

PARAMETER HETEROGENEITY IN THE NEOCLASSICAL GROWTH MODEL: A QUANTILE REGRESSION APPROACH

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In this study we examine the issue of parameter heterogeneity in the neoclassical growth model using a quantile regression estimator. Using cross-sectional data on 86 countries covering the period from 1960 to 2000, we estimate a version of the growth model of Mankiw, Romer and Weil (1992). We first estimate the model by OLS. We find that the model is quite successful in explaining the growth empirics of the “average” country. We next estimate the model using quantile regression. The results of quantile regression are at odds with the OLS results. We find evidence of partial parameter heterogeneity. Countries whose growth rates are in the higher quantiles respond differently to investment in human and physical capital than do countries whose growth rates are in the lower quantiles. The neoclassical model predicts conditional convergence. The results from the quantile regression do not fully confirm this prediction. We find that convergence is not a generalized phenomenon across the conditional growth distribution, and, in particular, is not characteristic of countries in the lower quantiles. This suggests that an endogenous growth model, where government policies play a more decisive role in shaping the growth process, may be more suitable to describe growth in the lower tail of the distribution, whereas growth in the middle and higher quantiles is better described by the neoclassical model.

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JEL classification: C14, C21, O41

1. INTRODUCTION

Much of the work in empirical growth research, as epitomized by Barro (1991, 1997), Barro and Sala-i-Martin (1995), and Mankiw, Romer and Weil (1992), among others, has focused on the parameter-invariant linear regression model

$$\Delta y_i = x_i' \gamma + z_i' \zeta + y_{0,i} \beta + \varepsilon_i \quad (1)$$

where Δy_i represents the rate of growth of income per capita in country i over a given period of time, i.e., from $t=0$ to $t=T$, x_i is a $k \times 1$ vector of explanatory variables designed to capture the transitional effects on the growth rate of economy i , $y_{0,i}$ denotes an initial condition, z_i is a $q \times 1$ vector of additional country specific variables, including proxies for technological development and/or other *ad hoc* variables deemed relevant, and ε_i is the unexplained residual. In empirical applications of the Solow growth model (Solow (1956), Mankiw, Romer and Weil (1992)) the initial condition is taken to be real per capita income of the country i at $t=0$, and the vector x_i typically includes three basic variables: investment, population growth, and schooling as a measure of human capital accumulation. The finding that the coefficient on initial level of per capita income is negative is generally interpreted as evidence that there exists a cross-country conditional convergence process,¹ known as β -convergence (Barro and Sala-i-Martin (1995)), which is the centerpiece of the neoclassical model. The neoclassical model predicts that growth rates should converge over time conditioned upon investment rates in physical and human capital and rates of growth of population.

Recently, however, researchers have begun to point out that there are substantial problems and pitfalls in estimating and interpreting growth regressions.² One recent criticism concerns the assumption of parameter homogeneity (Durlauf and Johnson (1995), Durlauf (2000), Durlauf, Kourtellos and Minkin (2001)).³ Parameter homogeneity means that the parameters of the model in Equation (1) are assumed to be country-invariant. As conventionally specified, cross-country growth regressions restrict the magnitude and the significance of the effects of population growth, physical and human

¹ This interpretation has been challenged by Friedman (1992), Quah (1993), and Bernard and Durlauf (1996), among others. Friedman (1992) and Quah (1993) argue that a negative coefficient on initial level of per capita income does not shed any light on whether convergence occurs or not. According to these authors, to make this association is equivalent to commit the classical Galton's fallacy, since the negative association between growth and initial level of per capita income may reflect regression to the mean and not convergence. Bernard and Durlauf (1996), instead, make the point that since the estimated coefficient on initial per capita income is a weighted average of the ratio of the differences of growth rates to differences of initial per capita incomes, both taken from their means, it is theoretically possible that some countries converge while others do not, despite the negative sign of the coefficient.

² See, for example, Temple (1999) and Durlauf (2000) for a general review.

³ An example is Durlauf, Kourtellos and Minkin (2001), p. 928: "As a careful reading of Solow [...] makes clear, the stylized facts for which this model was developed were not interpreted as universal properties for every country in the world. In contrast, the current literature imposes very strong homogeneity assumptions on the cross-country growth process as each country is assumed to have an identical... aggregate production function. This is surprising, as modern growth theory suggests that different countries should be described by distinct aggregate production functions [...]. To us, this suggests that for a given parsimonious growth regression [...] one should explicitly account for parameter heterogeneity."

capital investment as well as the initial level of per capita income on per capita growth rates to be the same for all countries. This assumption seems quite strong and unrealistic, and is neither an empirical regularity nor a theoretical result (Durlauf (2000)). It is unlikely, for example, that the impact of human capital on economic growth is the same for all countries irrespective of their level of development. Rather, it is a convenient choice, often dictated by the ease of applying and interpreting OLS regression.

The focus on the conditional mean is not dictated by theoretical considerations either. The theory of neoclassical growth does not give any guidance as to the parts of the distribution of per capita growth rates where the effects of investment, population growth, education, or other explanatory variables are likely to occur. Therefore, the question as to which part, if any, of the conditional growth distribution satisfies the neoclassical growth model can only be answered by examining whether there exist meaningful differences at different segments of the distribution. In fact, finding the magnitude of the effects of the explanatory variables at the tails of the conditional growth distribution is likely to be more interesting and useful than finding the magnitude of such effects at the conditional mean.

Durlauf (2000) suggests that modeling parameter heterogeneity is one of the crucial topics in the agenda for empirical growth analyses, and Durlauf, Kourtellos and Minkin's (2001), as well as Durlauf and Johnson (1995), Desdoigts (1999), Rappaport (2000), and Kourtellos (2003), among others, have been at the forefront in this area of research. Their findings, which rely on semi-parametric varying coefficient models along the lines of Hastie and Tibshirani (1993), suggest that there is ample evidence of parameter heterogeneity, especially among poorer countries.

While the Hastie and Tibshirani (1993) approach is undoubtedly persuasive and promising, in this paper we propose to examine the question of parameter heterogeneity in cross-country growth models using an alternative framework of analysis. We approach cross-country growth analysis using the concepts and tools of quantile regression, a methodology originally proposed by Koenker and Bassett (1978),⁴ which extends classical least squares estimation of conditional mean models to the estimation of conditional quantile models (Koenker and Hallock (2001)). As conventionally specified, cross-country growth regressions estimate the conditional mean function of the growth distribution. The conditional mean is an important location measure, but it does not fully characterize the conditional distribution. Economic relationships that are significant at the conditional mean may be insignificant over other segments of the conditional distribution; similarly, economic relationships that are insignificant at the conditional mean may be highly significant over other regions of the conditional distribution. Quantile regression provides a wider view of the conditional distribution as it estimates the entire family of quantile functions, which describe not only the central

⁴ See, for example, Koenker (2000), Koenker and Hallock (2001) as well as Buchinsky (1998) for recent reviews of the theory and empirical applications of quantile regression.

but also the tail characteristics of the conditional distribution.

The quantile regression model is especially relevant to the estimation of the neoclassical growth model because it provides an approach to the analysis of the issue of parameter heterogeneity and proposes a framework that overcomes the objections raised by Friedman (1992), Quah (1993) and Bernard and Durlauf (1996). Each estimated quantile portrays a particular segment of the conditional distribution, resulting in a broader and heterogeneous description of the relationship between per capita growth rates and investment in physical and human capital, population growth rates, and the initial level of per capita income.

Two additional features of quantile regression are relevant to the estimation of the neoclassical growth model. First, the classical properties of efficiency and minimum variance of the OLS estimator are obtained under the restrictive assumption of independently, identically and normally distributed errors. When the distribution of errors is non-normal, the quantile regression estimator may be more efficient than the OLS estimator (Buchinsky (1998)). Second, the quantile regression estimator is “robust” to outliers and “long tails” in the distribution of the residuals. Since the quantile regression estimator is derived from the minimization of a weighted sum of absolute deviations, the parameter estimates are less sensitive to a few large or small observations at the tails of the distribution.

The rest of the paper is organized as follows. The Section 2 sets out the methodology of quantile regression and reviews some relevant testing issues. Section 3 describes the data, while Section 4 outlines the quantile version of the neoclassical growth model. Empirical analyses of cross-country growth have generally relied on data drawn from sources such as Barro (1991) and Mankiw, Romer, and Weil (1992), which in turn are based mainly earlier versions of the Penn World Table’s international output estimates (Summers and Heston (1988, 1991)). They are also confined to the period 1960-1985. In this paper we use new data, obtained from the most recent version of the Penn World Table, released in October 2002 (Heston, Summers and Aten (2002)), which enables us to expand the analysis to the year 2000. Section 5 presents the quantile estimates and compares them to the OLS results. Section 6 draws some conclusions. The countries and their associated data series are presented in the Appendix.

2. QUANTILE REGRESSION: AN OVERVIEW

Quantile regression, as the name implies, is a statistical technique designed to estimate and conduct inferences about conditional quantile functions. Just as classical linear least squares methods based on minimizing sums of squares residuals offer a mechanism for estimating models for conditional mean functions, quantile regression methods estimate models for conditional quantile functions, including the conditional median function, thus providing a more complete statistical analysis of the relationships between the dependent variable and the explanatory variables.

