($i = 1, 2, \dots, p$) against the general alternative that H_0 is not valid.¹⁴ It is notable that Taylor *et al.* (2001) and Han and Min (2001) use the variable for the real exchange rate q_t instead of that for the deviations of PPP y_t in testing the linearity of adjustment process, and estimate the artificial regression (10) and test the null hypothesis $H_0 : \pi_{2i} = \pi_{3i} = \pi_{4i} = 0$ ($i = 1, 2, \dots, p$).

$$y_t = \pi_0 + \sum_{i=1}^p \pi_{1i} y_{t-i} + \sum_{i=1}^p \pi_{2i} y_{t-i} y_{t-d} + \sum_{i=1}^p \pi_{3i} y_{t-i} y_{t-d}^2 + \varepsilon_t \quad (9)$$

$$q_t = \pi_0 + \sum_{i=1}^p \pi_{1i} q_{t-i} + \sum_{i=1}^p \pi_{2i} q_{t-i} q_{t-d} + \sum_{i=1}^p \pi_{3i} q_{t-i} q_{t-d}^2 + \sum_{i=1}^p \pi_{4i} q_{t-i} q_{t-d}^3 + \varepsilon_t \quad (10)$$

If the null hypothesis H_0 is rejected, then the linearity is rejected. In practice the ordinary F -test is used as an approximation to the Lagrange multiplier-type test in order to improve the size and the power properties. We can choose the delay parameter as the

¹³ See Michael *et al.* (1997, pp. 866-867).

¹⁴ See Michael *et al.* (1997, pp. 867-868).

value \hat{d} that minimizes the p -value of the linearity test, repeating the linear test for a set of plausible values of d .

In order to choose between ESTAR and LSTAR, we carry out another F -tests based on (9) by testing the null hypotheses $H_1: \pi_{2i} = 0$ ($i = 1, 2, \dots, p$) and $H_2: \pi_{3i} = 0 \mid \pi_{2i} = 0$ ($i = 1, 2, \dots, p$). If the null hypothesis H_1 is rejected, then LSTAR model is selected. If the null hypothesis H_1 is not rejected but the null hypothesis H_2 is rejected, then ESTAR model is selected. Taylor *et al.* (2001) and Han and Min (2001) successively test the following null hypotheses:

$$\begin{aligned} H_5: \pi_{4i} &= 0, \quad (i = 1, 2, \dots, p). \\ H_4: \pi_{3i} &= 0 \mid \pi_{4i} = 0, \quad (i = 1, 2, \dots, p). \\ H_3: \pi_{2i} &= 0 \mid \pi_{3i} = \pi_{4i} = 0, \quad (i = 1, 2, \dots, p). \end{aligned}$$

If the null hypothesis H_5 is rejected, then the LSTAR model is selected. If the null hypothesis H_5 is not rejected but the null hypothesis H_4 is rejected, then the ESTAR model is selected. Also, if the null hypotheses H_5 and H_4 are not rejected, but the null hypothesis H_3 is rejected, then ESTAR model is selected.

Assuming that linearity is rejected, we can proceed to estimate and evaluate the non-linear model. Equation (8) is estimated by non-linear least squares (NLS), which provides estimators that are consistent and asymptotically normal. To make it easier to select a good starting value for γ , Terasvirta (1994) suggests standardizing the exponent of the transition function F by dividing it by the sample variance of y_t ; $\gamma = 1$ is then an appropriate starting value. Also, the initial value of each coefficient is set at the estimated value of the linear ADF model. To check whether a global minimum has been achieved, estimation may be carried out with fixed γ at different values.

We can estimate the ESTAR model after successively testing and imposing the following restrictions.¹⁵

$$\begin{aligned} R_1: k &= k^* = \mu = 0. \\ R_2: 1 + \lambda &= -\lambda^*, \pi_i = -\pi_i^*, \quad (i = 1, 2, \dots, p-1) \text{ given } R_1. \\ R_3: \lambda &= 0 \text{ given } R_1 \text{ and } R_2. \end{aligned}$$

We can reasonably expect the ESTAR model to satisfy R_1 because the series y_t represent the mean-corrected deviations from PPP. R_2 implies that the process for y_t

¹⁵ See Michael *et al.* (1997, p. 872) and Hasan (2006, p. 154).

in the outer regime, when $F = 1$, is white noise, whereas R_3 implies that in the middle regime, when $F = 0$, y_t has a unit root. Acceptance of these restrictions is supportive of the random walk behavior of small deviations from PPP with mean-reverting adjustment taking place for large deviations.

