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The Effect of Market Structure and Conduct on the Incentive for a Horizontal Merger

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In this paper, we examine how market structure and firms' conduct affect the private incentive and welfare effect of a merger. The main result of this paper is as follows. First, as market becomes more concentrated, the increase in the joint profit of merging firms becomes lower for the case of the same cost savings by a merger. This implies that as market concentration increases, it is necessary for a merger to attain larger cost reduction in order to be profitable. Second, the increase in welfare by a merger rises (or, the decrease in welfare by a merger falls) as the market structure goes toward competition. Third, As the collusion level among firms becomes higher, the increase in the joint profit of merging firms goes down. Fourth, As the collusion among firms decreases, the required level of cost savings for a merger to raise welfare also goes down. Fifth, a merger can induce a stable cartel which was not formed before merger.

I. Introduction

There has been a lot of research on the private incentive and welfare effect of a horizontal merger. For the private profitability of a horizontal merger, the competition type (that is, Cournot vs. Bertrand competition) becomes very important. Salant *et al*. (1983) showed that a horizontal merger can be unprofitable by using Cournot model. The main reason for this result is due to the response of outsiders. Merging firms usually reduce joint output in order to exercise market power. In reaction to this event, non-merging firms raise output according to their reaction functions, which has a negative influence on merging firms since output is a strategic substitute. Contrary to this result, Deneckere and Davidson (1985) showed that a merger of any size is privately profitable by using Bertrand model. They argued that a merged firm raises its price to raise market power. In response to this, outsiders increase their price, which has a positive effect on the profit of a merged firm since price is a strategic complement.[1]

Usually a horizontal merger of two firms have mixed effects on welfare because it affects market structure and efficiency of firms. The negative side of a merger is that it raises market power due to the concentration of a market and positive side of a merger is that it may enhance efficiency by synergy effect. Williamson (1968) developed a method by which we can evaluate the welfare effect of a merger. He argued that a merger can enhance efficiency through some sort of synergy effect. If this efficiency effect is large enough to

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1. Bulow *et al*. (1985) introduced the concept of strategic substitutes and complements.

outweigh market power effect, then a merger can raise social welfare. Farrell and Shapiro (1990) derived the condition under which a privately profitable merger is also socially desirable.

In this paper, we are concerned with the question how the private incentive and welfare effect of a merger are affected by the change in market structure. Suppose all the mergers can attain the same level of cost reduction, then as market structure becomes more concentrated, is it more likely (or unlikely) that a merger is privately profitable (or socially desirable)? If we can answer for this kind of question, then we can identify the relation between market structure and incentive for a merger. Here, we also examine the relation between the degree of competitiveness (or collusiveness) and the merger incentive. By this analysis, we can see how firms' conduct in terms of collusion can influence the merger incentive. Finally, we see the case where a merger generates a stable cartel. If a cartel is induced by a merger, negative welfare effect of a merger becomes dominant and so government had better adopt a strict merger allowance policy.

The structure of this paper is as follows. In Section II, we look at the change in the profitability and social desirability of a merger according to the change in market structure using Cournot model. In Section III, we introduce the conjectural variation terms in the model in order to examine the effect of collusion level on the merger incentive. By this way, we investigate how merger incentive is influenced by firms' conduct such as degree of collusion. In Section IV, we show the case where a merger can occur even without cost savings in a Cournot model. That is, we derive the condition under which a merger can trigger the formation of a cartel in a repeated game context. In Section V, we summarize main results of the paper and point out some limitations of this paper.

II. Market Structure and Merger Incentive

We assume that there are n identical firms in the market. Each firm has a constant returns to scale technology and has a marginal cost (MC) of $\overline{c}$ at first. The inverse market demand is linear; $P = a - bQ$, where P is the market price and Q is the total market output. All the firms in market are involved in Cournot competition. We assume that the MC of a merged firm goes down from $\overline{c}$ to $\underline{c}$.[2] (where, $\underline{c} < \overline{c}$, $d = \overline{c} - \underline{c}$) Here we would like to look at the change in private profitability and social desirability of a merger according to the change in market structure. In order to see this, we are going to derive the change in the joint profit of merging firms and social welfare by a merger.