Quantile regression was first introduced three decades ago by Koenker and Bassett (1978), but it was not until recently that researchers began to utilize this new tool, helped by the developments of Koenker and Bassett (1982), Buchinsky (1995), Koenker and Machado (1999), and Koenker and Xiao (2002). According to Koenker and Bassett (1982), one of the key factors that makes quantile regression's ability to characterize the entire conditional distribution so useful and interesting is the presence of heteroskedasticity in the data. When data are homoskedastic, the set of slope parameters of conditional quantile functions at each point of the distribution of the growth rates will be identical with each other and with the slope parameters of the conditional mean function. In such a case, the quantile regression at any point along the distribution of the growth rates reproduces the OLS slope coefficients, and the only difference is the intercepts. However, when data are heteroskedastic, the set of slope coefficients of the conditional quantile functions will differ from each other as well as from the OLS slope parameters. In such a case, estimating conditional quantiles at various points of the distribution of the growth rates will allow us to trace out different marginal responses of the growth rates to changes in the explanatory variables at these points. However, one should not conclude that any differences found in regressions at different quantiles are merely due to heteroskedasticity. As Buchinsky (1998, p. 89) observes, "...potentially different solutions at distinct quantiles may be interpreted as differences in the response of the dependent variable to changes in the regressors at various points in the conditional distribution of the dependent variable..." implying that it is possible to think about models that exhibit a linear relationship between conditional quantiles of a dependent variable and explanatory variables, but the relationship itself depends on the quantile under consideration. In such a case, similarly to the heteroskedasticity case, the conditional quantile functions are not necessarily just vertically shifted with respect to each other, and consequently, their estimation can provide a more complete description of the model under consideration than the traditional conditional mean regression. In the cross-sectional growth empirics framework, this means that it is possible to interpret changing coefficients across the conditional distribution as the result of systematic differences between countries.

In what follows we briefly summarize the linear quantile regression model. Let (y_i, x_i) , $i=1, \dots, n$, be a sample of observations from some population, where x_i is a vector of explanatory variables that correspond to y_i . Assuming that the θ th quantile of the conditional distribution of y_i is linear in x_i , we can write the conditional quantile regression model (Buchinsky (1998)) as follows:

$$y_i = x_i' \beta_\theta + u_{\theta i} \quad (2)$$

where β_θ is an unknown $k \times 1$ vector of regression parameters associated with the θ th quantile, x_i is a $k \times 1$ vector of independent variables, y_i is the dependent

variable and u_{θ_i} is the unknown error term. The θ th conditional quantile of y given x is $Quant_{\theta}(y_i|x_i) \equiv \inf\{y: F_i(y|x) \geq \theta\} = x_i'\beta_{\theta}$ and its estimate is given by $x_i'\hat{\beta}_{\theta}$. As θ increases from 0 to 1, the entire distribution of y conditional on x can be traced out. The quantile regression estimator⁵ of β_{θ} can be obtained by minimizing a weighted sum of the absolute errors, where the weights are symmetric for the median regression case ($\theta = 0.5$) and asymmetric otherwise, i.e., as the solution to the following linear programming minimization problem:

$$\min_{\beta_{\theta}} \left\{ \sum_{i: y \geq x_i'\beta} \theta |y_i - x_i'\beta_{\theta}| + \sum_{i: y < x_i'\beta} (1-\theta) |y_i - x_i'\beta_{\theta}| \right\} = \min_{\beta_{\theta}} \sum \rho_{\theta}(u_{\theta_i}) \quad (3)$$

where $\rho_{\theta}(u_{\theta_i})$ denotes the ‘‘check function’’ defined as $\rho_{\theta}(u_{\theta_i}) = \theta u_{\theta_i}$ if $u_{\theta_i} \geq 0$, and $\rho_{\theta}(u_{\theta_i}) = (\theta - 1)u_{\theta_i}$ if $u_{\theta_i} < 0$. The ‘‘check function’’ can be thought as a ‘‘loss function’’ which weights the values of u_{θ_i} at the estimated values of β_{θ} : if a residual is negative, it is weighted $(\theta - 1)$, and if the residual is positive it is weighted θ .⁶ Buchinsky (1998) discusses the consistency and asymptotic normality properties of $\hat{\beta}_{\theta}$ and notes that, under regularity conditions $\sqrt{n}(\hat{\beta}_{\theta} - \beta_{\theta}) \rightarrow N(0, \Omega_{\theta})$, where Ω_{θ} is the variance covariance matrix of $\hat{\beta}_{\theta}$ for a given value of θ . Two general approaches exist in the literature for the estimation of Ω_{θ} . The first, originally suggested by Koenker and Bassett (1978), derives analytically the asymptotic standard errors of the estimator based on the assumption identically independently distributed errors, while the second, suggested by Buchinsky (1995) uses bootstrap methods. The literature provides no definite answer as to which approach to follow. This, however, is not a serious problem. As noted by Koenker and Hallock (2001), the differences between competing methods of inference for quantile regression are very small in practice, and are more robust than other forms of inference in econometrics. There are several bootstrap methods that can be used to derive the variance-covariance matrix of $\hat{\beta}_{\theta}$. In this paper we employ the design matrix bootstrap method to obtain estimates of the standard errors for the coefficients in quantile regression (Buchinsky (1995, 1998)). Based on the results of a Monte Carlo study, Buchinsky recommends the use of this method as it performs well in relatively small samples and is robust to changes of the bootstrap sample size relative to

⁵ The quantile regression estimator of β_{θ} , unlike the OLS estimator of β , does not have an explicit form, but can be obtained by linear programming techniques (Koenker and Bassett (1978)).

⁶ To illustrate this point, suppose the quantile regression for the 75th quantile is estimated. The weight for negative residuals is -0.25 while the weight for positive residuals is 0.75. Minimizing the sum of the residuals using these weights is equivalent to using the absolute values of the residuals with weights 0.25 and 0.75.

the data sample size. More importantly, the design matrix bootstrap is valid under many forms of heteroskedasticity.

In the design matrix bootstrap, we treat our sample of n observations as if it were the population of interest. Specifically, let $(y_i^{BS}, x_i'^{BS})$, $i = 1, \dots, n$, be the bootstrap sample obtained by sampling with replacement from the original sample (y_i, x_i') . It then follows from Equation (2) that $y_i^{BS} = x_i'^{BS} \beta_\theta^{BS} + u_{\theta i}^{BS}$. Applying the simplex algorithm to this sample gives $\hat{\beta}_\theta^{BS}$, a bootstrap estimate of β_θ . Repeating this process B times yields bootstrap estimates $\hat{\beta}_{\theta 1}^{BS}, \dots, \hat{\beta}_{\theta B}^{BS}$. The bootstrap estimate of the asymptotic variance-covariance matrix of β_θ is then obtained as follows:

$$\hat{\Omega}_\theta^{BS} = \frac{n}{B} \sum_{j=1}^B (\hat{\beta}_{\theta j}^{BS} - \bar{\hat{\beta}}_\theta) (\hat{\beta}_{\theta j}^{BS} - \bar{\hat{\beta}}_\theta)' \quad \text{where} \quad \bar{\hat{\beta}}_\theta = \frac{1}{B} \sum_{j=1}^B \hat{\beta}_{\theta j}^{BS}. \quad (4)$$

Intuitively, one can see that $\hat{\beta}_\theta^{BS}$ has the same asymptotic distribution as does $\hat{\beta}_\theta$. That is, both $\sqrt{n}(\hat{\beta}_\theta - \beta_\theta)$ and $\sqrt{n}(\hat{\beta}_\theta^{BS} - \beta_\theta) \rightarrow N(0, \Omega_\theta)$.

3. THE NEOCLASSICAL GROWTH MODEL IN A QUANTILE FRAMEWORK

One way of thinking about a quantile generalization of the neoclassical model is to assume that each quantile of the transitional growth conditional distribution obeys the neoclassical model, but that the aggregate production function and the structural parameters that characterize the transitional path to steady state differ along quantiles. Specifically, we assume that aggregate output at time t in country i in the θ quantile of the distribution of per capita growth can be described by a Cobb-Douglas aggregate production function where total output, Y , is a function of effective labor, AL , the stock of physical capital, K , and the stock of human capital H :

$$Y_t = K_t^{\alpha(\theta)} H_t^{\beta(\theta)} (A_t L_t)^{1-\alpha(\theta)-\beta(\theta)} \quad (5)$$

