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INTERNAL LEARNING BY DOING AND ECONOMIC GROWTH

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This paper analyses the consequences for growth during the transitional period of considering the learning-by-doing process proposed by Arrow (1962) as internal, instead of as an externality. To do this, it develops a simple endogenous growth model with human capital accumulation through external and internal learning processes. The calibrated model delivers two features of the Japanese growth experience: slow convergence, and negative correlation between the growth rate of per capita GDP and investment share. The crucial implication of internal learning-by-doing is the double role of physical investment that operates reducing diminishing returns to physical capital.

Keywords: Internal LBD, Endogenous Growth, Return to Saving, Convergence Speed, Investment Share and Growth Rate
JEL classification: O30, O41, O50

1. INTRODUCTION

A strand of endogenous growth theory has shown that the introduction of positive externalities of investment in physical capital leads to sustained growth of per capita output (Sheshinski (1967); Romer (1986); Lucas (1988)). These models are based on Arrow's (1962) idea that knowledge and productivity gains come from investment in physical capital. More specifically, an increase in a firm's capital stock leads to a parallel increase in its stock of knowledge that incorporates to workers through an external learning process, or learning-by-doing (LBD) process.

Since firms decide physical investment and human capital is embodied in workers, this learning process has been traditionally considered as an externality. Moreover, a

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usual assumption is that workers' skills generated result in general human capital that is portable across jobs. Concepts of general and specific human capital are due to Becker (1975). Skills are general when a big part of them are portable across jobs, and firm-specific when a big part of them are useful only in the firm where they were generated. Nonetheless, the empirical study by Bessen (1998) reveals that LBD is largely a firm-specific human capital investment. He also finds that many plants invest in LBD as much as in physical capital, which means that this investment is quantitatively relevant.

Based on these empirical results, it seems reasonable to consider LBD investment as intentional or internal. To this respect, Álvarez Albelo (1999) shows that internal LBD extremely affects the length of the transitional period. More concretely, she finds that the introduction of internal LBD in a two-sector model of endogenous growth permits to reconcile the convergence speed delivered by the model with the 2% reported by empirical studies (e.g., Barro and Sala-i-Martin (1992); Mankiw *et al*. (1992)). However, she leaves aside the special features of specific human capital, and solves the planner problem.

Our aim in this paper is to analyse the effects of internal LBD on growth along the transitional period. To do this, we develop a simple model of endogenous growth with firm-specific and general human capital accumulation through internal and external learning processes, respectively. Differently from Álvarez-Albelo (1999), our results are based on a decentralised equilibrium. The predictions from our model confirm that internal LBD enlarges the transitional period of the economy. We also find that it extremely affects the time evolution of investment share of output (saving rate).

Our theoretical framework builds on Arrow's (1962). There are two types of homogenous agents in the economy: firms and households. Firms produce an aggregate good, hire labour and make decisions on investment in physical capital and specific human capital of their workers. The same amount of current investment results in higher levels of both physical and human capital in the next period and, hence, there exists complementarities between these two types of capital. Households work, consume and save. The human capital embodied in the worker is composed of general and firm-specific skills. General human capital is accumulated through an external learning process, in which total efficient time is the relevant variable.

Since in the model the firm carries out the investment, and specific skills are embodied in the worker, the wage per time unit must be higher than the competitive one to avoid that the employee quits the firm. This means that both parts must achieve an agreement to share the returns to investment. By simplicity, we do not explicitly formalise that agreement, and assume that the returns are split according to an exogenous and fixed parameter that determines the bargaining power of the parts.

Our analysis is mainly numerical and involves two steps. First, we calibrate the model to replicate some long-run observations of the American economy, and a measure of labour remuneration constructed from Japanese data. Then, we compute the model using the parameter values calibrated. In the computation, we assume that the initial ratio of physical to human capital is lower than its long-run value.

The main feature of internal LBD is that it reduces the diminishing return to physical capital accumulation. This is a consequence of the double role played by physical investment. Thus, the interest rate is higher than the usual net marginal productivity of physical capital, and its equation includes both capital and labour marginal productivities. As physical capital accumulates, the marginal productivity of physical capital falls, while the marginal productivity of labour raises, which prevents interest rate from declining to fast. This mechanism acts slowing the convergence of the economy towards its long-run equilibrium, and strengthening the incentives to save (invest).

The time paths obtained from the computation of the model illustrate the effects of internal LBD on the transitional behaviour of the economy. The most striking result refers to the upward sloping time path of saving rate. The time evolution of the saving rate is driving by the offsetting impacts of a substitution effect and an income effect. The first one relies on diminishing returns, and generates incentives to reduce the saving rate along time. The income effect is related to the gap between current and permanent income and the willingness to smooth consumption over time, and generates the opposite incentives. Since internal LBD reduces diminishing returns, the second effect imposes to the first one, and the saving rate rises during the transitional period. The growth rate of output, however, strictly decreases as a consequence of diminishing returns.

The predictions from our model are consistent with the empirical evidence by Jones (1995). This author points out that AK type of models predict a direct relationship between per capita GDP growth and investment share, while the data that he considers show that, for some countries, the growth variable decreases while investment share increases over time. This is especially true for Japan. Unlike other countries, investment share and the growth rate of Japan exhibit a notably high negative correlation. Several empirical findings suggest that the hypothesis of internal LBD could be a good explanation for the behaviour of the Japanese economy.

The rest of the paper is organized as follows. Section 2 presents and comments some empirical facts regarding investment share and growth. Section 3 describes the theoretical framework. Section 4 characterises the equilibrium of the economy. Section 5 contains the calibration and the computation of the model. Section 5 summarizes and concludes. Lastly, the Appendix establishes conditions for existence of the long-run equilibrium.