Model evaluation includes checking whether the estimates seem reasonable, examining the long-run properties of the model, and checking the residuals for autocorrelation, autoregressive conditional heteroscedasticity (ARCH), and normality.

3. REVIEW OF EARLIER LITERATURE

Michael *et al.* (1997) examine the non-linear adjustment process of real exchange rates of sterling/dollar, sterling/fran, sterling/mark, dollar/mark, dollar/fran, fran/mark using monthly data during the 1920s and those of sterling/dollar, sterling/fran using the annual data spanning two centuries. Real exchange rates are based on the WPI measure of price indices. Using Engle-Granger cointegration method, they find that for the three German pairs there is strong evidence to reject the null of non-cointegration. For non-German pairs, the evidence against the null is much weaker, particularly for monthly sterling/dollar. They assume that the failure to reject non-cointegration may be accounted for by the low power of the test or the non-linear behavior of y_t . Carrying out linearity tests on 5 real exchange rates except the German pairs, they conclude that linearity is rejected at the 5% level of significance, and the model selection procedure suggests the ESTAR model for every pair except monthly sterling/dollar. Estimating all of 5 real exchange rates by the ESTAR model, they find the results strongly supporting all the restrictions except for monthly sterling/dollar. They fit the LSTAR model for monthly sterling/dollar. The residual diagnostic statistics are satisfactory apart from the rejection of normality for annual sterling/dollar, and annual sterling/fran and evidence of ARCH for annual sterling/fran. The reduction in the residual variances compared to the linear models ranges from 3% to 25%. The standardized transition parameter shows that the speed of transition between regimes is much slower for the annual series than for the monthly series. They finally conclude that the adjustment process of real exchange rates can be parsimoniously represented in terms of an ESTAR model subject to appropriate parameter restrictions.

Taylor *et al.* (2001) examine the non-linear adjustment process of real exchange rates of sterling, fran, mark, and yen against dollar using monthly data from January, 1973 to December, 1996. Real exchange rates are based on the CPI indices. As a preliminary exercise, they test for unit root behavior of each of the real exchange rate series by ADF unit root test and they are in each case unable to reject the unit root null hypothesis at conventional levels of significance. Applying the Terasvirta rule to estimations of the nonlinear model, they find that an ESTAR model with $p = d = 1$ is their preferred model for all of the series. In no case they could reject at the 5%

significance level the restrictions of $\lambda = 0$ and $\lambda^* = -1$. These restrictions imply that the real exchange rate is close to a random walk in the neighborhood of the equilibrium level, becoming increasingly mean reverting with the absolute size of the deviation from equilibrium. The residual diagnostic statistics are satisfactory in all cases. The estimated standardized transition parameter in each case appears to be strongly significantly different from zero.

Kilian and Taylor (2001) apply the ESTAR model to quarterly data from the first quarter of 1973 to the fourth quarter of 1998 on bilateral US dollar real exchange rates for Canada, France, Germany, Italy, Japan, Switzerland, and UK. They conclude that the ESTAR model performs well in terms of providing good fits, statistically significant coefficients and the residual diagnostic statistics are satisfactory in all cases. The estimated standardized transition parameter in each case appears to be strongly significantly different from zero.

Hasan (2006) examines the non-linear adjustment process of real exchange rates of Canada and Australia against UK using annual data spanning over a century. This study employs both CPI and WPI measures of price indices. First he tests for unit root procedures of real exchange rates using various methods including the cointegration method, then he empirically verifies the weak version of PPP in the long-run in all cases except the real exchange rate of Canada against UK based on the WPI measure. The linearity test shows that the null hypothesis of linearity is rejected for Canada while the null hypothesis of linearity is upheld for Australia. Therefore, he applies the non-linear estimation routine to the ESTAR model only in the case of Canada. The γ estimates vary widely between regressions based on CPI and WPI indices, with the speed of transition being higher in the WPI-based system. Values of γ are estimated to be significantly different from zero, which in general accords well with the adequacy of the ESTAR model specification. He concludes that, estimated with the restriction $1 + \lambda = -\lambda^*$, the negative and statistically significant coefficient $\hat{\lambda}$ satisfies the global stability condition of $\lambda + \lambda^* < 0$, and that, most importantly, it indicates a nonlinear mean reversion.