The parameters $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are implied shares of physical and human capital (i.e., the output elasticities with respect to physical and human capital) for countries in the θ quantile, respectively. In contrast to the conditional mean model, α and β are indexed by θ . There are constant returns with respect to all inputs and decreasing returns to both labor and the two stocks of capital. As in the conditional mean model, however, technology and labor are assumed to grow at a constant exogenous rate g and n , respectively. Gross investment in physical and human capital comes from

households saving a fixed percentage, s_k and s_h , respectively, of output and both stocks of capital depreciate at constant rate δ . Approximating around the steady state and imposing a cross-section specification, the stochastic representation of the transitional growth rate of output per worker at the θ quantile can be found to be

$$\begin{aligned} \ln(y_t) - \ln(y_0) = & \xi(\theta) + (1 - e^{-\lambda(\theta)t}) \ln(A_0) - (1 - e^{-\lambda(\theta)t}) \ln(y_0) \\ & + (1 - e^{-\lambda(\theta)t}) \frac{\alpha(\theta)}{1 - \alpha(\theta) - \beta(\theta)} \ln(s_k) + (1 - e^{-\lambda(\theta)t}) \frac{\beta(\theta)}{1 - \alpha(\theta) - \beta(\theta)} \ln(s_h) \\ & - (1 - e^{-\lambda(\theta)t}) \frac{\alpha(\theta) + \beta(\theta)}{1 - \alpha(\theta) - \beta(\theta)} \ln(n + g + \delta) + u_\theta \end{aligned} \quad (6)$$

where $\lambda(\theta) = (n + g + \delta)(1 - \alpha(\theta) - \beta(\theta))$ is the rate of convergence of the countries in the θ quantile of the conditional growth distribution and u_θ is the error term at the θ quantile.⁷

⁷ Equation (6) is a dynamic equation that is obtained by linearizing the transition path around the steady state. The steady state version of Equation (6), following Mankiw, Romer and Weil (1992), is given by

$$\begin{aligned} \ln(\tilde{y}^*) = & \xi(\theta) + \ln(A_0) + \frac{\alpha(\theta)}{1 - \alpha(\theta) - \beta(\theta)} \ln(s_k) + \frac{\beta(\theta)}{1 - \alpha(\theta) - \beta(\theta)} \ln(s_h) \\ & - \frac{\alpha(\theta) + \beta(\theta)}{1 - \alpha(\theta) - \beta(\theta)} \ln(n + g + \delta) + u_\theta \end{aligned} \quad (i)$$

where $\tilde{y}^* = (Y_t/A_t L_t)^*$ is the steady state level of output per effective worker. Linearizing the transition path around \tilde{y}^* by means of a Taylor series expansion yields

$$d(\ln(\tilde{y}^*)/dt) = \lambda(\theta) [\ln(\tilde{y}^*) - \ln(\tilde{y}_t)] \quad (ii)$$

where \tilde{y}_t is the level of output per effective worker at time t . The first-order differential equation in (ii) implies that

$$\ln(\tilde{y}_t) = (1 - e^{-\lambda(\theta)t}) \ln(\tilde{y}^*) + e^{-\lambda(\theta)t} \ln(\tilde{y}_0) \quad (iii)$$

where \tilde{y}_0 is output per effective worker at some initial point in time. Subtracting $\ln(\tilde{y}_0)$ from both sides of (iii) gives

Equation (6) incorporates the two basic predictions of the neoclassical model. First, the model predicts that the transitional growth rate of output per worker is positively affected by investment in human capital, s_h , and investment in physical capital, s_k , and negatively affected by population growth, n , physical capital depreciation, g , and technological progress, δ . Second, the model predicts conditional convergence, i.e., countries with low initial output per worker possess faster transitional growth rates than countries with higher initial output per worker, conditioned upon s_k, s_h, n, g, δ , but, unlike its Mankiw, Romer and Weil's (1992) conditional mean counterpart, also on θ , thus explicitly accounting for the possibility of parameter heterogeneity of the model. Equation (6) places the restriction that the coefficients of $\ln(s_k)$, $\ln(s_h)$, and $\ln(n + g + \delta)$ at each quantile add to zero. Following the conventional terminology of growth empirics, this equation will be referred to as the "restricted" quantile version of the neoclassical model. Relaxing the restrictions yields the "unrestricted" version:

$$\begin{aligned} \ln(y_t) - \ln(y_0) = & \gamma_0(\theta) + \gamma_1(\theta)\ln(y_0) + \gamma_2(\theta)\ln(s_k) + \gamma_3(\theta)\ln(s_h) \\ & + \gamma_4(\theta)\ln(n + g + \delta) + \gamma_5(\theta)\ln(A_0) + v_\theta. \end{aligned} \quad (7)$$

It should be noted that, unlike its conditional mean counterpart, the quantile representation of the neoclassical model distinguishes between "local" and "global" conditional convergence (Durlauf and Johnson (1995)). A negative value for $\gamma_1(\theta)$ in Equation (7) or for $-(1 - e^{-\lambda(\theta)t})$ in Equation (6) for some θ is taken as evidence supporting the convergence hypothesis locally, while a negative value for $\gamma_1(\theta)$ in Equation (7) or for $-(1 - e^{-\lambda(\theta)t})$ in Equation (6) for all θ is taken as evidence supporting the convergence hypothesis globally.

4. DATA, VARIABLES AND DESCRIPTIVE STATISTICS

Empirical analyses of cross-country growth rely heavily on data obtained from the Penn World Table. Typically, previous research has utilized either the Penn World Table 4.0 (Summers and Heston (1988)) or the Penn World Table 5.0 (Summers and Heston (1991)). One major characteristic that distinguishes our empirical work from previous research is that our data are obtained from the Penn World Table 6.1 (Heston, Summers

$$\ln(\tilde{y}_t) - \ln(\tilde{y}_0) = (1 - e^{-\lambda(\theta)t}) [\ln(\tilde{y}^*) - \ln(\tilde{y}_0)]. \quad (\text{iv})$$

Finally, substituting (i) into (iv) and rearranging (iv) in terms of output per worker (since output per effective worker is not observable) yields the transitional growth equation in (6).

and Aten (2002)). This latest version of the Penn World Table, released on October 2002, takes into account quality improvements in the national income accounts data which were not available in the earlier versions, such as national accounts revisions, updates to reflect 1996 benchmarks, and changes in base year prices. In addition, the Penn World Table 6.1 extends international output estimates up to the year 2000, whereas the previous versions were confined to the period from 1960 to 1985.

Originally, we intended our sample of countries to include all the 98 countries used in Barro (1991) and Mankiw, Romer and Weil (1992). However, one problem with the Penn World Table 6.1 is that real per capita income for the year 2000 is not available for eleven countries (Angola, Botswana, Burma, Central African Republic, Haiti, Liberia, Mauritania, Papua New Guinea, Somalia, Sudan, and Zaire) and real per capita income in 1960 is not available for Germany. Removal of these countries yields a sample of 86 countries with complete data on the variables of the model.

All the variables are constructed following as close as possible the methodology and the definitions used by Mankiw, Romer and Weil (1992). The rate of growth of real per capita GDP for a given country is computed as $\ln(y_{2000}) - \ln(y_{1960})$. The variable y in year 2000, i.e., y_{2000} , or in year 1960, i.e., y_{1960} , for a given country is measured by Y/L , real GDP per person of working age (defined as the age cohort 15 to 64 years) in year 2000 (or year 1960). This variable is obtained by multiplying Y/P , real GDP per capita adjusted for terms of trade changes for a given country in 1996 international prices (source: variable *rgdptt*, Penn World Table 6.1) by P/L , where P is total population (source: variable *POP*, Penn World Table 6.1) and L is working population, age 15 to 64 years (source: World Bank Development Indicators Online (December 2002)) in year 2000 (or year 1960). The variable s_k for a given country is measured by I/GDP , the average share of GDP devoted to investment in physical capital between 1960 and 2000 (source: variable *ki*, Penn World Table 6.1). The variable s_h for a given country is constructed exactly as the Mankiw, Romer and Weil's (1992) SCHOOL variable, i.e., the average fraction of working age population in secondary school (age 15 to 19 years) between 1960 and 2000. The variable n for a given country refers to the average annual rate of growth of the working age population (ages 15 to 64 years) between 1960 and 2000 (source: World Bank Development Indicators Online (December 2002)).⁸

The right-hand side of Equations (6) and (7) contains three variables, the rate of technological progress, g , the rate of depreciation, δ , and the initial level of

⁸ Data for s_h are obtained by updating the SCHOOL measure for the years 1990, 1995 and 2000. We follow the procedure utilized by Mankiw, Romer and Weil (1992). Data on gross enrollment for secondary education (the ratio, expressed as a percentage, of the number of students enrolled in secondary school to the population in the age cohort 12 to 17 years) are from the World Bank's Edstats database. Data on the cohort 15 years to 19 years are from the International Data Base (IDB) of the United States Census Bureau, Population Division, International Programs Center. Data on working population, ages 15 to 64 years, are from the World Bank's Development Indicators Online.

technological efficiency, A_0 , which are not directly observable. In the case of g and δ , the problem is not very crucial. The usual procedure in the literature is to impose “reasonable values” on these parameters prior to estimation. We follow Mankiw, Romer and Weil (1992) in assuming that the rate of depreciation δ and the rate of technological progress g are constant and equal to 0.03 and 0.02, respectively. The lack of direct observability of A_0 , instead, raises a more difficult issue. This is particularly important in the context of quantile regression, since any observed difference in the estimated parameters at different points of the conditional growth distribution is interpreted as evidence of parameter heterogeneity.