2. THE EMPIRICAL FACTS REGARDING INVESTMENT AND GROWTH

The empirical evidence by Jones (1995) shows that total investment shares have increased by several percentage points for a number of developed countries during the post World War II era, while per capita GDP growth rates have fallen. Here, we present some evidence on this empirical fact. We look at data in a very simple way, by correlating real per capita GDP growth rate and real investment share of GDP. We consider data from six European countries and Japan since 1950, after World War II, until 2000.[1] These economies lost an important part of their physical capital during World War II, and during Civil War in the case of Spain. Hence, we can ensure that they were in transition during that period. Table 1 contains the results from actual and smoothed data.

Table 1. Correlations between Per Capita GDP Growth Rate (1951-2000) and Investment Share of GDP (1950-1999)

Country	Actual Data	Smoothed Data
Austria	-0.4161	-0.4736
France	-0.1230	-0.1949
Great Britain	-0.1729	-0.1759
Italy	0.3116	0.8405
Spain	-0.1879	-0.4992
Japan	-0.4349	-0.6616
West Germany	0.0971	0.5007

Notes: Data for West Germany correspond to the period 1950-1992. Data have been smoothed using the HP filter. (smoothing parameter=400)

Source: PWT Mark 6.1 and 5.6.

We find that correlation coefficients calculated with both actual and smoothed data are negative for all considered economies, except for Italy and West Germany. Moreover, correlations (in absolute value) increase when business cycle effects are removed, and especially for Spain and Japan.

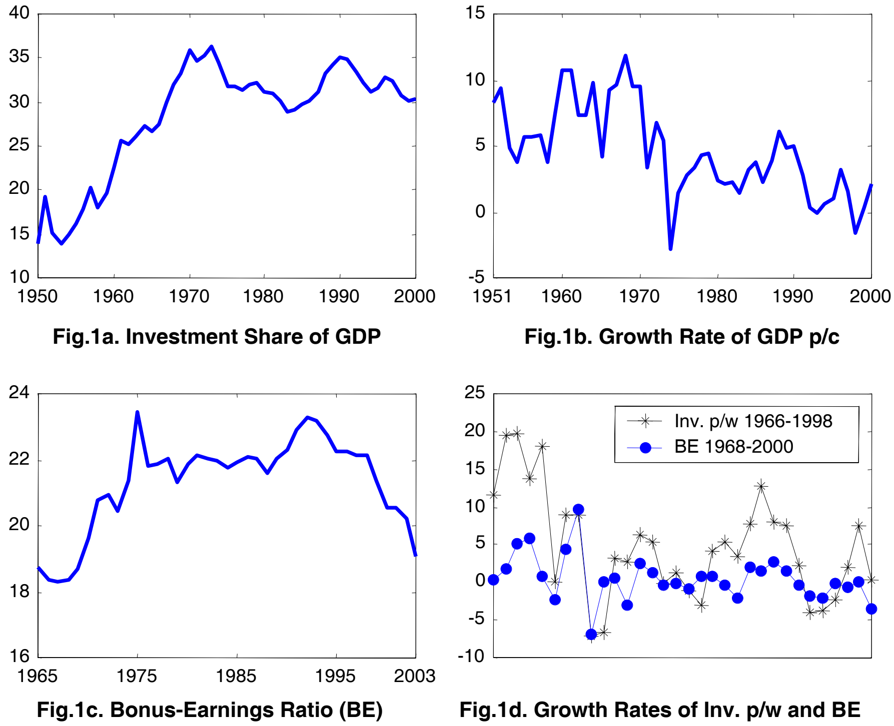
The negative correlations seem to be related to the facts that time paths of both variables are hump shaped, and increases in growth significantly precede increases in investment share. Contrary to the traditional view that investment causes growth, these observations suggest that the causality runs in the opposite direction. Several empirical studies support this hypothesis (e.g., Edwards (1995); Barro and Sala-i-Martin (1995); Loayza *et al*. (2000)), but there exists also evidence of a two-ways causality (Madsen (2002)).[2]

These empirical findings are problematic for neoclassical growth models and also for AK type of models, in which the feedback from growth to saving is absent. To this respect, Carroll *et al*. (2000) showed that the introduction of internal habit formation in

[1] We use variables KI and GRGDPL in the PWT Mark 5.6a and 6.1.

[2] More concretely, growth is caused by investment in machinery and equipment, whereas investment in structures and non-residential buildings is caused by growth.

the AK model permits to reconcile the predictions from the model with the data.[3] Nevertheless, growth-to-saving causality might be not all that counts for understanding the negative correlations in Table 1, and specially the high one of the Japanese economy. As shown in Figure 1, the Japanese growth rate presents a trend to decreasing (Fig. 1b). However, it is difficult to infer from the data the future evolution of investment share (Fig. 1a). What is clear is that it rapidly rose until the early 1970s, and has being fluctuating around a remarkably high percentage (32%) since then.



Notes: BE in Figure 1c refers to the ratio of special earnings to total earnings. Inv. p/w in Figure 1d refers to real investment per worker. All variables are in percentage form.
Sources: PWT Mark 6.1 and Historical Statistics of Japan.

Figure 1. Japanese Growth Experience

[3] Their analysis considers saving rate instead of investment share of output. Nevertheless, the data reveal that there is a powerful relationship between both variables.

One could wonder why the Japanese economy has been investing such a big part of its GDP, while investment shares of other countries have been falling. Here, we propose that the answer lies on the existence of investment in firm-specific human capital through internal or intentional LBD. As we will show later on, internal LBD prevents the return to physical investment (savings) from declining too fast, which generates incentives to postpone today consumption for a longer period than with an external LBD process. Under internal learning, investment leads to growth, and the growth rate declines through time as a consequence of decreasing returns to physical capital.

The peculiar features of the Japanese labour market, also called *nenko* system,[4] are consistent with the existence of investment in firm-specific skills. Indeed, two pervasive characteristics of this system are lifetime employment (Hashimoto and Raisian (1985))[5] and flexible wages in the form of variable bonus payments, which add to base wages (Fig. 1c). The theoretical and empirical analysis by Hashimoto (1979) reveals that these features are associated to a high profitability of investment in firm-specific human capital, and the low transaction costs facing the worker and the employer in evaluating fluctuations in productivity. Thus, the bonus would reflect the part of returns to investment that corresponds to the worker.