1. Pre-merger case

Before a merger, each firm has the same MC of $\overline{c}$. We can easily derive each firm's Cournot equilibrium output and profit as follows.

2. Salant *et al.* (1983) showed that a merger lowers joint profit of merging firms for the case of linear demand and constant MC using Cournot model. So in order for a profitable merger to happen, it is necessary to assume MC reduction by a merger.

$$q_i = \alpha / b(n+1), \quad \Pi_i = \alpha^2 / b(n+1)^2 \quad (\text{where}, \quad \alpha = a - \bar{c}, \; i = 1, 2, \ldots, n) \tag{1}$$

2. Post-merger case

We consider the case where (n-1)th firm and (n-2)th firm merge into (n-1)th firm. After a merger, a merged firm's MC becomes $\underline{c}$, while the remaining (n-2) firms have MC's of $\bar{c}$. Then problems of a non-merging firm and a merged firm are as follows.

$$\max_{q_i} (a - bQ - \bar{c}) q_i, \; (\text{where} \quad i = 1, 2, \ldots, n-2) \tag{2}$$

$$\max_{q_{n-1}} (a - bQ - \underline{c}) q_{n-1}.$$

Now by solving (n-1) first order conditions (FOC's) simultaneously, we get the post-merger Cournot equilibrium output and profit of each firm as follows.

$$q_i^m = \frac{\alpha - d}{bn} (i = 1, 2, \ldots, n-2), \quad q_{n-1}^m = \frac{\alpha + (n-1)d}{bn} \tag{3}$$

$$\Pi_{n-1}^m = \frac{[\alpha + (n-1)d]^2}{bn^2}, \quad \Pi_i^m = \frac{(\alpha - d)^2}{bn^2}, \quad (i = 1, 2, \ldots, n-2).$$

By using the pre-merger profit and post-merger profit in (1) and (3), we can get the change in the joint profit of merging firms by a merger.

$$\Delta\Pi_{n-1} = \Pi_{n-1}^m - (\Pi_{n-1} + \Pi_n) \tag{4}$$

$$= \frac{1}{bn^2(n+1)^2} \{[\alpha + (n-1)d]^2 (n+1)^2 - 2\alpha^2 n^2\}$$

$$= \frac{1}{bn^2(n+1)^2} \{(n^2 - 1)^2 d^2 + 2(n+1)^2 (n-1)\alpha d + (-n^2 + 2n + 1)\alpha^2\}$$

Now we are in a position to analyze the effect of change in market structure on the change in the joint profit of merging firms by the following proposition.

***Proposition 1**: Given the same level of cost savings by a merger, if a merger is privately profitable when there are n_1 firms in the market, then it is also privately profitable when there are n_2 firms in the market. (where, $n_1 < n_2$, $n \geq 4$)*

Pf) The change in the joint profit of merging firms when the amount of cost savings is d is in Equation (4), which is

$$\Delta\Pi_{n-1} = \frac{1}{bn^2(n+1)^2}\{(n^2-1)^2 d^2 + 2(n+1)^2(n-1)\alpha d + (-n^2+2n+1)\alpha^2\} = \frac{1}{b} f(n)$$

Now we want to show $f'(n) > 0$. Let $K = n^2(n+1)^2$. Then

$$K^2 f'(n) = [4n(n^2-1)d^2 + 2(3n-1)(n+1)\alpha d + (-2n+2)\alpha^2]n^2(n+1)^2$$

$$-[(n^2-1)^2 d^2 + 2(n+1)^2(n-1)\alpha d + (-n^2+2n+1)\alpha^2]2n(2n+1)(n+1)$$

From this equation, we get $f'(n)$ as below.

$$f'(n) = \frac{1}{K^2} 2n(n+1) g(n)$$

(where, $g(n) = d^2(n-1)(n+1)^3 + \alpha d\,(n+1)^2[n(7n-3)-2] + \alpha^2[n(n^2-3n-3)-1]$)

So the sign of $f'(n)$ is the same as that of $g(n)$.