Han and Min (2001) examine the non-linear adjustment process of real exchange rates and real effective exchange rates of won/dollar and won/yen using monthly data from March, 1980 to December, 1998. They cannot reject the unit root null hypothesis at 1% level of significance in each case. They conclude that linearity is strongly rejected and the LSTAR model is preferable to the ESTAR model for every real exchange rate. Estimating all of 6 real exchange rates by the LSTAR model, they find that in all cases estimated coefficients including the standardized transition parameter appear to be strongly significantly different from zero and the residual diagnostic statistics on autocorrelation, ARCH, are satisfactory.

Chung and Kim (2004) apply the ESTAR model on US dollar real exchange rates against the currencies of Germany, France, Italy, UK, Switzerland, Japan and Canada. They use monthly data from January, 1975 to November, 2002. They test for unit root

behavior of each of the real exchange rate series by ADF unit root test and they are in each case unable to reject the unit root null hypothesis strongly. Therefore carrying out linearity tests on all of real exchange rates, they find that linearity is strongly rejected. They conclude that in each case the estimated standardized transition parameter $\hat{\gamma}$ appears to be strongly significantly different from zero and the global stability condition of $\lambda + \lambda^* < 0$ is satisfied, which in general accords well with the adequacy of the ESTAR model specification.

Kim (2007) examines the non-linear adjustment process of real exchange rates of won against dollar and yen using monthly data from January, 1995 to December, 2004. He tests for unit root behavior of two real exchange rate series by the ADF unit root test and he is in each case unable to reject the unit root null hypothesis. Then, he carries out the inf-t test in which the null hypothesis is a linear unit root process and the alternative hypothesis is a non-linear mean-reverting process following the ESTAR model.¹⁶ Rejecting the null hypothesis, he concludes that the real exchange rate in each case follows a non-linear mean-reverting process.

4. EMPIRICAL RESULTS

4.1. Data and Definition of Variables

We use the monthly average data on wholesale price indices and spot exchange rates for Korea, the US, and Japan. The data set is taken from Ecos (economic statistics system) of the Bank of Korea, and covers the period from January, 1980 to December, 2007, during which the exchange rates of Korean won floated. Exchange rates are expressed as the domestic price of foreign currency and real exchange rate are defined in terms of wholesale price indices. Computer programmes such as GRETL and PcGive are used for econometric analyses. The definitions of the variables used in this study are as follows.

ERUS: won/dollar nominal exchange rate, monthly average

ERJN: won/yen nominal exchange rate (100yen), monthly average

WPI: Korea wholesale price index, monthly average, base year: 2000

WPIUS: US wholesale price index, monthly average, base year: 1982

WPIJN: Japan wholesale price index, monthly average, base year: 2005

EUS: won/dollar nominal exchange rate index, base year: 1993

EJN: won/yen nominal exchange rate index, base year: 1993

¹⁶ Though he tests the null hypothesis of unit root process against the alternative hypothesis of non-linear mean-reverting process following the D-TAR (Doubly TAR) model or D-LSTAR (Doubly LSTAR) model as well as the ESTAR model, he can reject the null hypothesis of unit root process in every case.