The next question to be clarified is whether a fraction of this investment takes the form of LBD. The empirical evidence provided by Bessen (1998) strongly supports this hypothesis. He also finds that this investment is quantitatively relevant; many plants invest in LBD as much as in physical capital and more than they invest in formal on-the-job-training. Though Bessen uses American plant-level data, we do not find any reason to think that his results cannot be extrapolated to the Japanese economy.

To achieve some intuition regarding the relationship between physical investment and bonus payments in Japan, we plot the growth rates of per worker investment and the bonus-earnings ratio (Fig. 1d). Considering that LBD is related to new capital goods introduced in the production process (e.g., Dunne *et al*. (2000)), one would expect a strong investment-to-bonus causality. The data suggest the existence of this causal channel. More concretely, the growth rate of bonus-earnings ratio mimics fairly well the two-lagged growth rate of investment per worker.

In light of these empirical findings, internal LBD can be considered as plausible explanation for the negative correlation between investment share and the growth rate. However, internal LBD must not be viewed as a competitor for the theory of habit formation, but as another factor that may coexist with habit formation and that generates incentives to increase physical investment for a longer period.

[4] See Cole (1971, p. 134) for a characterisation of this system.

[5] Hashimoto and Raisian (1985) find that long-term employment is more prevalent and earnings-tenure profiles are more steeply sloped in Japan than in the United States. They argue that their results are consistent with there being more specific human capital in Japan than in the American economy.

3. THE MODEL AND AGENTS' DECISIONS

This section presents the theoretical framework and outlines agents' decisions. To offer a clearer exposition, we first make a sketch of the model and then describe it with more detail in the two next subsections.

The basic structure of the model is as follows. Time is discrete and endless. All variables are expressed in per capita terms. As it is usual, we assume that agents have perfect foresight. There are two types of agents: firms and households. Firms produce a final aggregate good, which can be either accumulated as physical capital or consumed. Firms hire labour, and make decisions on investing in physical capital and specific human capital of their workers. The economy is inhabited by a continuum of infinitely lived identical households, which grows at an exogenous rate *n*. Each household is endowed with a unit of time at every period that can only be devoted to work. Households work, consume and save. Total human capital of households is composed of specific skills provided by firms through an intentional investment process (internal LBD), and general skills that accumulate while working through an external learning process.

3.1. Firms

There is a continuum of measure one of identical firms that produce a homogenous good using the same Cobb-Douglas production function with constant returns to scale:

$$\begin{aligned} &Y_t = K_t^{\alpha} H_t^{1-\alpha}, \; 0<\alpha<1, \\ &H_t \equiv h_t + e_t, \end{aligned} \tag{1}$$

where Y_t is the production, K_t denotes the stock of physical capital and H_t is the labour input. The labour input is measured in efficiency units, and results from multiplying working time by total human capital or skills per worker. Each worker possesses general and firm-specific skills, h_t and e_t, respectively.

We adopt Arrow (1962) technology to model the accumulation of firm-specific human capital. This formalization implies that knowledge and productivity gains come from investment in physical capital, but here this investment is intentional or internal, instead of an external effect. Concretely,

$$\begin{aligned} &(1+n)K_{t+1} = I_t + (1-\delta_K)K_t, \\ &(1+n)e_{t+1} = \gamma_1 I_t + (1-\delta_H)e_t, \end{aligned} \tag{2}$$

where I_t is gross investment in physical capital, $\gamma_1 > 0$ is a productivity parameter, and $\delta_K \geq 0$ and $\delta_H \geq 0$ are physical and human capital depreciation rates.

The investment decision in our model corresponds to the firm. However, to prevent workers from quitting, it must offer a higher remuneration than competitors. Note that only general human capital is remunerated if workers quit. In this case, the worker would receive the competitive wage. Therefore, employer and employee must achieve an agreement to share the returns to investment in specific human capital. This agreement may take the form of an implicit or explicit contract. The contract is mutually beneficial, since it permits the firm to retain a part of the returns, while the worker receives a higher remuneration than in any other job.

Equation (1) implies that general and specific skills are perfect substitute and, hence, have the same marginal productivity. Since the remuneration to general human capital is competitively determined, the marginal productivity of both types of human capital will be equal to the competitive wage, w_t. The total return to investment in specific skills is $w_t e_t$ in the firm where it took place, and equals zero in any other firm. The total value of a worker in the firm is given by $w_t(h_t + e_t)$, while the value in an alternative employment is $w_t h_t$. Thus, the worker and the firm must agree on how to share $w_t e_t$.

By simplicity, we do not explicitly formalise any contract. Instead, we follow Fukuda and Owen (2004) and assume that the returns are shared according to the bargaining power of both parties. The bargaining power is exogenously given and remains constant over time.[6] Worker's wage per time unit, w_t^T, may be viewed as consisting of the alternative wage per time unit, plus a wage premium or bonus representing his share in total returns to investment in specific human capital. In addition, we define the ratio of bonus to total earnings, BE_t, which will be useful later on. Thus:

$$\begin{aligned} w_t^T &= w_t h_t + (1-\eta) w_t e_t, \ 0 < \eta < 1, \\ BE_t &\equiv \frac{(1-\eta)e_t}{h_t + (1-\eta)e_t}, \end{aligned} \tag{3}$$

where η is the firm's sharing ratio in total returns to investment or bargaining power of the firm. Given that η lies between zero and one, the separation never takes place. Note that the criteria for quit is $w_t^T \le w_t h_t$ and for dismissal is $\eta\, w_t e_t \le 0$.