For $n \geq 4$, $g(n) > 0$ holds. Therefore $f'(n) > 0$ holds. Q.E.D.

According to the above proposition, with the same level of cost savings, the profit increase by a merger becomes lessened as the number of firms decreases. That is, as a market becomes more concentrated, it is more difficult for a merger to be privately profitable. We can interpret this result as follows. As a market becomes more concentrated, pre-merger profit of each firm increases due to the reduction in competition. So large cost savings is necessary in order for a merger to raise joint profit of merging firms in this case. In this respect, it is more difficult for a merger to occur as market concentration goes up. This kind of phenomena has a desirable aspect from the view-points of social welfare. In general, the increase in market concentration gives a negative impact on welfare. But, in a concentrated market, a merger can occur only when efficiency gain is high enough, which raises the possibility that a merger can have a positive welfare effect.

Next, we would like to see how the market structure affects the welfare effect of a merger. In order to examine this, we need to know the change in social welfare by a merger. We provide the derivation of this change in the appendix.

Proposition 2: *Given the same level of cost savings by a merger, if a merger is socially desirable when there are* n_1 *firms in the market, then it is also socially desirable when there are* n_2 *firms in the market. (where,* $n_1 < n_2$, $n \geq 3$)

Pf) From the appendix, the change in social welfare by a merger is

$$\Delta SW = \frac{1}{2bn^2(n+1)^2}[(n+1)^2(2n^2-2n-1)d^2 + 2\alpha d(n^3+3n^2-n-1) - (2n+1)\alpha^2]$$

$$= \frac{1}{2b} f(n)$$

Now, we would like to show $f'(n) > 0$. Let $K = n^2(n+1)^2$. Then

$$K^2 f'(n) = [2(n+1)(4n^2-n-2)d^2 + 2\alpha d(3n^2+6n-1) - 2\alpha^2]n^2(n+1)^2$$

$$-[(n+1)^2(2n^2-2n-1)d^2 + 2\alpha d(n^3+3n^2-n-1) - (2n+1)\alpha^2]2n(2n+1)(n+1)$$

$$= 2n(n+1)g(n)$$

(where, $g(n) = d^2(n+1)^2(8n^3-3n^2-6n-1) + \alpha d[n(n+1)(3n^2+6n-1)$

$+ 2(2n+1)(n^3+3n^2-n-1)] + \alpha^2[3n^2+3n+1]$)

From the above relation, we get $f'(n)$ as below.

$$f'(n) = \frac{1}{K^2} 2n(n+1)g(n)$$

So the sign of $f'(n)$ is the same as that of $g(n)$.

Now, let $h(n) = 8n^3 - 3n^2 - 6n - 1$. Then $h(n)$ is an increasing function of n since $h'(n) = 6n(4n-1) - 6 > 0$ (where, $n \geq 3$).

Therefore, $h(n) \geq h(3) = 192 > 0$.

Next, let $k(n) = n^3 + 3n^2 - n - 1$. Then, $k(n)$ is an increasing function of n since $k'(n) = 3n(n+2) - 1 > 0$ (where, $n \geq 3$).

Therefore, $k(n) \geq k(3) = 50 > 0$.

From the above result, it is true that $g(n) > 0$ (where, $n \geq 3$). So $f'(n) > 0$ holds. Q.E.D.