PK: Korea wholesale price index, base year: 1993
 PUS: US wholesale price index, base year: 1993
 PJN: Japan wholesale price index, base year: 1993
 REUS: won/dollar real exchange rate index, $EUS*PUS/PK$
 REJN: won/yen real exchange rate index, $EJN*PJN/PK$
 LREUS= $\log(REUS)$, LREJN= $\log(REJN)$
 LEUS= $\log(EUS)$, LEJN= $\log(EJN)$
 LPK= $\log(PK)$, LPUS= $\log(PUS)$, LPJN= $\log(PJN)$
 DLREUS, DLREJN=first difference of LREUS, LREJN
 DLPK, DLPUS, DLPJN=first difference of LPK, LPUS, LPJN
 DDLREUS, DDLREJN=second difference of LREUS, LREJN
 DDLPK, DDLPUS, DDLPJN=second difference of LPK, LPUS, LPJN

4.2. Linear Cointegration Test

As a preliminary procedure for cointegration analysis, the integration order of the variables is examined by ADF unit root tests. In testing for unit root behavior of won/dollar nominal exchange rate (LEUS), won/yen nominal exchange rate (LEJN), domestic price (LPK), US price (LPUS), and Japan price (LPJN) by the ADF equation such as (2), we include a trend variable as well as a constant. We use F-tests to specify the autoregressive order. The results of ADF unit root tests on these variables are summarized in Table 1.

Table 1. Unit Root Tests on Nominal Exchange Rates and Prices

Dependent Variables	Constant	Trend	γ	Critical Values	Lags	DW
DLEUS	0.0548 (2.036)		-0.0115 (-2.012)	5%=-2.871 1%=-3.452	8	1.99
DLEJN	0.0428 (2.465)		-0.0090 (-2.320)	5%=-2.871 1%=-3.452	2	2.00
DLPK	0.0912 (2.674)	0.0000 (2.432)	-0.0208 (-2.618)	5%=-3.425 1%=-3.989	1	1.66
DLPUS	0.1447 (2.636)	0.0001 (2.742)	-0.0334 (-2.627)	5%=-3.425 1%=-3.989	12	1.95
DLPJN	0.0573 (1.716)	-0.0000 (-1.120)	-0.0123 (-1.731)	5%=-3.425 1%=-3.989	12	1.98

Note: The numbers in parentheses are *t*-values.

Excluding the trend variables from the ADF equations for unit root tests of LEUS and LEJN due to insignificance of the estimated coefficients, we could not reject the null hypothesis of a unit root in LEUS and LEJN, because the estimated values of γ were

larger than the critical values of ADF tests. In testing for unit root behavior of LPK, LPUS, LPJN, including the trend variables in the ADF equations, we also could not reject the null hypothesis of a unit root in each variable, because the estimated values of γ were larger than the critical values of ADF tests.

Next, we tested for unit root behavior of the first differences of the variables to determine the order of integration. The results of ADF unit root tests on the first differences of these variables are summarized in Table 2.

Table 2. Unit Root Tests on First Differences of Variables

Dependence Variables	Constant	γ	Critical Values	Lags	DW
DDLEUS		-0.5618 (-5.462)	5%=-1.941 1%=-2.572	7	1.99
DDLEJN	0.0027 (1.586)	-0.7164 (-12.388)	5%=-2.871 1%=-3.452	1	2.00
DDLPK	0.0014 (3.814)	-0.5319 (-11.261)	5%=-3.425 1%=-3.989	1	1.98
DDLpus	0.0012 (2.952)	-0.6765 (-4.398)	5%=-3.425 1%=-3.989	12	1.97
DDLpJN		-0.3223 (-3.422)	5%=-1.941 1%=-2.572	11	1.98

Note: The numbers in parentheses are t -values.

We could safely reject the null hypothesis of a unit root in each variable because the estimated values of γ were smaller than the critical values of ADF tests. Therefore, all variables are estimated to be I(1) series, hence we can proceed to test cointegration between these variables.

Following the cointegration method suggested by Engle and Granger (1987), we first estimated the long-run equilibrium relationship in the form of (11) in won/dollar case and (12) in won/yen case.

$$LEUS_t = \beta_0 + \beta_1 LPK_t + \beta_2 LPUS_t + y_t. \quad (11)$$

$$LEJN_t = \beta_0 + \beta_1 LPK_t + \beta_2 LPJN_t + y_t. \quad (12)$$

The estimated results of the long-run relationship between the nominal exchange rate and relative prices are as follows. The numbers in parentheses are the t -values.

$$LEUS_t = 4.0315 + 2.6277 LPK_t - 2.5162 LPUS_t. \quad (13)$$

(20.415) (26.554) (-18.931)

$$LEJN_t = 22.261 + 0.7171LPK_t - 4.5804LPJN_t. \quad (14)$$

(14.47) (9.722) (-17.122)

It is noted that the estimated coefficients are all significantly different from zero and are correctly signed as expected, but the proportionality conditions that $\beta_1 = 1$, $\beta_2 = -1$ are not met in both cases. This means that even though the null hypothesis of non-cointegration between the nominal exchange rate and relative prices could be rejected, strong form of the PPP could not be accepted and only weak form of the PPP could be accepted.