Given these assumptions, the problem of the representative firm consists of maximising the sum of total profits discounted at the market interest rate r_t,

$$\sum_{t=0}^{\infty} \frac{(1+n)^t}{\prod_{\tau=0}^{t}(1+r_\tau)} \left\{ K_t^{\alpha}(h_t + e_t)^{1-\alpha} - I_t - w_t(h_t + (1-\eta)e_t) \right\}, \tag{4}$$

[6] To analyse how the parties achieve an agreement is far from the scope of this paper. Further discussions on this issue can be found in Hashimoto (1979, 1981) and Rosen (1985).

subject to the accumulation of physical capital and specific human capital, (2), the split of labour returns, (3), and the initial endowments of physical and specific human capital, $K_0 > 0$ and $e_0 = 0,$ respectively. Although it is not necessary, we assume that there are not specific skills embodied in the worker at the initial period. The price of the aggregate good is taken as numerary.

This is a standard problem of dynamic optimisation, with first order conditions for the case of interior solution:

$$-\frac{(1+n)^t}{\prod_{\tau=0}^{t}(1+r_\tau)} + \lambda_t^K + \gamma_1 \lambda_t^e = 0, \tag{5}$$

$$\frac{(1+n)^{t+1}}{\prod_{t=0}^{t+1}(1+r_\tau)} \alpha K_{t+1}^{\alpha-1}(h_{t+1}+e_{t+1})^{1-\alpha} - (1+n)\lambda_t^K + \lambda_{t+1}^K(1-\delta_K) = 0, \tag{6}$$

$$\frac{(1+n)^{t+1}}{\prod_{t=0}^{t+1}(1+r_\tau)} \left((1-\alpha)K_{t+1}^{\alpha}(h_{t+1}+e_{t+1})^{-\alpha} - (1-\eta)w_{t+1}\right) - (1+n)\lambda_t^e + \lambda_{t+1}^e(1-\delta_H) = 0, \tag{7}$$

$$w_t = (1-\alpha)K_t^{\alpha}(h_t+e_t)^{-\alpha}, \tag{8}$$

and equations in (2), where λ_t^K and λ_t^e denote shadow prices for physical and specific human capital, respectively.

3.2. Households

Each household derives utility from the consumption of the aggregate good, C_t, and maximizes its intertemporal utility discounted at a positive rate β:

$$\sum_{t=0}^{\infty}\left(\beta(1+n)\right)^t U(C_t), \ t = 0,1,2,\ldots,$$
$$U(C_t) = \begin{cases} \dfrac{C_t^{1-\sigma}-1}{1-\sigma} & if \quad \sigma \in (0,1)\cup(1,\infty) \\ \ln\left|C_t\right| & if \quad \sigma = 1 \end{cases} \tag{9}$$

The households' assets equal the market value of the firm, A_t. At each period, households earn the rate of return r_t on assets, and receive labour income. They face the budget constraint:

$$(1+n)A_{t+1} = (1+r_t)A_t + w_t(h_t + (1-\eta)e_t) - C_t. \tag{10}$$

As before indicated, general skills come from an external LBD process, in which total efficient time embodied in the worker is the relevant variable. More specifically, the technology for general human capital takes the form:

$$(1+n)h_{t+1} = \gamma_0(h_t + e_t) + (1-\delta_H)h_t, \tag{11}$$

where $\gamma_0 > 0$ is a productivity parameter, and general and specific skills depreciate at the same rate. Additionally, we assume that the growth rate of general skills is always positive, which requires that $\gamma_0 - \delta_H > 0.$ It seems to us reasonable that specific human capital contributes to produce general skills. For instance, it may be argued that once the worker has learned how to use a particular type of machine, it is easier for him to make work other type machines. In the example, the first task acts as a kind of learning that generates more general abilities.[7]

The representative household maximises its total discounted utility (9), subject to the budget constraint (10), and given the time path of firm-specific human capital, $\{e_t\}_{t=0,1,2,\ldots}$, the accumulation of general human capital (11) and the initial endowment of this capital, $h_0 > 0.$ The first order conditions of this problem are:

$$\left(\beta(1+n)\right)^t C_t^{-\sigma} - \mu_t = 0, \tag{12}$$

$$-(1+n)\mu_t + \mu_{t+1}(1+r_{t+1}) = 0, \tag{13}$$

and Equation (10), where μ_t is the shadow value of household's assets.

4. EQUILIBRIUM OF THE ECONOMY

Markets of consumption good, assets and labour with just general skills are perfectly competitive. However, the wage of workers with specific skills is not competitively determined. In our simple model, agents are homogenous and do not face any kind of uncertainty or costs of agreeing. Therefore, the equilibrium implies that workers are hired at the initial period, and stay in the firm forever. Next, we characterised the equilibrium of the economy.

Given the initial values for physical capital, $K_0 > 0,$ and general and specific

[7] Of course, this technology extremely simplifies the model, since it permits to solve it just considering total human capital H_t.

human capital, $h_0 > 0$ and $e_0 = 0,$ the split of labour returns, (3), and the technology to accumulate general skills, (11), the equilibrium of the economy is characterised by a set of allocations $\{C_t, K_{t+1}, H_{t+1}, e_{t+1}\}_{t=0,1,2,...}$ and prices $\{w_t, r_t\}_{t=0,1,2,...}$ such that they satisfy firm's and household's problems, and that clear all markets in the economy. In particular, the feasibility condition, $Y_t = I_t + C_t,$ must hold.

Manipulating the first order conditions of firm's and household's problems, we can construct the dynamic system driving the behaviour of the economy over time. Notice that adding up the second equation in (2) and (11), the model can be solved just considering the variable H_t. As it is standard in the literature, we first define intensive variables that will remain stationary along the balanced growth path (BGP), or long-run equilibrium. It is easy to check that along the BGP all shadow prices, λ_t^K, λ_t^K and μ_t must grow at the same constant rate, and K_t, h_t, e_t, H_t and C_t must grow at a common constant rate (long-run growth rate). Hence, we define the new variables:

$$Z_t \equiv \frac{K_t}{H_t}; X_t \equiv \frac{C_t}{K_t}; G_t \equiv \frac{\lambda_t^K}{\lambda_t^e}, E_t \equiv \frac{e_t}{h_t}. \tag{14}$$