The above proposition explains the relation between the change in market structure and the welfare effect of a merger. Suppose that the level of cost reduction by a merger is the same irrespective of a market structure. Then, as the number of firms increases, the welfare increase by a merger becomes larger (or the welfare decrease becomes smaller). Therefore, it is possible that a merger lowers welfare for a concentrated market but it raises welfare for a more competitive market. So even if the efficiency enhancement effect by a merger is the same, it is necessary to adopt different policy direction for a merger depending on the market structure. Namely, as the market becomes more competitive, the government policy toward a merger needs to be generous.

III. Degree of Competitiveness and Merger Incentive

Until now we assumed that each firm in an industry conjectures that the output of the other firms does not change with the change of its own output; i.e., $s = dQ_{-i} / dq_i = 0$ (where, $Q_i = \sum_{j \neq i}^{n} q_j$) was assumed. Now we introduce the non-zero conjectural variation terms in the model to examine how the different degree of competitiveness changes the welfare implication of a merger. We assume that s does not change before and after a merger.[3] In this section, we are going to analyze how the change in competitiveness in terms of conduct affects the private incentive for a merger. In the appendix, we derive the change in the joint profit of merging firms with non-zero conjectural variation terms. And by using this result, we get the following proposition that represents the relation between the degree of collusion and the incentive for a merger.

Proposition 3*: Given the same level of cost savings by a merger, the increase in the joint profit of merging firms becomes lessened (or the decrease in the joint profit becomes enlarged) as the degree of collusion goes up in the market.*

Pf) From the appendix, we have the change in the joint profit of merging firms when the conjectural variation is s as follows.

$$\Delta \prod_{n-1}(s) = \frac{1}{Z}[((n+s)^2 - 1)^2 d^2 + 2(1+s)(n+s+1)^2(n+s-1)\boldsymbol{a}d$$

$$+ ((n+1+s)^2 - 2(n+s)^2)(1+s)^2 \boldsymbol{a}^2] = f(s)$$

We want to show $f'(s) < 0$

3. Here, the range of s that we consider is from 0 to (n-2) because we are interested in the case where the degree of collusion is between Cournot behavior and perfect collusion. Also, the upper bound of s is (n-2) because the number of the rival firms for each firm becomes (n-2) after a merger.

Now,

$$Z^2 f'(\sigma) = (n+\sigma)(n+1+\sigma)[Ad^2 + B\alpha d + C\alpha^2(1+\sigma)^2]$$

(where, $A = (m+1)[4tm^2 - (m-1)(m^2 + (4t+1)m + 2t)]$

$B = 2(m+1)^3 t^2 (2-m)$, $C = (m+1)(1+2m-m^2)(m-2t) + 2tm(-2m)$

and where, $m = n+\sigma$, $t = 1+\sigma$, $0 \le \sigma \le n-2$)

Now it is sufficient to show $A < 0$, $B < 0$ and $C < 0$.

step 1) From A, let $g(m) = 4tm^2 - (m-1)(m^2 + (4t+1)m + 2t)$
By simplification, we get $g(m) = -m^3 - 2m^2 + (4t+1)m + 2t.$
Then, $g(m)$ is a decreasing function of m since
$g'(m) = -3m^2 - 4m + (4t+1) = -3(n+\sigma)^2 - 4n + 5 < 0$, (where, $n \ge 3$, $\sigma \ge 0$)
Since $m \ge 3$, $g(m)$ attains the maximum when $m = 3$ (which is the case where $n = 3$ and $\sigma = 0$).
So, $g(m) \le g(3) = 14t - 42 = 14(1+\sigma) - 42 = -28 < 0$, since $\sigma = 0$ for $m = 3$.
Therefore, $A = (m+1)g(m) < 0$.

step 2) From B, $(2-m)$ is negative since $m \ge 3$. Therefore,
$B = 2(m+1)^3 t^2 (2-m) < 0.$

step 3) Now, from $C = (m+1)(1+2m-m^2)(m-2t) + 2tm(-2m)$,
$(1+2m-m^2) = -(m-1)^2 + 2 < 0$, since $m \ge 3$.
$m - 2t = (n+\sigma) - 2(1+\sigma) = n - 2 - \sigma \ge 0$, since $0 \le \sigma \le n-2$.
Therefore, $C < 0$ holds. Q.E.D.