Then we tested the stationarity of the residuals from (13) and (14). The augmented Dickey-Fuller (ADF) test was used to test the stationarity of the residual series. The appropriate lag length of the ADF equation in won/dollar case was estimated to be 9 and that in won/yen case was estimated to be 2 through the F -tests. Because the estimate of γ was -2.888 (p -value is 0.289) in won/dollar case and -3.503 (p -value is 0.089) in won/yen case, we could not reject the null hypothesis of a unit root at the 5% level of significance. Therefore, we conclude that the residual series are not stationary and that the variables are not cointegrated. It is noted that we could reject the null hypothesis of a unit root at the 10% level of significance in won/yen case.

4.3. Estimations of Non-linear Model

It is assumed that the failure to reject non-cointegration by linear cointegration tests could be a consequence of the non-linear behavior of the residuals that are the estimated values of the deviations from the long-run relationship. If the real exchange rate is assumed to follow a non-linear adjustment process, the first hypothesis we need to test is that of linearity.

Using the approach of Michael *et al.* (1997), we carried out linearity tests on the residuals estimated from (13) and (14). We estimated the artificial regression (15) in won/dollar case and (16) in won/yen case by OLS and tested the null hypothesis $H_0 : \pi_{2i} = \pi_{3i} = 0$ ($i = 1, 2, \dots, p$).

$$RESUS_t = \pi_0 + \sum_{i=1}^9 \pi_{1i} RESUS_{t-i} + \sum_{i=1}^9 \pi_{2i} RESUS_{t-i}^* RESUS_{t-d} + \sum_{i=1}^9 \pi_{3i} RESUS_{t-i}^* RESUS_{t-d}^2 + \varepsilon_t. \quad (15)$$

$$RESJN_t = \pi_0 + \sum_{i=1}^2 \pi_{1i} RESJN_{t-i} + \sum_{i=1}^2 \pi_{2i} RESJN_{t-i}^* RESJN_{t-d} + \sum_{i=1}^2 \pi_{3i} RESJN_{t-i}^* RESJN_{t-d}^2 + \varepsilon_t. \quad (16)$$

In (15) RESUS represents the estimated residuals from (13) and the lag length of 9, which was selected in ADF test for unit roots of the residuals, is used. In (16) RESJN represents the estimated residuals from (14) and the lag length of 2, which was selected in ADF test for unit roots of the residuals, is used. We chose the delay parameter as the value \hat{d} that minimized the p -value of the linearity test, repeating the linear test for a set of plausible values of $d = 1, 2, 3$.

Table 3 reports the results of linearity tests. Linearity is rejected most strongly when $d = 2$ for won/dollar and when $d = 1$ for won/yen. We could reject the null hypothesis of linearity even at the 1% level of significance. The numbers in Table 3 are p -values of F -tests.

Table 3. Results of Linearity Test

	$d = 1$	$d = 2$	$d = 3$
Won/Dollar	0.0335	0.0095	0.1079
Won/Yen	0.0001	0.0274	0.2461

In order to choose the appropriate model between ESTAR and LSTAR for capturing the non-linear adjustment process, we carried out another F -tests based on (15) and (16) by testing the null hypotheses $H_1: \pi_{2i} = 0$ ($i = 1, 2, \dots, p$) and $H_2: \pi_{3i} = 0 | \pi_{2i} = 0$ ($i = 1, 2, \dots, p$).

As shown in Table 4, we could not reject the null hypothesis H_1 at the 10% level of significance but could reject the null hypothesis H_2 at the 1% level of significance. Therefore, the ESTAR model is preferred to the LSTAR model.