Considering (14), we construct the dynamic system composed of the three following difference equations:

$$\frac{Z_{t+1}}{Z_t} = \frac{Z_t^{\alpha-1} - X_t + 1 - \delta_K}{\gamma_0 + \gamma_1 Z_t (Z_t^{\alpha-1} - X_t) + 1 - \delta_H}, \tag{15}$$

$$\frac{X_{t+1}}{X_t} = \frac{(1+n)(\beta(1+r_{t+1}))^{\frac{1}{\sigma}}}{Z_t^{\alpha-1} - X_t + 1 - \delta_K}, \tag{16}$$

$$\frac{G_t}{G_{t+1}} = \frac{\frac{G_{t+1} + \gamma_1}{G_{t+1}} \alpha Z_{t+1}^{\alpha-1} + 1 - \delta_K}{(G_{t+1} + \gamma_1)\eta(1-\alpha) Z_{t+1}^{\alpha} + 1 - \delta_H}, \tag{17}$$

where interest rate in equilibrium equals:

$$\begin{aligned} r_{t+1} &= \alpha Z_{t+1}^{\alpha-1} + \gamma_1 \eta (1-\alpha) Z_{t+1}^{\alpha} - \left(\frac{\lambda_{t+1}^K}{\lambda_{t+1}^K + \gamma_1 \lambda_{t+1}^e} \delta_K + \frac{\lambda_{t+1}^e}{\lambda_{t+1}^K + \gamma_1 \lambda_{t+1}^e} \gamma_1 \delta_H \right) \\ &= \alpha Z_{t+1}^{\alpha-1} + \gamma_1 \eta (1-\alpha) Z_{t+1}^{\alpha} - \frac{G_{t+1}\delta_K + \gamma_1 \delta_H}{G_{t+1} + \gamma_1}. \end{aligned} \tag{18}$$

Equation (15) to (18), along with the initial values K_0 and H_0, and the transversality conditions:

$$\lim_{t\to\infty} \lambda_t^K K_t = 0, \quad \lim_{t\to\infty} \lambda_t^e e_t = 0, \tag{19}$$

fully characterise the dynamics of the economy.

The evolution of the bonus-earnings ratio can be obtained considering definitions in (14), and dividing the second equation in (2) by (11). This operation yields:

$$E_{t+1} = \frac{\gamma_1 (Z_t^{\alpha-1} - X_t) Z_t (1+E_t) + (1-\delta_H) E_t}{\gamma_0 (1+E_t) + 1 - \delta_H}, \ E_0 = 0, \quad BE_t = \frac{(1-\eta)E_t}{1+(1-\eta)E_t}. \tag{20}$$

Internal LBD implies the existence of complementarities between physical and human capital, which extremely affects the evolution of interest rate, or return to savings, through time. More specifically, current savings (investment) have a double return: one from increasing physical capital and the other from improving efficiency units of labour. Thus, interest rate (Equation (18)) is composed of marginal productivity of physical capital, plus the return from increasing the marginal productivity of labour due to investment in human capital. The latter return is multiplied by η, reflecting that the firm retains that proportion of the returns to investment in specific human capital. The depreciation rates of physical and human capital are weighted by the shadow value of the respective capital over total shadow value. Lastly, notice that an additional unit of physical investment produces γ_1 units of specific human capital, which depreciates by $\gamma_1 \delta_H$.

As we will discuss in the next section, this feature of the model is crucial in determining the length of the transitional period and the shape of saving rate time path.

5. BALANCED GROWTH PATH AND DYNAMICS: A NUMERICAL ANALYSIS

Our purpose in this section is to illustrate through a numerical example the predictions of the model. This exercise will be useful to understand the implications of internal LBD for growth during the transition. In particular, we will show that the calibrated model delivers two observed features of the Japanese growth experience: slow convergence, and negative correlation between per capita output growth rate and saving rate. First, we calibrate the model to match some long-run observations of American economy, and a measure of the bonus-earnings ratio constructed from Japanese data. Then, we compute the model using the parameter values calibrated.[8]

5.1. Calibration

We choose the American economy to carry out the calibration of the model. The assumption that we make is to consider that this economy is over its long-run equilibrium. Indeed, if we look at the values of relevant variables of the United States throughout the twentieth century (Madisson (1991, 1995)), it may be assumed as in a BGP. In addition, we use observations regarding the bonus-earnings ratio in Japan (in Fig. 1c).

To calibrate the model, we evaluate the dynamic system over the BGP, which yields:

$$(1+n)(1+\theta)=\frac{Y_t}{K_t}-\frac{C_t}{K_t}+1-\delta_K, \tag{21}$$

$$(1+n)(1+\theta)=\gamma_0+\gamma_1 Z\big((1+n)(1+\theta)-(1-\delta_K)\big)+1-\delta_H, \tag{22}$$

$$\left(\frac{\gamma_0+\delta_K-\delta_H}{(1-\gamma_1 Z)(1+n)}+\frac{1-\delta_K}{1+n}\right)^{\sigma}=\beta(1+r), \tag{23}$$

$$\frac{G+\gamma_1}{G}\alpha Z^{\alpha-1}-(G+\gamma_1)\eta(1-\alpha)Z^{\alpha}+\delta_H-\delta_K=0, \tag{24}$$

$$r=\alpha Z^{\alpha-1}+\gamma_1\eta(1-\alpha)Z^{\alpha}-\frac{G\delta_K+\gamma_1\delta_H}{G+\gamma_1}, \tag{25}$$

where θ is the long-run growth rate, and the equations of physical and human capital accumulation ((21) and (22), respectively) have been considered separately. From now on, the omission of time sub-indexes will denote stationary values over the BGP. In addition, Equation (20) in the long-run equilibrium becomes:

$$\begin{aligned} E&=\frac{(1+n)(1+\theta)-1-(\gamma_0-\delta_H)}{\gamma_0},\\ BE&=\frac{(1-\eta)E}{1+(1-\eta)E}. \end{aligned} \tag{26}$$

[8] The data and codes are available upon request via e-mail to the authors.