Casual observation suggests that levels of technological development differ widely across countries. Yet, Mankiw, Romer and Weil (1992) omit A_0 from the regression and subsume technological differences across countries in the error term, justifying the omission with the assumption that the error term is independent of the explanatory variables. This, however, biases the estimates of the remaining parameters, since A_0 is correlated with one of the explanatory variables, the initial level of per capita income. The panel data growth literature (Islam (1995), Caselli *et al.* (1996), Bond, Hoeffler and Temple (1998)) resolves the issue⁹ by treating A_0 as an unobserved country specific effect. This literature represents the transitional growth Equations (6) and (7) as dynamic panel models which are estimated on differenced data. One might naively consider quantile regression on a differenced equation since $Quant_\theta(\Delta u_{it} | \Delta x_i) = 0$. However, this approach has a fundamental drawback. Unlike conditional mean estimators, differencing in the quantile regression framework is not equivalent to a fixed effect estimator. The quantile operator is not linear, namely, $Quant_\theta(X + Y) \neq Quant_\theta(X) + Quant_\theta(Y)$. The determination of a quantile requires sorting; consequently, in quantile regression the order of the countries matters when differencing, and thus it is not possible to recover the quantile estimates from the estimates on differenced data.

A more pragmatic approach is suggested by Temple (1998). Based on the findings of Koop, Osiewalski and Steel (1995) that most of the variation in technical efficiency is between regional country groupings rather than within them, Temple (1998) suggests proxying A_0 with a set of regional dummy variables. As Temple (1998, p. 363) observes “it is natural to think of the industrialized countries sharing a high level of efficiency in 1960 and sub-Saharan Africa a low one.” We follow Temple’s (1998) approach. Although *ad hoc* and imperfect, nonetheless it helps to lessen the omitted variable bias and reduces the risk of inducing parameter heterogeneity by omission of a relevant variable. Accordingly, to account for technological differences across countries we include in both Equation (6) and (7) four dummy variables, defined as follows: sub-Saharan Africa (26 countries), Latin America and the Caribbean (20 countries), Eastern Asia (7 countries), and the industrialized countries (21 countries), as defined by

⁹ Yet, Barro (1997) points out that this approach has problems of its own, particularly when the data contain important cyclical components or other short-term noise.

OECD membership in 1960. The omitted region includes North Africa, the Middle East, and the Indian sub-continent (12 countries). Table 1 provides a brief summary of the distributional properties of the data in logarithmic form.

Table 1. Summary Statistics

	$\ln(y_{1960})$	$\ln(s_k)$	$\ln(n + g + \delta)$	$\ln(s_h)$	$\ln(y_{2000}/y_{1960})$
Mean	8.3977	-2.0000	-2.6388	-3.0576	0.5872
Standard Deviation	0.8177	0.6258	0.1382	0.8232	0.6270
Median	8.3869	-1.9328	-2.5854	-2.8110	0.6335
1st quantile	7.8268	-2.3001	-2.7017	-3.4793	0.1998
3rd quantile	8.9847	-1.5042	-2.5478	-2.4507	0.9770
Skewness	0.0316	-0.9038	-0.7758	-1.1455	0.0078
Kurtosis	2.2062	3.5796	2.6569	3.4489	3.2286

As can be seen from the table, the mean and the median of $\ln(y_{1960})$ are very close, which indicates that there is no leftward or rightward skewness. Similarly, the mean and the median of $\ln(y_{2000}) - \ln(y_{1960})$ do not seem to indicate any problem with skewness. Conversely, $\ln(s_k)$, $\ln(s_h)$ and $\ln(n + g + \delta)$ are all moderately skewed to the left, namely, they have long left tails. Departure from normality is also quite apparent in the kurtosis of all of the variables. Platykurticity is observable in both $\ln(y_{1960})$ and $\ln(n + g + \delta)$, whereas $\ln(y_{2000}) - \ln(y_{1960})$, $\ln(s_k)$, and $\ln(s_h)$ exhibit a moderate degree of leptokurticity. Distributional characteristics of the data tend to bear some impact on the results of OLS estimation. In our case, it is apparent that using OLS on these variables has the potential for problems. OLS is moderately robust to departures from normality in the presence of kurtosis, but does not perform as well in the presence of skewness.

5. ESTIMATION RESULTS

5.1. OLS Regression Results: Restricted and Unrestricted Estimates

In Table 2 we present the OLS estimates of the transitional growth Equations (6) and (7), augmented with regional variables, using White's (1980) consistent covariance matrix to correct for any unknown form of heteroskedasticity. Although the primary concern of this study is not the estimated effects at the mean, it is useful to discuss these results for they provide a benchmark against which the quantile regression estimates might be compared. Also, they are useful to indicate in which direction the parameter heterogeneity bias induced by OLS methods operates.

Table 2. OLS Regression Results

Dependent Variable: log difference GDP per working-age person, 1960-2000		
	Unrestricted Regression	Restricted Regression
Constant	2.0075 (1.1541)	3.6270 (0.8032)
$\ln(y_0)$	-0.3577 (0.0997)	-0.3636 (0.0986)
$\ln(s_k)$	0.2565 (0.1376)	0.2680 (0.1375)
$\ln(n + g + \delta)$	-1.1468 (0.4235)	-0.5448 (0.1518)
$\ln(s_h)$	0.2732 (0.1134)	0.2768 (0.1146)
<i>East Asia</i>	0.5763 (0.2624)	0.5743 (0.2625)
<i>Latin America and Caribbean</i>	-0.1831 (0.1385)	-0.1625 (0.1349)
<i>Sub-Saharan Africa</i>	-0.4318 (0.1995)	-0.4097 (0.1993)
<i>Industrialized Countries</i>	0.1317 (0.2309)	0.2980 (0.2076)
R^2	0.7009	0.6946
Test of Joint Significance of Regional Variables		
<i>F</i> -statistic	3.5666	3.9554
<i>p</i> -value	0.0101	0.0057
Wald Test of Restrictions		
<i>F</i> -statistic	2.6200	
<i>p</i> -value	0.1098	
Implied Structural Parameters		
λ		0.0113 (0.0039)
α		0.2951 (0.1325)
β		0.3047 (0.1070)
Jarque-Bera Test	12.7411	14.4999
<i>p</i> -value	0.0017	0.0007
Shapiro-Wilk Test	0.9640	0.9630
<i>p</i> -value	0.0169	0.0147
Shapiro-Francia Test	0.9580	0.9566
<i>p</i> -value	0.0088	0.0075
White Test	1.5998	1.7821
<i>p</i> -value	0.0631	0.03509
Breusch-Pagan Test	32.5200	33.1700
<i>p</i> -value	0.0001	0.00001

Note: Robust standard errors in parentheses.

The results are generally supportive of the predictions of the neoclassical model. In both regressions, the coefficients of physical and human capital investment and population growth have the expected sign and are highly significant. Growth of per capita income is positively related to physical and human capital and negatively related to the rate of population growth. The coefficient on the initial level of income is negative and highly significant, providing strong evidence of conditional convergence. The magnitude of the coefficients is also resoundingly plausible.

The country regional location is also correlated to growth. However, although the F tests reveal that the four regional variables are jointly significantly different from zero, only two of the coefficients are individually significant, namely, the coefficient on East Asian countries, which is positive, and the coefficient on sub-Saharan African countries, which is negative. This implies that technology, climate, institutions, and resource endowments have a positive impact on East Asian countries, but a negative impact on sub-Saharan African countries. On average, because of these, growth is more than 50 percent higher in East Asian countries and about 40 percent lower in sub-Saharan African countries than in the countries of the omitted region (North Africa, the Middle East, and the Indian sub-continent). Conversely, the OLS estimates fail to indicate that such impact is present for Latin America and Caribbean countries or the industrialized countries. The presence of the regional variables has a significant effect on R^2 , which increases by approximately 15 percent, but has little effect on the remaining coefficients. The coefficient of determination R^2 indicates that the model, in both its unrestricted and restricted specification, explains approximately 60 percent of cross-country variations in growth of per capita income. This is a customary result found in the literature. The restriction that the sum of the coefficients of $\ln(s_k)$ and $\ln(s_h)$, and the coefficient of $\ln(n + g + \delta)$ are equal in magnitude and opposite in sign is also not rejected by the Wald test, which is asymptotically distributed as an F statistic with 1 and 77 degrees of freedom. This is also a standard result that is found often in the literature. The magnitude of the coefficient of human capital, however, is larger than what previous studies have found over the period 1960-1985, which suggests that the role of human capital in the process of economic growth has become increasingly relevant in the last twenty years.

The results of the restricted specification also reveal that the implied values of α and β are much closer to the values predicted by the model than those obtained in empirical analyses of the model over the period 1960-1985, and imply a convergence rate, λ , of approximately 1 percent, which is not substantially different from the estimate obtained by Mankiw, Romer and Weil (1992) for their larger sample.¹⁰ Taken as a

¹⁰ The presence of the regional variables does not appear to have any significant effect on the estimates of the structural parameters or the results of the tests of normality or heteroskedasticity of the residuals. Dropping these variables from the regression yields the following implied values: $\alpha = 0.3463$ (t -statistic = 5.1152), $\beta = 0.3431$ (t -statistic = 6.0941), and $\lambda = 0.0135$ (t -statistic = 3.7500).

whole, the results in Table 2 indicate that the OLS regression performs quite well in explaining the growth performance of the “average” country.