Table 4. Test Results on Hypotheses H_1 and H_2

	Value of d	Test on H_1 (p -value)	Test on H_2 (p -value)
Won/Dollar	2	0.1819	0.0073
Won/Yen	1	0.2461	0.0000

Linearity being rejected, we can proceed to estimate and evaluate the non-linear model. We estimated the ESTAR model by nonlinear least squares (NLS). To select a good starting value for γ , we standardized the exponent of the transition function by dividing it by the sample variance of the residuals y_i , then chose $\gamma = 1$ as an

appropriate starting value.¹⁷ Also, the initial value for each coefficient was set at the estimated value of the linear ADF model. We estimated the ESTAR model after successively testing and imposing the following restrictions and we excluded successively the lagged variables whose estimated coefficients were not significant through F -tests based on the general to specific approach.

$$R_1 : k = k^* = \mu = 0.$$

$$R_2 : 1 + \lambda = -\lambda^*, \pi_i = -\pi_i^* \quad (i = 1, 2, \dots, p - 1) \quad \text{given } R_1.$$

$$R_3 : \lambda = 0 \quad \text{given } R_1 \text{ and } R_2.$$

The final estimated equations by the ESTAR model are (17) for won/dollar and (18) for won/yen. And the significance levels of estimated coefficients are shown in Table 5 for won/dollar and in Table 6 for won/yen.

$$\begin{aligned} \Delta RESUS_t = & 0.220\Delta RESUS_{t-1} + 0.117\Delta RESUS_{t-5} + 0.153\Delta RESUS_{t-9} \\ & - (RESUS_{t-1} + 0.220\Delta RESUS_{t-1} + 0.117\Delta RESUS_{t-5} \\ & + 0.153\Delta RESUS_{t-9})(1 - \exp(-0.023NRESUS_{t-2}^2)). \end{aligned} \quad (17)$$

$$\begin{aligned} \Delta RESJN_t = & 0.489\Delta RESJN_{t-1} - 0.108\Delta RESUS_{t-2} - (RESJN_{t-1} \\ & + 0.489\Delta RESJN_{t-1} - 0.108\Delta RESJN_{t-2})(1 - \exp(-0.022NRESJN_{t-1}^2)). \end{aligned} \quad (18)$$

Table 5. Significance Level of Estimated Coefficients (Won/Dollar)

Variables	Coefficient	Standard Error	t -value	p -value
$\Delta RESUS_{t-1}$	0.22004	0.05322	4.135	0.0000
$\Delta RESUS_{t-5}$	0.11652	0.05358	2.175	0.0304
$\Delta RESUS_{t-9}$	0.15275	0.04596	3.323	0.0010
γ	0.02260	0.00496	4.555	0.0000

Table 6. Significance Level of Estimated Coefficients (Won/Yen)

Variables	Coefficient	Standard Error	t -value	p -value
$\Delta RESJN_{t-1}$	0.48898	0.05417	9.027	0.0000
$\Delta RESJN_{t-2}$	-0.10821	0.05643	-1.918	0.0560
γ	0.021526	0.00402	5.360	0.0000

¹⁷ We defined $NRESUS$ as $(RESUS_{t-2} - \mu_{RESUS}) / \sigma_{RESUS}^2$ and defined $NRESJN$ as $(RESJN_{t-1} - \mu_{RESJN}) / \sigma_{RESJN}^2$.

As shown in Tables 5 and 6, the estimates of standardized transition parameter and the coefficients of lagged variables appear to be strongly significantly different from zero in each case. The finding that the γ estimate of won/dollar is larger than that of won/yen suggests that the speed of mean reversion of won/dollar is quicker than that of won/yen.

The residual diagnostic statistics of autocorrelation and ARCH are listed in table 7.

Table 7. Residual Diagnostic Statistics

Test	Won/Dollar, p -value	Won/Yen, p -value
No Autocorrelation (1-24)	0.4560	0.2753
No Autocorrelation (1-12)	0.4065	0.0506*
No Autocorrelation (1-6)	0.8259	0.7262
No Autocorrelation (1)	0.3078	0.2934
No ARCH (1-12)	0.0739*	0.1815
No ARCH (1-6)	0.0035***	0.9852
No ARCH (1-3)	0.0010***	0.8174
Normality	0.0000***	0.0000***

Note: ***significant at 1%, ** significant at 5%, * significant at 10%

For both exchange rates, the null hypothesis of non-autocorrelation of the residuals is not rejected in most of lags in the ESTAR model. Compared with the linear ADF model, evidence of autocorrelation of the residuals almost disappears in the non-linear model, as shown in Tables 8 and 9.