We must also impose that any parameterisation of the model fulfils the transversality conditions, which requires that:

$$r > (1+\theta)(1+n)-1. \qquad (27)$$

We show in the Appendix that if $\gamma_0 - \delta_H > 0,$ the existence of a BGP implies that it is unique. Furthermore, it can be proved by means of a continuous time version of the model that the BGP is locally saddle-path stable.[9]

The calibration process classifies the parameter into two groups. Those in the first group are taken directly from Cooley and Prescott (1995), and refer to the population growth rate, $n = 0.012,$ and physical capital share in good technology, $\alpha = 0.4,$ of the American economy. The rest of parameters $(\delta_K, \gamma_0, \gamma_1, \delta_H, \eta, \beta, \sigma)$ are calibrated to match some long-run observations of the American economy, and a measure of the bonus-earnings ratio that we construct from Japanese data (calibration targets).

The targets and sources are as follows. We choose three measures of long-run variables constructed by Cooley and Prescott (1995): long-run growth rate, $\theta = 0.0156,$ capital-output ratio, $K_t / Y_t = 3.32,$ and consumption-output ratio, $C_t / Y_t = 0.7518.$ Note that these observations and Equation (21) allow obtaining physical capital depreciation rate, $\delta_K = 0.04697.$ The calibration strategy implies that consumption-physical capital ratio, $SR = 0.2482.$ physical capital-labour ratio, $Z = 7.3886,$ net marginal productivity of physical capital, $NMPK = \alpha Z^{\alpha-1} - \delta_K = 7.35\%,$ and saving rate $SR = 0.2482$ can be obtained independently from human capital, preference parameters and firm's bargaining power.

The fourth target is based on the average ratio of annual special earnings to annual total cash earnings in Japan during the period 1965-2003 (21.3%).[10] Since the Japanese economy was not over its BGP during that period, we use the average ratio as an approximation. Special earnings include mainly bonuses and also special allowances. As reported by Hashimoto (1979), bonus payments predominate and constitute the most systematic component of special earnings. Here, we choose a compromise value of 20% for this ratio $(BE = 20\%).$

We would need three additional targets to calibrate the seven parameters. To solve this problem, we use the estimates of the speed of convergence as a target. Plenty of empirical studies find that the rate at which economies approach their long-run equilibrium ranges between 2 and 3 percent per year. Thus, we calibrate the parameters

[9] To prove local stability we follow the same strategy as Greiner (2003). The proof is available upon request via e-mail to the authors.

[10] The bonus-earnings ratio has been computed as the mean of the ratio of annual special earnings to annual total cash earnings during the period 1965-2003, in establishments with 10 or more regular workers. For more details on the data, see http://www.stat.go.jp/data/chouki/zuhyou/19-34.xls.

for them to deliver an asymptotic speed of convergence as close as possible to that range $(SC \in [2,3])$.

Table 2. Calibration Targets, Parameter Values and Long-Run Equilibrium

Calibration Targets
$\theta = 0.0156^a$; $K_t / Y_t = 3.32^a$; $C_t / Y_t = 0.7518^a$; $BE = 0.2^b$;
$SC \in [2,3]^c$; $\delta_H \in [0,0.09]^d$; $\sigma \in [0.9,10]^e$; $\beta \in (0,1)^f$
Parameter Values
Population growth rate
$n = 0.012^a$
Output technology
$\alpha = 0.4^a$; $\delta_K = 0.04697$
Firm's bargaining power
$\eta = 0.5825$
Human capital technology
$\gamma_0 = 0.01738$; $\gamma_1 = 0.01884$; $\delta_H = 0$
Preferences
$\beta = 0.9999$; $\sigma = 5.8791$
Long-run Equilibrium
Variables in the dynamic system
$Z = 7.3886$; $X = 0.2264$; $G = 0.1038$
Interest rate and net marginal productivity of capital (%)
$r = 9.5384$; $NMPK = 7.351$
Saving rate (%)
$SR = 24.82$
Asymptotic speed of convergence (%)
$SC = 4.0908$

Sources: [a)] Cooley and Prescott (1995); [b)] compromise value based on own elaboration from Historical Statistics of Japan; [c)] Barro and Sala (1995); [d)] Jones, *et al*. (2000); [e)] Auerbach, *et al*. (1983); [f)] compromise range based on quantitative studies.

We also impose certain conditions on the parameter values to ensure that they are empirically plausible. As reported in Jones, *et al*. (2000), the estimates of human capital depreciation rate range from 0 to 0.09. Auerbach, *et al*. (1983) survey the estimates of the elasticity of intertemporal substitution and find that it ranges from 0.1 to lightly above 1. Therefore, σ may take values between lightly below 1 and 10. Lastly, looking at quantitative works (e.g., Christiano (1989); Cooley and Prescott (1995)) it seems reasonable to consider values for β below one.

Table 2 contains a summary of calibration targets, parameter values calibrated and the long-run equilibrium.

Some results in the table deserve some comments. The calibration delivers a firm's bargaining power of 58.3%, which means that the worker receives the 41.75% of total returns to investment in specific human capital. Human capital depreciation corresponds to the lower bound of the considered range. Preference parameters take quite high values. More specifically, the implied elasticity of intertemporal substitution is 0.17 and β is almost one.

All targets are replicated, with the exception of the asymptotic convergence rate. The lowest rate delivered by the model is about 4%. Nevertheless, we consider that this result is satisfactory, since the convergence speed achieved by two-sector models of endogenous growth, including those with external LBD, is around 20% (Ortigueira and Santos (1997)).