The above result represents the relation between the degree of competitiveness (or collusion) in conduct and private profitability of a merger. Suppose that the amount of cost savings are the same irrespective of the competitiveness in a market, then as the intensity of cooperation among firms becomes stronger, the private gain by a merger becomes lower. Usually, the gain by a merger comes from the increase in market power due to the reduction in competition. When the cooperation among firms becomes prevalent in a market, each firm already has a significant market power. In this situation, the additional gain by a merger becomes lower. This kind of relation has a desirable welfare implication in the following respect. That is, from the social welfare view-points, it becomes more difficult to go to the worse state (the state where market becomes more concentrated by a merger) from the bad state (the state where degree of collusion among firms is large).

Next, we would like to look at the relation between the degree of collusion and the welfare change of a merger. Since the differentiation of the welfare change on the number of firms becomes so complex, we could not get the definite sign. Instead of this, we are going to derive the minimum level of cost savings that guarantees the increase in welfare by a merger. And by simulation, we will get the relation between the degree of collusion and the required cost savings, which gives us indirect information about the relation between the competitiveness in conduct and welfare change by a merger. We obtain the following result by simulation. (Refer to the Table 1 in appendix.)

Simulation Result 1*: As the degree of collusion goes up, the required level of cost savings to raise social welfare rises.*

We can explain above result as follows. Usually a merger provides a negative effect on consumer surplus and positive effect on producer surplus. The positive effect of a merger on producer surplus is diminished as s goes up, because for the high value of s, each firm already got high pre-merger profit by cooperation. Therefore we need more cost savings to raise positive effect and to lower negative effect to get a socially desirable merger as s goes up. One implication of this result is that even for the case of the same market structure, the policy toward a merger needs to reflect the degree of collusion in the market.

IV. Relation between a Merger and Cartel Stability

Until now, we deal with the case where cost reduction is necessary for a merger to be profitable. But if a merger induces the industry-wide cartel which was not formed before a merger, then a merger can be profitable even without cost savings. The reason why this is possible comes from the fact that a merger changes the market structure, which in tum affects the condition for the stable cartel in a repeated game context. In the appendix, we derive the condition for the stable cartel in an infinitely repeated game.

Proposition 4*: As the number of firms decreases, the critical level of discount factor above which cartel is stable also decreases.*

Pf) From the appendix, the critical discount factor above which cartel is stable when there are n firms is as follows.

$$d_c(n) = \frac{(n+1)^2}{(n^2+6n+1)} = f(n). \text{ It is sufficient to show } f'(n) > 0. \text{ Then}$$

$$f'(n) = \frac{1}{K^2}[(2n+2)(n^2+6n+1) - (n^2+2n+1)(2n+6)] \quad (\text{where, } K = n^2+6n+1)$$

$$= \frac{4}{K^2}(n+1)(n-1) > 0 \quad \text{since} \quad n \geq 3. \quad \text{Q.E.D.}$$

According to the above result, as market becomes more concentrated, it is more likely that a stable cartel is formed. In some case, it is possible that industry-wide cartel is unstable for the competitive market, but it becomes stable for the concentrated market. In this sense, a merger can generate a stable cartel which was not formed before a merger because market concentration increases due to a merger. Therefore, we need to consider the possibility of cartel formation due to a merger as well as market power and efficiency effect. Thus if the discount factor is in the range where a stable cartel is formed by a merger, it is necessary to adopt a strict merger allowance policy.

V. Conclusion

Until now, we have examined the following issues; (1) the relation between a market structure and the merger incentive, (2) the relation between a degree of competitiveness (or collusiveness) in terms of conduct and the merger incentive, (3) the relation between a merger and industry-wide cartel formation. We can have some implication for the government policy toward a horizontal merger by this kind of analysis. The main result of this paper can be summarized as follows.