Table 8. Comparison of Residual Diagnostic Statistics (Won/Dollar)

Test	Linear Model, p -value	Non-linear Model, p -value
No Autocorrelation (1-24)	0.0061***	0.4560
No Autocorrelation (1-12)	0.0147**	0.4065
No Autocorrelation (1-6)	0.0111**	0.8259
No Autocorrelation (1)	0.4234	0.3078
No ARCH (1-12)	0.0678*	0.0739*
No ARCH (1-6)	0.0034***	0.0035***
No ARCH (1-3)	0.0007***	0.0010***
Normality	0.0000***	0.0000***

Note: *** significant at 1%, ** significant at 5%, * significant at 10%

Table 9. Comparison of Residual Diagnostic Statistics (Won/Yen)

Test	Linear Model, p -value	Non-linear Model, p -value
No Autocorrelation (1-24)	0.0404**	0.2753
No Autocorrelation (1-12)	0.0019***	0.0506*
No Autocorrelation (1-6)	0.0502*	0.7262
No Autocorrelation (1)	0.6367	0.2934
No ARCH (1-12)	0.3289	0.1815
No ARCH (1-6)	0.9832	0.9852
No ARCH (1-3)	0.8745	0.8174
Normality	0.0000***	0.0000***

Note: *** significant at 1%, ** significant at 5%, * significant at 10%

The residual variances in the non-linear model decreases by 4-8% compared with those in the linear model, as shown in Table 10.

Table 10. Comparison of Variance Ratio

	Linear Model (A)	Non-linear Model (B)	Variance Ratio (B/A)
Won/Dollar	0.0005613	0.0005491	0.978
Won/Yen	0.0009752	0.0009226	0.946

5. CONCLUSIONS

As Rogoff (1996) remarked, many economists instinctively believe in some variant of PPP as an anchor for long-run real exchange rates. Notwithstanding the simplicity and intuitive appeal of PPP, many empirical studies on the validity of PPP have failed in finding clear support for it. This might be due to the non-linearity of adjustment process in real exchange rates. In fact, many studies using non-linear models provide confirmation that real exchange rates are well characterized by non-linearly mean reverting processes.

This paper examined the possibility of non-linearly mean reverting processes in won/dollar and won/yen real exchange rates. This paper first tested the PPP hypothesis using the linear cointegration method suggested by Engle and Granger (1987). The estimated residuals, which are deviations from the long-run PPP equilibrium, are found to be non-stationary, and hence it fails to reject the null hypothesis of non-cointegration at the 5% level of significance. Because it could be a consequence of non-linear adjustment process of real exchange rates, the null hypothesis of linearity was tested. For both of exchange rates considered, linearity is clearly rejected and the ESTAR model is assumed to be an appropriate model. The results of estimation by the ESTAR model show that the estimated parameters appear to be strongly significantly different from

zero in each case and the restrictions of $\lambda = 0$ and $\lambda^* = -1$, implying random walk behavior for small deviations but fast adjustment for large deviations, are strongly accepted. These results confirm that adjustment process in real exchange rates can be parsimoniously represented in terms of an ESTAR model subject to appropriate parameter restrictions. The residual diagnostic statistics are in general satisfactory and especially, evidence of autocorrelation shown in the linear model almost disappears in the ESTAR model. Also, the residual variances in the non-linear model show the reduction of 4-8% compared to the linear model. All these findings verify the adequacy of ESTAR model specification.

The results of this paper accord well with those of other studies like Michael *et al.* (1997), Taylor *et al.* (2001), Hasan (2006), and Chung and Kim (2004), all of which examine the real exchange rates of industrial countries. Also, the findings of Kim (2007), who studied the real won exchange rates but used a different testing method, are similar to those of this paper. All these studies verify the adequacy of non-linear ESTAR model specification.

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