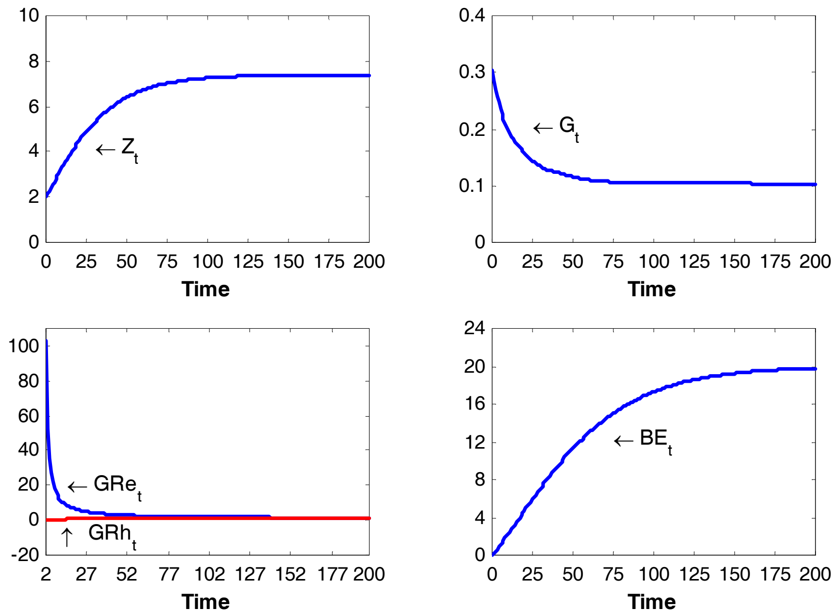
Lastly, interest rate is about 2% higher than net marginal productivity of physical capital. As it is clear from (25), the explanation for this result comes from the double return to savings. Moreover, since the calibration implies that $\delta_H = 0,$ the depreciation of physical capital in (25) is multiplied by less than one.

5.2. Transitional Dynamics

In this subsection, we expose and comment the results from the computation of the model using the parameter values in Table 2.[11] We assume an initial ratio of physical to human capital lower than the one over the BGP, to be consistent with the fact that economies in Table 1 lost a big amount of physical capital during the war period. The Figures 2 and 3 display time paths for significant variables.

Figure 2 shows the time paths for two variables in the dynamic system and the bonus-earnings ratio. The ratio of physical to human capital increases during the transitional period. Accordingly, the ratio of physical capital to specific human capital shadow value declines. The underlying source of these results is in the initial imbalance of capitals. Under LBD, the productions of physical and human capital do not compete for resources. On the contrary, current investment leads to higher physical and human capital in the next period. Thus, resources allocation is not what matters for explaining the increase in $Z_t,$ but the fact that $Z_0 < Z.$

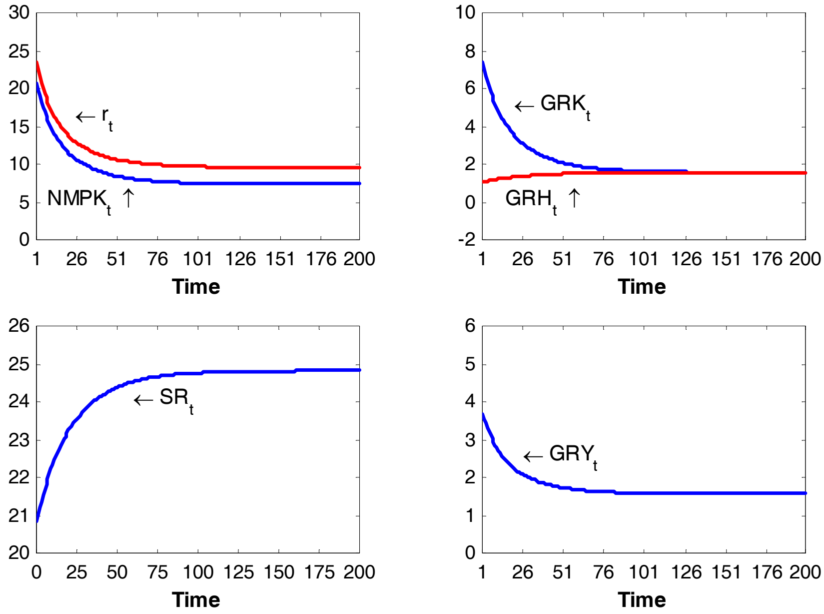
[11] The transitional paths have been computed using the weighted residuals method. See McGrattan (1998) for a detailed description of this method.



Notes: GRe_t: growth rate of specific human capital; GRh_t: growth rate of general human capital.

Figure 2. Transitional Dynamics: Variables in the Dynamic System and Bonus-Earnings Ratio

The behaviour of the bonus-earnings ratio depends on the dynamics of E_t (Equation (20)). Since specific human capital is nil at the initial period, it starts growing at a high rate, which declines as e_t accumulates. The production of general human capital positively depends on specific skills and, thus, its growth rate increases as e_t raises. Consequently, the growth rate of specific skills is always above the one of general skills and, hence, E_t and BE_t rise along the transition. The time evolution of the latter variable agrees with the one in Figure 1c, except for the observed fall from 1997. Nevertheless, the decrease in the data may be interpreted as temporary adjustments, out of the increasing trend, to face the Japanese crisis that started at that year.



Notes: $NMPK_t$: net marginal productivity of physical capital; r_t: interest rate; GRK_t: growth rate of physical capita; GRH_t: growth rate of human capital; SR_t: saving rate; GRY_t: growth rate of output. Capitals and output are expressed in per capita terms.

Figure 3. Transitional Dynamics: Interest Rate, Saving Rate and Growth Rates of Capitals and Output

Figure 3 displays the time path of the return to investment (saving), saving rate and growth rates of capitals and output. According to our theoretical analysis, the time path of interest rate is always above the one of net marginal productivity of physical capital ($NMPK_t$). If the learning process were an external effect, the return to saving would be equal to $NMPK_t$. Internal LBD, however, implies that saving has a double return: the usual return for increasing physical capital, and a second for producing specific skills.

The second component of interest rate acts reducing diminishing returns to physical capital. Indeed, as physical capital accumulates, rises in labour productivity compensates the fall of marginal productivity of physical capital, and makes interest rate time path flatter than $NMPK_t$ one. This is the reason why the model exhibits slow convergence to the BGP. As shown by Barro and Sala-i-Martin (1995, p. 81-82) diminishing returns play an important role in determining the length of the transitional period. More concretely, they find that the capital share has to reach implausibly large values (in the neighbourhood of 0.7) for the Ramsey model to deliver a convergence rate of 4%. Our model, instead, is able to slow convergence with an empirically reasonable value of capital share.