One pitfall, however, with these results is that the OLS residuals, from both the unrestricted and restricted regression, are non-normal and heteroskedastic. In Table 2 we report the results of the Jarque-Bera test (Jarque and Bera (1980)), the Shapiro-Wilk test (Shapiro and Wilk (1965)) and the Shapiro-Francia test (Shapiro and Francia (1972)). The residuals from the unrestricted regression have a skewness of -0.4345 and a kurtosis of 4.6734. The Jarque-Bera test statistic is 12.7411 (p -value = 0.0017). Similarly, the residuals from the restricted regression have a skewness of -0.4637 and a kurtosis of 4.7850. The Jarque-Bera test statistic is 14.4999 (p -value = 0.0007). The Shapiro-Wilk and Shapiro-Francia tests further confirm the non-normality of the residuals. The Shapiro-Wilk test statistic W is 0.9640 (p -value = 0.0169) in the unrestricted regression and 0.9630 (p -value = 0.0147) in the restricted regression. Similarly, the Shapiro-Francia test statistic W' is 0.9580 (p -value = 0.0088) in the unrestricted regression and 0.9566 (p -value = 0.0075) in the restricted regression. These findings provide solid evidence against the null hypothesis that the residuals are normally distributed. In addition, the Breusch-Pagan test (Breusch and Pagan (1979)), also reported in Table 2, rejects the hypothesis that the residuals are homoskedastic. The Breusch-Pagan test statistic is 32.52 (p -value = 0.0001) in the unrestricted regression and 33.17 (p -value = 0.0001) in the restricted regression. The same conclusion can be attained using the White test (White (1980)).

5.2. Quantile Regression Results: Unrestricted Estimates

The results of the unrestricted quantile regressions of the transitional growth model, augmented with regional variables, are contained in Table 3. We estimate Equation (7) at 19 quantiles, using simultaneous quantile regression, from the 5th to the 95th quantile by increments of 0.05. Figure 1 plots the quantile coefficient estimates. The dashed line locates the point estimates $\hat{\gamma}_{\theta_j}$, $j=1,\dots,4$ with the two dotted lines depicting a 90 percent point-wise confidence band. The superimposed solid horizontal line refers to the unrestricted OLS estimate.

In Table 3 we report the coefficient estimates and related statistics for the five most relevant quantiles, namely, the 10th, 25th, 75th, and 90th quantiles. The estimated standard errors in parenthesis are obtained using the design matrix bootstrap approach; hence, they are robust to heteroskedasticity and any general dependence between the explanatory variables and the error term. We use 2000 replications and, to assure reproducibility of the results, set the seed of the pseudo-random number generator of the bootstrap process at 1001.¹¹

¹¹ The problem of choosing the number of bootstrap repetitions has been studied in the literature. See

The interpretation of quantile regression coefficients is straightforward. In OLS growth regressions the coefficient of, say $\ln(s_k)$, represents, other things equal, the impact of a change in $\ln(s_k)$ on expected, i.e., average, growth. In the quantile model, instead, the quantile coefficient of $\ln(s_k)$ represents the change, other things equal, in the θ th conditional quantile of the growth distribution due to a change in $\ln(s_k)$. Thus, one should note that when the k th element of x changes, it does not follow that y_i associated with a specific quantile θ would remain in that quantile.¹²

The results are quite revealing. The empirical validity of the neoclassical growth model impinges upon the non-rejection of the restriction that the sum of the coefficients of $\ln(s_k)$, $\ln(s_h)$, and $\ln(n + g + \delta)$ adds to zero. In Table 3 the F statistics, with 1 and 77 degrees of freedom, corresponding to the Wald test of the restriction are reported. They indicate that this restriction hold across all the five quantiles. Associated with the quantile estimates Table 3 reports the *pseudo* R^2 , a quantile measure of goodness of fit.¹³ The *pseudo* R^2 increases from the lower to the higher quantiles, which indicates that the model explains growth for countries whose growth rates are in the higher quantiles better than for countries whose growth rates are in the lower quantiles. Inspection of the quantile estimates in Figure 1 reveals that the slope coefficients estimated at different quantiles are not flat but follow nonlinear patterns, which suggests, at least at an informal level, the existence of parameter heterogeneity across quantiles.

Andrews and Buchinsky (1998) and Davidson and McKinnon (1997) for a discussion. The problem is that one can obtain “different answers” from the same data merely by using different replications if B is too small, but computational costs can be great if B is chosen to be extremely large. In this paper the number of replications was chosen as follows. First we set a large but tolerable number of replications and obtained the bootstrap estimates. Next, we changed the random number seed a few times, and, using the same number of replications, we obtained the bootstrap estimates again. If the results changed in any meaningful way, the number of replication was deemed too small and a higher number was used. This process was continued until the results were deemed stable. One should note that the number of replications used in this paper is quite large compared to what is found in the empirical literature, which usually reports a number of bootstrap replications around 100.

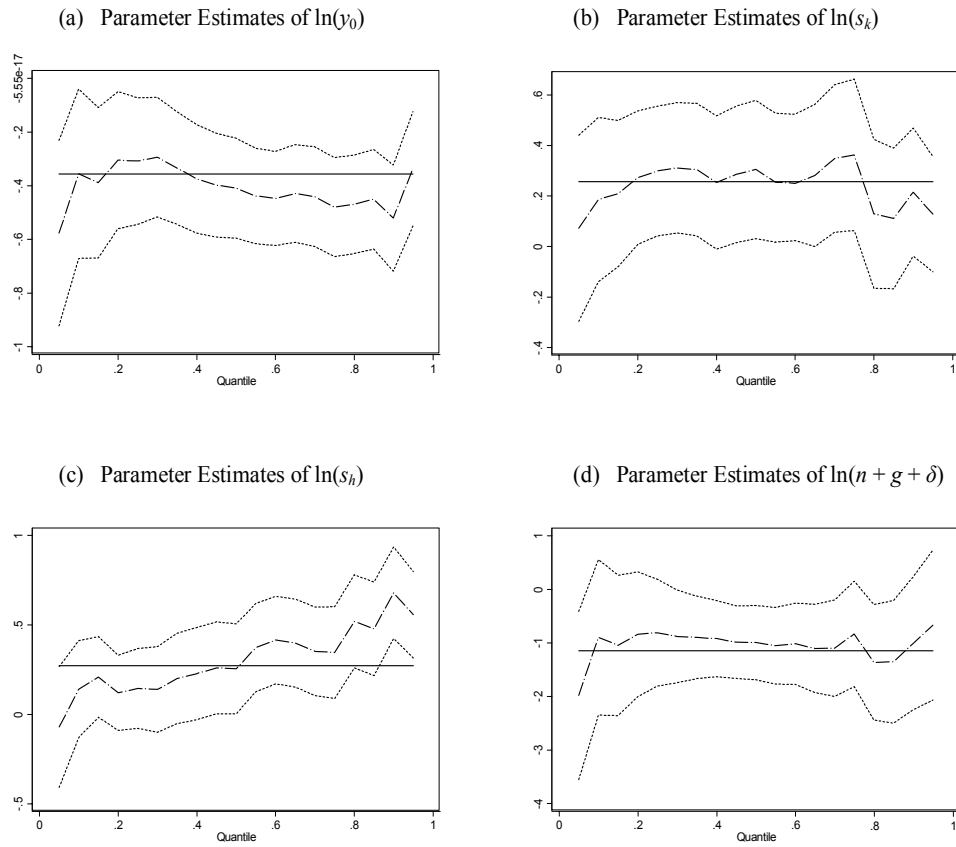
¹² This is because a change in the quantile coefficient changes the shape of the conditional distribution, unlike the change in the OLS coefficient, which only shifts the distribution.

¹³ The statistic, developed by Koenker and Machado (1999), is analogous to the conventional R^2 statistic, and is obtained in a similar manner. Let \hat{V}_θ be the solution to Equation (2) and let \tilde{V}_θ be the solution to Equation (2) when x_i is restricted to include only the intercept. Then the *pseudo* R^2 for the θ th quantile is defined as $1 - \hat{V}_\theta / \tilde{V}_\theta$. Unlike the conventional R^2 , which is a “global” measure of goodness-of-fit over the entire conditional distribution, the *pseudo* R^2 is only a “local” measure of goodness-of-fit at a specific quantile.