First, for the case where the cost reduction by any merger is the same, the increase in joint profit of merging firms becomes lower as market becomes more concentrated. Thus, as market concentration increases, it is necessary for a merger to attain larger cost reduction in order to be privately profitable. This is somewhat good news to the economy since it is more difficult for a merger to occur as market structure goes away from the perfect competition. Second, for the same cost savings by a merger, the increase in social welfare by a merger rises (or the decrease in social welfare falls) as the market structure goes toward competition. That is, as market becomes competitive, it is more likely that a merger has a positive effect on welfare. Third, the effect of a merger on the joint profit also depends on the degree of competition (or collusion) among firms. As the collusion level becomes higher among firms, the profitability of a merger goes down. So it becomes more difficult for a merger to come out in this case, which reduces the negative effect of a merger. Fourth, As the collusion among firms goes down, the required level of cost savings for a merger to raise welfare also goes down. So the policy toward a merger needs to consider the degree of collusion in terms of firms' conduct. Fifth, a merger can induce a industry-wide cartel which was not formed before a merger. In this case, a merger is always privately profitable even if there is no cost savings by a merger, which has a negative effect on welfare.

One implication of this paper is that the policy toward a merger had better take into account the market structure, the type of firms' conduct and the possibility of cartel formation. So even if a merger achieves the same efficiency enhancement, the government needs to take different policies depending on the state of market structure and firm's conduct.

Finally, limitations of this paper and further research directions can be stated as follows. First, we use Cournot model to analyze the merger effect. It would be better to check the robustness (or sensitivity) of this result by using another model like Bertrand competition. Second, here linear demand and constant returns to scale technology are assumed. So we need to see how the result changes as the demand and cost functions are generalized. Third, here we deal with the case of symmetric firms. Usually firms in a real market are in asymmetric positions. If we can encompass the case of asymmetric firms, then the application of the result can be enlarged.

Appendix

1. The derivation of the change in social welfare by a merger

[1] Pre-merger case

Before a merger, we can derive the producer surplus by summing up the pre-merger profit of each firm as below.

$$PS = \sum_{i=1}^{n} \Pi_i = \frac{n\boldsymbol{a}^2}{b(n+1)^2}$$

Also, the consumer surplus is given below.

$$CS = \frac{b}{2}Q^2 = \frac{n^2\boldsymbol{a}^2}{2b(n+1)^2}$$

[2] Post-merger case

By using Equation (3) in Section II, we get the post-merger producer surplus as below.

$$PS^m = \sum_{i=1}^{n-2} \Pi_i^m + \Pi_{n-1}^m = \frac{(n^2 - n - 1)d^2 + 2ad + (n-1)\boldsymbol{a}^2}{bn^2}$$

Also, we can get the post-merger total industry output as follows.

$$Q^m = \sum_{i=1}^{n-2} q_i^m + q_{n-1}^m = \frac{(n+1)\boldsymbol{a} + d}{bn}$$

So, we get the post-merger consumer surplus.

$$CS^m = \frac{b}{2}(Q^m)^2 = \frac{[(n-1)\boldsymbol{a} + d]^2}{2bn^2}$$

[3] social welfare change by a merger

Now we get the welfare change by a merger by using the results of [1] and [2] as follows.

$$\Delta SW = (PS^m + CS^m) - (PS + CS)$$

$$= \frac{1}{2bn^2(n+1)^2}[(n+1)^2(2n^2 - 2n - 1)d^2 + 2\boldsymbol{a}d(n^3 + 3n^2 - n - 1) - (2n+1)\boldsymbol{a}^2]$$

2. The derivation of the change in the joint profit of merging firms and social welfare when there is a conjectural variation parameter.