The behaviour of the saving rate in the model involves a substitution effect and an income effect that generate incentives to reduce and increase, respectively, the saving rate through time. The first one is related to diminishing returns to physical capital. As capital rises, interest rate and, thus, the incentives to save decline. The second effect has to do with the gap between current and permanent income. Initially, the current income is far below the permanent or long-term income. The preference for smoothing consumption implies that households would like to devote a big (small) part of income to consumption (saving) when they are poor. As capital accumulates, the gap reduces and saving tends to rise in relation to income. From Figure 3 it follows that in our calibrated model the second effect imposes to the first one. This result comes from the fact that internal LBD acts reducing diminishing returns to physical capital accumulation and, hence, weakening the substitution effect.

As we commented before, the initial imbalance between capitals leads to a higher growth rate for physical (GRK_t) than for human capital (GRK_t). Additionally, physical capital grows at a decreasing rate along the transition as a consequence of diminishing returns. However, the time path of GRH_t is positively sloped and also flatter than the one of GRK_t. The explanation for this behaviour relies on the productivity of investment in the accumulation of both types of capital. An additional unit of investment produces one unit of physical capital and $\gamma < 1$ units of human capital. Thus, the ratio of investment to physical capital decreases through time, whereas investment relative to total human capital barely rises.

The time evolution of the growth rate of per capita output immediately follows from the latter considerations. The fall of the growth rate during the transition is due to diminishing returns to physical capital accumulation.

6. CONCLUSION

Traditionally, the literature on economic growth has treated the LBD process proposed by Arrow (1962) as an externality. However, the empirical evidence shows that LBD is an intentional, and quantitatively relevant, investment in firm-specific human capital. Since internal LBD reduces the diminishing returns to physical capital accumulation, it has important implications for growth during the transitional period.

In this paper we have analysed those implications by developing a simple model of endogenous growth with human capital accumulation through internal and external learning processes. The main feature of the model is that physical investment plays a double role, which results in the return to saving being higher, and its time path flatter, than the net marginal productivity of physical capital. As a consequence, the convergence speed of the economy towards its long-run equilibrium considerably slows.

Additionally, saving rate (investment share of output) increases along the transitional period, while the growth rate of per capita output falls, which agrees with observed facts of some economies, and especially of the Japanese economy.

We can then conclude that the introduction of internal LBD extremely alters the predictions of the model, making them closer to real observations. In light of these findings, understanding the implications of internal LBD for economic growth seems a promising research line. In particular, explicitly modelling how the parties achieve an agreement to share investment returns, and the exploration with more general human capital technologies constitute an interesting matter for future research.

APPENDIX

To prove existence and uniqueness of the BGP, we proceed as Greiner (2003). We start by evaluating (15) and $\frac{H_{t+1}}{H_t}$ over the BGP, and inserting the former equation in the latter one, which gives:

$$\frac{H_{t+1}}{H_t} = 1+\theta = \frac{1}{1+n}\left(\gamma_0 + \gamma_1(\gamma_0 - \delta_H + \delta_K)\frac{Z}{1-\gamma_1 Z}\right). \quad (28)$$

From (5), (12) and (13) it follows that:

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta(1+n)\frac{\mu_t}{\mu_{t+1}} = \beta(1+n)\frac{\lambda_t^K + \gamma_1\lambda_t^r}{\lambda_{t+1}^K + \gamma_1\lambda_{t+1}^e}. \quad (29)$$

Using (6), (7) and (8) and considering that $\frac{\lambda_t^K}{\lambda_{t+1}^K} = \frac{\lambda_t^e}{\lambda_{t+1}^e} = \frac{1+\theta}{\beta(1+n)}$ over the BGP, we can write the latter ratio in (29) as:

$$\frac{\lambda_t^K + \gamma_1\lambda_t^e}{\lambda_{t+1}^K + \gamma_1\lambda_{t+1}^e} = \frac{1}{1+n}(\alpha Z^{\alpha-1} + \gamma_1(1-\alpha)\eta Z^{\alpha} + 1 - \Omega), \quad (30)$$

where $\Omega \equiv \frac{c^K\delta_K + c^e\gamma_1\delta_H}{c^K\delta_K + c^e\gamma_1}$ with c^K and c^e constants. Finally, inserting (30) in (29) and imposing that $\frac{H_{t+1}}{H_t} = \frac{C_{t+1}}{C_t} = 1+\theta,$ we find that the BGP exists if the following auxiliary function has at least a solution:

$$F\left(\tilde{Z},\cdot\right)=\left(\gamma_0+\gamma_1(\gamma_0-\delta_H+\delta_K)\frac{\tilde{Z}}{1-\gamma_1\tilde{Z}}+1-\delta_H\right)^{\sigma}$$
$$-(1+n)^{\sigma}\beta(\alpha\tilde{Z}_{t+1}^{\alpha-1}+\gamma_1(1-\alpha)\eta\tilde{Z}_{t+1}^{\alpha}+1-B), \qquad (31)$$

$$F\left(\tilde{Z},\cdot\right)=0 \text{ if } \tilde{Z}=Z.$$

Note that $F\left(\tilde{Z},\cdot\right)$ is a continuous function for $\tilde{Z}\in\left(0,\gamma_1^{-1}\right)\cup\left(\gamma_1^{-1},\infty\right)$. However, Z must be lower than γ_1^{-1} in order for the long-run growth rate to be positive, since we have assumed that $\gamma_0-\delta_H>0$. Further, $\lim_{\tilde{Z}\to\infty}F\left(\tilde{Z},\cdot\right)=-\infty$, $\lim_{\tilde{Z}\to\gamma_1^{-1}}F\left(\tilde{Z},\cdot\right)=\infty$, and $\frac{\partial F\left(\tilde{Z},\cdot\right)}{\partial\tilde{Z}}>0$. Thus, the BGP exists and it is unique.

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