Table 3. Unrestricted Quantile Regression Estimates

Dependent Variable: log difference GDP per working-age person, 1960-2000					
	$\theta = 0.10$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 0.90$
Constant	1.7431 (2.3741)	2.1621 (1.7013)	2.8479 (1.3871)	4.3364 (1.8369)	5.0740 (2.2808)
$\ln(y_0)$	-0.3563 (0.1893)	-0.3091 (0.1414)	-0.4100 (0.1117)	-0.4803 (0.1105)	-0.5213 (0.1186)
$\ln(s_k)$	0.1862 (0.1951)	0.2990 (0.1542)	0.3049 (0.1642)	0.3627 (0.1801)	0.2154 (0.1524)
$\ln(n + g + \delta)$	-0.8978 (0.8716)	-0.8118 (0.5982)	-0.9972 (0.4158)	-0.8350 (0.5913)	-1.0082 (0.7459)
$\ln(s_h)$	0.1426 (0.1621)	0.1456 (0.1339)	0.2552 (0.1509)	0.3455 (0.1540)	0.6796 (0.1535)
<i>East Asia</i>	0.0776 (0.3901)	0.3792 (0.3491)	0.3821 (0.3055)	1.0215 (0.3715)	0.9317 (0.3129)
<i>Latin America and Caribbean</i>	-0.1243 (0.3263)	-0.4523 (0.2292)	-0.0858 (0.1855)	0.0347 (0.1611)	-0.0122 (0.1551)
<i>Sub-Saharan Africa</i>	-0.6439 (0.2957)	-0.7757 (0.2593)	-0.4003 (0.2369)	-0.2221 (0.2315)	0.2575 (0.2844)
<i>Industrialized Countries</i>	0.5071 (0.5067)	0.1325 (0.3538)	0.2073 (0.2644)	0.2522 (0.2781)	0.0631 (0.2872)
<i>Pseudo R²</i>	0.496	0.4993	0.5131	0.5034	0.5755
Wald Test of Restrictions					
<i>F</i> -statistic	0.4300	0.4200	1.1800	0.0400	0.0200
<i>p</i> -value	0.5149	0.5204	0.2811	0.8342	0.8834
Test of Joint Significance of Regional Variables					
<i>F</i> -statistic	1.5600	3.4900	1.2200	2.1800	2.9800
<i>p</i> -value	0.1930	0.0113	0.3077	0.0786	0.0243
Interquantile Tests					
$\theta = 0.10$		0.7300	0.8400	1.8800	2.5200
<i>p</i> -value		0.6669	0.5689	0.0755	0.0172
$\theta = 0.25$			1.1200	1.7100	2.4200
<i>p</i> -value			0.3580	0.1091	0.0218
$\theta = 0.50$				0.8300	1.4200
<i>p</i> -value				0.5830	0.2002
$\theta = 0.75$					0.9400
<i>p</i> -value					0.4899

Note: Bootstrapped standard errors in parentheses.



Notes: The graphs plot the unrestricted parameter estimates associated with each variable, together with a 90% confidence band, against the quantile at which the model is estimated. The superimposed horizontal solid line refers to the corresponding OLS (invariant) parameter estimate.

Figure 1. Parameter Estimates of the Unrestricted Model

Significant differences exist between the estimated coefficients, especially between the lower and the upper quantiles. The country regional location is correlated with growth rates but not at all quantiles across the distribution. In contrast with the OLS results, the test statistic for the joint significance of the regional variables, which is asymptotically distributed as an F statistic with 4 and 77 degrees of freedom, cannot reject the null hypothesis at the 10th and the 50th quantiles. The coefficient on Latin American and Caribbean countries is negative, but statistically significant only at the 25th quantile. The coefficient on industrialized countries is positive, and its magnitude tends to decline in the upper quantiles, but is not statistically significant. The coefficient on sub-Saharan African countries, instead, is mostly significant at the lower quantiles, but insignificant at the middle and higher quantiles. Conversely, the coefficient on East Asian countries is mostly insignificant at the lower quantiles, and highly significant at the higher quantiles.

The additional information revealed by the quantile estimates compared to the OLS estimates comes into sharper focus when we consider the quantile estimates for physical and human capital investment, population growth and the initial level of per capita income. The OLS estimates indicate that population growth and the initial level of per capita income have a significant negative impact on growth, while investment in physical and human capital have a significant positive effect. The quantile estimates, however, reveal that these effects are not uniformly distributed across the estimated quantiles. The quantile estimates for investment in human capital are smaller for countries in the lower quantiles and larger for countries in the higher quantiles. In the lower quantiles the effect of human capital investment on growth is positive but insignificant. Therefore, it does appear that after controlling for differences in technology and institutions, only countries in the upper region of the conditional distribution have the ability to translate investment in human capital into economic growth. The quantile estimates for population growth, instead, indicate that the negative effect of this variable on growth is almost entirely located at the median; at all the remaining quantiles the quantile estimates on population growth are insignificant, although they do not differ in magnitude from the OLS estimate. Conversely, the quantile estimates for investment in physical capital are significant in the interquantile range but insignificant in the lower and higher quantiles. Thus, it does appear that, after controlling for differences in technology and institutions, countries whose growth rates are in the lower and higher quantiles have difficulty in translating investment in physical capital into economic growth.

The quantile estimates for initial per capita income are the only estimates that are significant across all estimated quantiles, which provides some evidence of conditional convergence. This evidence, however, is not as persuasive as the OLS results imply, since the phenomenon of convergence is not as strong at the 10th and 25th quantiles as it is at the median and higher quantiles. The estimates of the coefficient on initial per capita income increase monotonically from the lower to the higher quantiles, implying that the speed of convergence is higher in countries whose growth rates are in the higher quantiles than in countries whose growth rates are in the lower quantiles.

Formally, parameter heterogeneity can be examined by means of interquantile tests. Interquantile tests are designed to examine whether the observed differences along the estimated coefficients are statistically significant across quantiles, i.e., whether the position in the growth distribution differentially affects how investment in physical and human capital, initial level of per capita income, population growth, and regional location are related to growth. This differentiation across quantiles is important for the analysis and the formulation of policies that may alter growth patterns. Since the coefficient estimates are not independent, however, a test of interquantile restrictions requires an estimate of the variance-covariance matrix Λ of the quantile regression coefficients estimated at different values of θ , which can be obtained by means of the design matrix bootstrap. Then a test of the interquantile restrictions $H_0 : R\beta = r$ against $H_1 : R\beta \neq r$ can be performed using the Wald test statistic $W = (R\hat{\beta} - r)'(R\hat{\Lambda}R')(R\hat{\beta} - r)$, where W is distributed as a χ^2 statistic with q degrees of freedom, equal to the number of restrictions, and $\hat{\Lambda}$ is the bootstrapped estimate of Λ . W/q is distributed as an F statistic with q and $n - k - 1$ degrees of freedom, where k is the number of coefficients estimated in each quantile regression. In the lower part of Table 3 we report the results of the parameter heterogeneity tests based on the design matrix bootstrap with 2000 replications and the random number seed set equal to 1001. The table presents, for each pair of interquantile comparisons, the derived p -value along with the test statistic, which is asymptotically distributed as an F statistic with 8 and 77 degrees of freedom. The tests confirm the visual impression from Figure 1, but provide evidence of only partial parameter heterogeneity.¹⁴ The tests of equality of the coefficients between the lower quantiles and the higher quantiles reject the hypothesis of parameter homogeneity. For instance, there is a statistically significant difference between the parameter estimates at the 10th quantiles and the parameter estimates at the 90th quantile. Note, however, that differences within the higher quantiles are not significant. Similarly, the differences within the lower quantiles are not significant. This indicates that countries whose growth performance is in the lower quantiles, namely, at or below the 25th quantile, exhibit parameter homogeneity; similarly, countries whose growth performance is in the higher quantiles, namely, at or above the 75th quantile, also exhibit parameter homogeneity. This pattern of parameter heterogeneity between lower and higher quantiles, combined with parameter homogeneity within the lower as well as the higher quantiles, is a strong indication that countries in the lower quantiles exhibit a growth dynamics that is different from the one displayed by countries in the higher quantiles.

¹⁴ Results of the interquantile tests for all 19 quantiles based on the unrestricted regression are available upon request.

5.3. Quantile Regression Results: Restricted Estimates

The results of the restricted quantile regressions of the transitional growth model, augmented with regional variables, are contained in Table 4 and illustrated in Figure 2. We report in Table 4 the quantile estimates for the five quantiles as in Table 3 as well as the estimates of the implied shares of physical and human capital and the rate of convergence.¹⁵