[1] pre-merger case

The first order condition for each firm' s maximization problem is as follows.

$$-b(1+s)q_i+(a-bQ-\overline{c})=0,\ \text{(where}\ \ s=\frac{dQ_{-i}}{dq_i},\ \ Q_i=\sum_{j\neq i}^{n}q_j)\ \ (i=1,2,...,n)$$

From this condition, we get equilibrium output and profit as follows.

$$q_i=\frac{\boldsymbol{a}}{b(n+1+s)},\ \ \Pi_i=\frac{(1+s)\boldsymbol{a}^2}{b(n+1+s)^2},\ \ Q=\frac{n\boldsymbol{a}}{b(n+1+s)}$$

So we get pre-merger producer surplus and consumer surplus as below.

$$PS=\sum_{i=1}^{n}\Pi_i=\frac{n(1+s)\boldsymbol{a}^2}{b(n+1+s)^2}$$

$$CS=\frac{b}{2}Q^2=\frac{n^2\boldsymbol{a}^2}{2b(n+1+s)^2}$$

[2] post-merger case

After (n-1)th and nth firm merger into a (n-1)th firm, the problem of a non-merging firm and a merged firm is the same as the one given in Equation (2) in Section II.

The first order condition for this problems is given as below.

$$-b(1+s)q_i^m+(a-bQ-\overline{c})=0,\ \ (i=1,2,...,n-2)$$

$$-b(1+s)q_{n-1}+(a-bQ-\underline{c})=0$$

Now using the (n-1) equations, we get the post-merger output and profit as follows.

$$q_i^m=\frac{(1+s)\boldsymbol{a}-d}{b(1+s)(n+s)}\ \ (i=1,2,...,n-2),\ \ q_{n-1}^m=\frac{(1+s)\boldsymbol{a}+(n+s-1)d}{b(1+s)(n+s)}$$

$$Q^m=\sum_{i=1}^{n-2}q_i^m+q_{n-1}^m=\frac{(n-1)\boldsymbol{a}+d}{b(n+s)}$$

$$\Pi_i^m = \frac{[(1+\sigma)\alpha - d]^2}{b(1+\sigma)(n+\sigma)^2} \quad (i = 1, 2, ..., n-2)\,, \quad \Pi_{n-1}^m = \frac{[(1+\sigma)\alpha + (n+\sigma-1)d]^2}{b(1+\sigma)(n+\sigma)^2}$$

Now using the equilibrium profit and output, we get the post-merger producer surplus and consumer surplus.

$$PS^m = \sum_{i=1}^{n-2} \Pi_i^m + \Pi_{n-1}^m$$

$$= \frac{1}{K}[(n-2)((1+\sigma)\alpha - d)^2 + ((1+\sigma)\alpha + (n+\sigma-1)d)]^2.$$

(where, $K = b(1+\sigma)(n+\sigma)^2$)

$$CS^m = \frac{b}{2}(Q^m)^2 = \frac{[(n-1)\alpha + d]^2}{2b(n+\sigma)^2}$$

[3] Change in the joint profit of merging firms and social welfare

Now the change in the joint profit of merging firms is given below.

$$\Delta\Pi_{n-1} = \Pi_{n-1}^m - (\Pi_{n-1} + \Pi_n)$$

$$= \frac{1}{Z}\{((n+\sigma)^2 - 1)^2 d^2 + 2(1+\sigma)(n+\sigma+1)^2(n+\sigma-1)\alpha d$$

$$+ ((n+1+\sigma)^2 - 2(n+\sigma)^2)(1+\sigma)^2\alpha^2\}$$

(where, $Z = b(1+\sigma)(n+\sigma)^2(n+1+\sigma)^2$)

Also, the change in the producer surplus ($\Delta PS = PS^m - PS$) is given below.

$$\Delta PS = \frac{1}{Z}\{((n+\sigma-1)^2 + (n-2))(n+\sigma-1)^2 d^2 + 2(1+\sigma)^2(n+\sigma+1)^2\alpha d$$

$$+ ((n-1)(n+1+\sigma)^2 - n(n+\sigma)^2)\alpha^2(1+\sigma)^2\}$$

Next, the change in consumer surplus ($\Delta CS = CS^m - CS$) is given below.