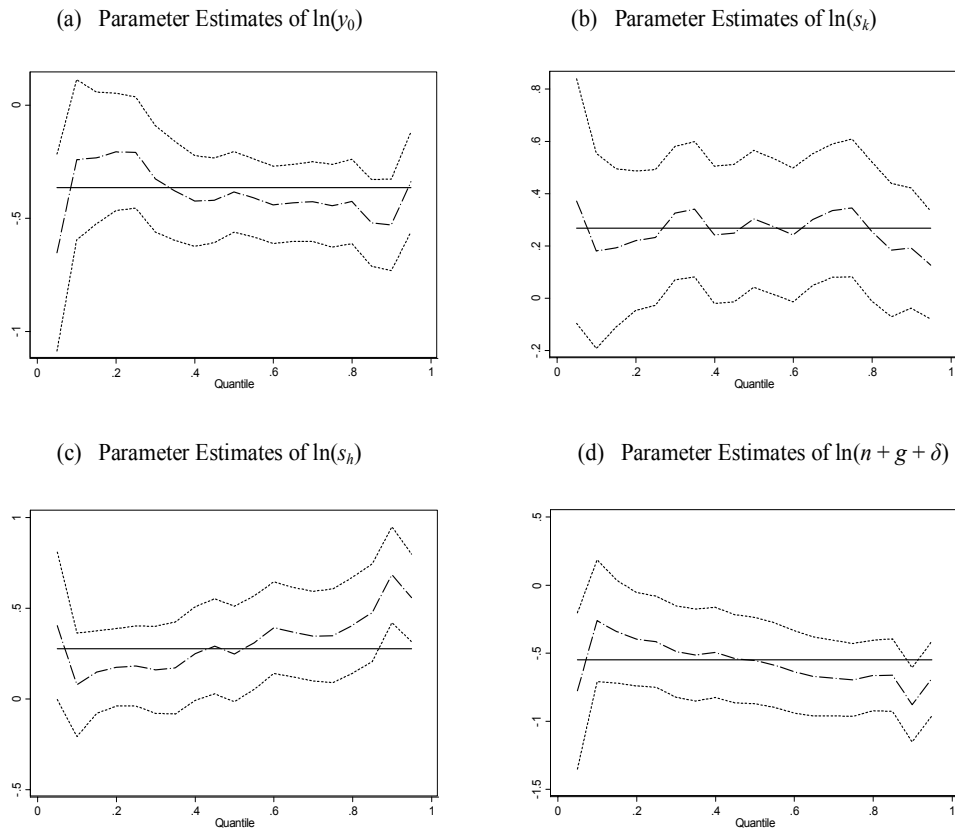
Note that, as in the unrestricted case, not all explanatory variables are significant across the entire conditional distribution. In particular, we find again that significant differences exist between the estimated coefficients in the lower and the upper quantiles. At the 50th quantile the regional variables are insignificant, both individually and as a group. The coefficient on East Asian countries is positive and significant at the higher quantiles, while the coefficient on Sub-Saharan Africa is negative and significant at the lower quantiles. Conversely, the coefficients on Latin America and Caribbean countries, and Industrialized countries are not significant across all the quantiles. The coefficient on initial per capita income is negative, as predicted by the conditional convergence hypothesis, but is now insignificant at the 10th and 25th quantiles. It also appears that the rate of convergence presents a notable threshold at the 25th quantile. Above the 25th quantile the rate of convergence is significant and generally increasing, while below the 25th quantile the rate of convergence is insignificant. At the 50th quantile, the implied value of λ is 0.0121 (t -statistic = 2.8139), which compares to the OLS value of 0.0113 (t -statistic = 2.8974) and the 90th quantile implied value of 0.0188 (t -statistic = 2.8923). At the 10th and 25th quantiles, however, the estimates are much lower and also insignificant, which indicates that the rate of convergence increases with growth. This result is also found in Rappaport (2000). However, only at the 90th quantile the rate of convergence is consistent with the conventional wisdom of a 2 percent rule. Taken as a whole, this heterogeneous dynamics imply that convergence is not a characteristic of countries whose growth rates are in the lower quantiles. Countries whose growth performance is in the lower quantiles appear to have unobserved structural characteristics that are not conducive to convergence in the conventional cross-sectional growth empirics framework. Unless they reach and cross over the threshold of divergence, they do not converge and cannot catch up with the rest of the world. This finding is consistent with Desdoigts (1999). Hence, it appears that conditional convergence is only a local phenomenon. From this perspective, the concept of “conditional” convergence introduced in Mankiw, Romer and Weil (1992) takes on an additional dimension of “conditionality,” in that convergence is conditional, in addition to s_k , s_h , n , g and δ , also on θ , namely on the countries not being in the lower quantiles of the growth distribution.

¹⁵ The standard errors of the implied shares and the rate of convergence are computed using the delta method.

Table 4. Restricted Quantile Regression Estimates

Dependent Variable: log difference GDP per working-age person, 1960-2000					
	$\theta = 0.10$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 0.90$
Constant	2.3729 (1.6525)	2.2306 (1.1471)	3.7484 (0.8902)	4.3758 (0.9133)	5.4408 (1.0740)
$\ln(y_0)$	-0.2408 (0.2121)	-0.2091 (0.1441)	-0.3834 (0.1066)	-0.4445 (0.1083)	-0.5289 (0.1233)
$\ln(s_k)$	0.1817 (0.2319)	0.2330 (0.1516)	0.3043 (0.1570)	0.3463 (0.1615)	0.1925 (0.1401)
$\ln(n + g + \delta)$	-0.2603 (0.2780)	-0.4154 (0.2010)	-0.5531 (0.1922)	-0.6958 (0.1712)	-0.8786 (0.1633)
$\ln(s_h)$	0.0786 (0.1723)	0.1824 (0.1305)	0.2487 (0.1542)	0.3496 (0.1538)	0.6861 (0.1582)
<i>East Asia</i>	-0.0496 (0.4075)	0.4563 (0.3528)	0.4288 (0.3240)	1.0237 (0.3692)	0.9530 (0.2896)
<i>Latin America and Caribbean</i>	-0.3094 (0.2836)	-0.2988 (0.2103)	-0.0864 (0.1818)	0.0475 (0.1567)	0.0016 (0.1505)
<i>Sub-Saharan Africa</i>	-0.7865 (0.2673)	-0.6314 (0.2528)	-0.3724 (0.2402)	-0.1681 (0.2450)	0.2679 (0.2987)
<i>Industrialized Countries</i>	0.3604 (0.4713)	0.2106 (0.3147)	0.3062 (0.2515)	0.2230 (0.2383)	0.1191 (0.2048)
<i>Pseudo R²</i>	0.4812	0.4931	0.5050	0.5031	0.5733
Implied Structural Parameters					
λ	0.0069 (0.0070)	0.0059 (0.0046)	0.0121 (0.0043)	0.0147 (0.0049)	0.0188 (0.0065)
α	0.3627 (0.3875)	0.3732 (0.1998)	0.3250 (0.1514)	0.3037 (0.1351)	0.1367 (0.0968)
β	0.1568 (0.2878)	0.2920 (0.1638)	0.2656 (0.1323)	0.3065 (0.1133)	0.4875 (0.0836)
Test of Joint Significance of Regional Variables					
<i>F</i> -statistic	2.9500	2.4400	1.4700	2.1100	3.5500
<i>p</i> -value	0.025	0.054	0.220	0.088	0.010
Interquantile Tests					
$\theta = 0.10$		0.5000	0.7700	2.2100	2.8700
<i>p</i> -value		0.8292	0.6165	0.0425	0.0101
$\theta = 0.25$			1.1900	2.3300	2.6500
<i>p</i> -value			0.3193	0.0329	0.0163
$\theta = 0.50$				1.1300	1.6800
<i>p</i> -value				0.3561	0.1265
$\theta = 0.75$					0.9500
<i>p</i> -value					0.4751

Note: Bootstrapped standard errors in parentheses.



Notes: The graphs plot the restricted parameter estimates associated with each variable, together with a 90% confidence band, against the quantile at which the model is estimated. The superimposed horizontal solid line refers to the corresponding OLS (invariant) parameter estimate.

Figure 2. Parameter Estimates of the Restricted Model

The magnitude and significance of the coefficient on investment in physical capital also vary across the quantiles. For example, at the 75th quantile the value of the estimated coefficient is 0.3463 (t -statistic = 2.1442), which compares to the OLS coefficient of 0.2680 (t -statistic = 1.9490) and the 25th quantile regression coefficient of 0.2330 (t -statistic = 1.5369). However, this coefficient is also not significant at the higher quantiles. At the 90th quantile, for example, the value of the estimated coefficient is 0.1925 (t -statistic = 1.3740). This implies that countries whose growth rates are in the lowest or the highest quantiles are less able to translate investment in physical capital into growth than countries whose growth rates are in the middle quantiles. The OLS estimate underestimates the impact of physical capital investment at the 75th quantile and overestimates it at the 25th quantile. This pattern is reflected in the estimates of the output elasticity of physical capital. As shown in Table 4, the estimates for the implied share of physical capital are smaller for countries in the higher quantiles and larger for countries in the lower quantiles. At the 75th quantile the estimate of α is 0.3037 (t -statistic = 2.2479), which compares to the OLS estimate of 0.2951 (t -statistic = 2.2271) and the 25th quantile estimate of 0.3732 (t -statistic = 1.8678). However, at quantiles above or below the interquantile range, the value of α is insignificant. This pattern is reversed for the estimates of the coefficient on human capital investment and the implied share of human capital. The coefficient on human capital investment is significant at the higher quantiles but insignificant at the lower quantiles. At the 90th quantile the coefficient is 0.6861 and highly significant (t -statistic = 4.3369), which compares to the OLS estimate of 0.2768 (t -statistic = 2.4153) and the 10th quantile estimate of 0.0786 (t -statistic = 0.1723). Thus the OLS estimate underestimates the impact of human capital at the higher quantiles and overestimates its impact at the lower quantiles. The implied value of β is also highly heterogeneous in nature. At the 90th quantile, the estimate of β is 0.4875 and is highly significant (t -statistic = 5.8313), which compares to the OLS estimate of 0.3047 (t -statistic = 2.8476) and the estimate of 0.1568 at the 10th quantile, which is insignificant (t -statistic = 0.5448). Note that the output elasticity of total capital, $\alpha + \beta$, also exhibit a significant variation, though more in statistical significance than in magnitude. At the 10th quantile the estimate of the implied share of total capital is 0.5195 (t -statistic = 1.6455), which is significantly different from the corresponding estimate of 0.6243 (t -statistic = 10.1458) at the 90th quantile. In comparison, the corresponding OLS estimate is 0.5997 (t -statistic = 6.5829), which implies that the output elasticity of labor is 0.4002 (t -statistic = 4.3932). Conversely, the estimate of the output elasticity of labor is 0.4805 (t -statistic = 1.5220) at the 10th quantile, and 0.3757 (t -statistic = 6.1067) at the 90th quantile. We present in the lower part of Table 4 the results of the interquantile tests of heterogeneity.¹⁶ The table contains, for each pair of interquantile comparisons, the derived p -value along with

¹⁶ Results of the interquantile tests for all 19 quantiles based on the restricted regression are available upon request.

the test statistic, which is asymptotically distributed as an F statistic with 7 and 78 degrees of freedom. As in the unrestricted case, these tests are based on the bootstrap approach with 2000 replications. The results reinforce the findings obtained in Table 3 and once more provide evidence of only partial heterogeneity of the parameters. Parameter heterogeneity is evident between the lower and the higher quantiles. For example, the test of parameter homogeneity between the 10th and the 90th quantile yields a value of the F statistic of 2.8700 (p -value = 0.0101), which is significant at the 5 percent level. However, countries whose growth performance is in the lower quantiles exhibit parameter homogeneity, and, similarly, countries whose growth performance is in the higher quantiles also exhibit parameter homogeneity. The F statistics between the 10th and the 25th quantiles and between the 75th and 90th quantiles are, respectively, 0.5000 and 0.9500, which cannot reject the hypothesis of parameter homogeneity at any conventional level of significance. This pattern of parameter heterogeneity between lower and higher quantiles, combined with parameter homogeneity within the lower as well as the higher quantiles, confirms the hypothesis that countries in the lower quantiles exhibit a growth dynamics that is different from countries in the higher quantiles. One possible explanation that these findings suggest is that an endogenous growth model may be more suitable to describe the growth characteristics of countries in the lower tail of the conditional growth distribution, whereas the growth dynamics of countries in the middle and higher quantiles is better described by neoclassical growth model.

6. CONCLUSION

The issue of parameter heterogeneity has been heralded in the new growth literature as one of the crucial topics in the agenda of growth empirics. In this study we examined this issue by adopting a quantile regression estimator, based on Koenker and Bassett's (1978) framework. Using cross-sectional data on 86 countries covering the period from 1960 to 2000, we estimated the transitional growth model of Mankiw, Romer and Weil (1992). As a baseline, we first estimated the model by applying OLS. We found that the model is quite successful in explaining the growth empirics of the "average" country. We next estimated the model using quantile regression. In contrast to OLS regression, quantile regression imposes fewer restrictions on the data, is robust to outliers in the data, relaxes the assumption of parameter homogeneity and provides more complete information on the conditional distribution.

In a nutshell, the main findings of this research can be summarized as follows. First, the results indicate that there is evidence of parameter heterogeneity between lower and higher quantiles, combined with parameter homogeneity within the lower quantiles as well as within the higher quantiles. We interpret this evidence as a strong indication that countries whose growth performance is in the lower quantiles exhibit a growth dynamics that is different from the one displayed by countries whose growth performance is in the higher quantiles. Thus, empirical analyses of cross-country growth that do not take into

account parameter heterogeneity are likely to be misspecified, leading to misleading inferences.

Second, the neoclassical model predicts conditional convergence. Our findings, however, do not fully confirm the conditional convergence prediction of the neoclassical model. We found evidence that convergence is a local but not a global growth experience. Convergence is not a generalized phenomenon across the conditional growth distribution, and, in particular, is not characteristic of economies in the lower quantiles. Countries whose growth performance is in the lower quantiles appear to have unobserved structural characteristics that are not conducive to convergence in the conventional growth empirics framework. We found evidence of conditional convergence only for the top 75 percent of the countries, but we also found evidence of conditional divergence for the bottom 25 percent. Countries in the bottom 25 percent do not converge and cannot catch up with the rest of the world. On a normative level, this suggests that programs of international economic aid to countries in the lower quantiles that are piecemeal in nature and not sufficiently aggressive to put these countries over the threshold of divergence are likely to fail. Finally, our findings suggest that an endogenous growth model, where monetary and fiscal policies as well as policies designed to increase investment in physical and human capital play a more decisive role in shaping the growth dynamics, may be more suitable to explain growth in the lower tail of the distribution, whereas growth in the middle and higher quantiles are better described by neoclassical growth model.

DATA APPENDIX

Country	y_{1960}	y_{2000}	n	s_k	s_h
Argentina	11747	17494	1.4	17.6	5.7
Australia	17245	38079	1.8	24.7	11.1
Austria	11263	34566	0.4	26.1	8.3
Burundi	1073	1158	1.9	5.0	0.5
Belgium	12065	35625	0.3	24.0	10.2
Benin	2140	2248	2.5	6.5	1.9
Burkin Faso	1281	1898	1.9	8.4	0.7
Bangladesh	1964	2855	2.5	10.0	3.0
Bolivia	3975	4854	2.4	10.1	5.0
Brazil	4644	10901	2.7	20.6	4.8
Canada	17247	39512	1.7	21.9	10.5
Switzerland	21154	39476	0.8	27.8	6.0
Chile	7103	15097	2.1	16.0	7.3
Cote d'Ivoire	3686	3464	3.7	8.2	2.6
Cameroon	3158	4051	2.4	6.9	3.5
Congo	1191	4394	2.6	23.0	5.1
Colombia	4840	8634	2.8	11.5	6.0
Costa Rica	7158	8753	3.5	14.2	6.3
Denmark	16786	40343	0.5	23.4	10.8
Dominican Rep.	3354	8472	2.9	12.4	5.6
Algeria	4729	9247	3.0	17.9	5.8
Ecuador	4113	5650	3.0	20.1	7.0
Egypt	3006	6813	2.5	7.0	8.1
Spain	7222	26556	0.8	24.5	9.6
Ethiopia	1026	1294	2.5	4.4	1.3
Finland	11850	34318	0.6	26.4	11.4
France	12903	34931	0.8	24.7	9.6
United Kingdom	14819	35009	0.3	18.3	9.8
Ghana	3273	2297	2.8	10.1	4.8
Greece	6178	21747	0.7	25.8	8.6
Guatemala	5090	7656	2.7	8.0	2.9
Hong Kong	4946	36058	2.6	25.8	6.6
Honduras	3214	3678	3.2	12.3	4.0
Indonesia	1354	5811	2.3	12.2	4.7
India	1521	4028	2.3	11.6	5.2
Ireland	9029	38195	1.1	17.9	12.8
Israel	10283	28838	2.8	28.2	9.2
Italy	10328	31764	0.4	24.9	7.3
Jamaica	4343	5249	1.5	19.1	10.2
Jordan	3911	6940	4.7	13.1	9.7
Japan	7386	35982	0.9	31.0	10.6
Kenya	1764	2277	3.4	11.1	2.9
Korea	2685	19552	2.3	27.3	9.7

(Continued)

Country	y_{1960}	y_{2000}	n	s_k	s_h
Sri Lanka	2771	5271	2.2	10.2	8.1
Morocco	2507	6535	2.6	12.9	4.0
Madagascar	2824	1565	2.6	2.9	2.5
Mexico	7980	14548	3.0	18.3	6.9
Mali	1739	1861	2.1	7.4	1.1
Mozambique	3602	1950	2.0	2.5	0.8
Mauritius	6382	19806	2.2	12.3	6.7
Malawi	994	1488	2.6	13.2	1.0
Malaysia	4623	13956	3.1	20.2	6.8
Niger	3059	1743	2.9	7.0	0.7
Nigeria	1780	1481	2.8	7.6	2.8
Nicaragua	6274	3267	3.2	10.9	6.0
Netherlands	15340	35735	1.1	24.2	11.0
Norway	13922	46671	0.6	31.9	10.4
Nepal	1371	2645	2.0	11.2	3.2
New Zealand	19559	28838	1.5	21.0	11.9
Pakistan	1552	3686	2.7	13.1	3.1
Panama	4463	9877	2.8	20.2	10.2
Peru	5843	7258	2.8	20.0	8.2
Philippines	4094	6449	2.9	14.7	10.4
Portugal	5271	23676	0.5	21.0	7.5
Paraguay	3983	7864	3.1	10.7	4.4
Rwanda	1767	1703	2.9	3.4	0.8
Senegal	3903	2953	2.7	7.0	1.9
Singapore	3666	38338	2.9	41.2	7.6
Sierra Leone	1990	1294	1.9	2.8	1.9
El Salvador	6456	7254	2.6	7.0	4.0
Sweden	15853	35666	0.4	22.2	9.2
Syria	2585	7152	3.4	12.4	8.3
Chad	2266	1768	2.0	9.6	0.7
Togo	1917	1635	2.7	7.1	3.2
Thailand	2239	9213	2.7	29.5	4.4
Trinidad & Tobago	7067	18773	1.7	10.0	8.3
Tunisia	4831	10376	2.6	18.2	5.0
Turkey	5003	11080	2.5	14.8	6.7
Tanzania	922	870	3.0	24.6	0.6
Uganda	1151	1978	2.9	2.1	1.4
Uruguay	9048	15384	0.6	11.8	7.1
United States	21005	49640	1.4	18.7	11.4
Venezuela	8374	10962	3.3	16.4	6.1
South Africa	8449	11920	2.6	12.3	5.4
Zambia	4436	1556	2.8	18.7	2.9
Zimbabwe	4148	4700	3.1	24.7	5.3

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