$$\Delta CS = \frac{1}{\theta}\{(n+1+\sigma)^2 d^2 + 2(n-1)(n+1+\sigma)^2\alpha d$$

$$+((n+1+\sigma)^2(n-1)^2 - n^2(n+\sigma)^2)\alpha^2\}$$

(where, $\theta = 2b(n+\sigma)^2(n+1+\sigma)^2$)

Now we get the change in social welfare ($\Delta SW = \Delta PS + \Delta CS$) as follows.

$$\Delta SW = \frac{1}{2Z}\{(2n^2 + (4\sigma - 2)n + (2\sigma^2 - 3\sigma - 1))(n+1+\sigma)^2 d^2 + 2\alpha(1+\sigma)(n+2\sigma+1)$$

$$(n+\sigma+1)^2 d + ((n+2\sigma+1)(n-1)(n+1+\sigma)^2 - n(n+2\sigma+2)(n+\sigma)^2)(1+\sigma)\alpha^2\}$$

Let λ be the minimum d that satisfies $\Delta SW \geq 0$. Then, the value of λ is

$$\lambda = \frac{\alpha\{-(1+\sigma)(n+\sigma+1)(n+2\sigma+1) + \sqrt{(1+\sigma)^2(n+2\sigma+1)^2(n+\sigma+1)^2 - E}\}}{(n+\sigma+1)\{2(n+\sigma-1)^2 + 2(n-2) + (1+\sigma)\}}$$

where, $E = (1+\sigma)\{2(n+\sigma-1)^2 + 2(n-2) + (1+\sigma)\}\{(n+2\sigma+1)(n-1)(n+\sigma+1)^2$

$$- n(n+2\sigma+2)(n+\sigma)^2\}.$$

3. Simulation result

Now we use the simulation method in order to see the relation between s and l which is derived in (Section II) in this appendix. The simulation result is summarized in the following table.

Table 1

N	s	l	N	s	l
3	0	0.0511	8	2	0.0536
	1	0.1091		3	0.0750
4	0	0.0334		4	0.0950
	1	0.0789		5	0.1133
	2	0.1180		6	0.1299
5	0	0.0236	9	0	0.0089
	1	0.0598		1	0.0264
	2	0.0936		2	0.0459
	3	0.1229		3	0.0653
6	0	0.0176		4	0.0837
	1	0.0470		5	0.1008
	2	0.0763		6	0.1166
	3	0.1027		7	0.1311
	4	0.1261	10	0	0.0074
7	0	0.0137		1	0.0225
	1	0.0380		2	0.0398
	2	0.0634		3	0.0574
	3	0.0872		4	0.0744
	4	0.1089		5	0.0904
	5	0.1283		6	0.1053
8	0	0.0109		7	0.1192
	1	0.0314		8	0.1321

N is the number of firms in the industry.
s is the conjectural variation parameter.
l is the required cost savings for a socially beneficial merger.
a is set to 1.

4. The derivation of a critical discount factor for the stable cartel in a repeated game.

We consider the following strategy of each firm in a repeated game; (1) At period t, each firm produces cartel output if no one has deviated from it before period t. (2) At period t, if some firm has deviated from the cartel output before, then each firm produces Cournot-Nash equilibrium output forever. From this strategy, we are going to derive the condition for the stability of cartel.

First, the total cartel output level is equal to monopoly output level. Since there are n identical firms initially, each firm produces $1/n$ of total cartel output and get $1/n$ of the total profit; $q_i^c = \alpha/2bn$, $\Pi_i^c = \alpha^2/4bn$. Now let's look at the gain that a firm can get by deviating from a cartel output. Given that other firms stick to the cartel output level, a defecting firm would produce $q_i^d = \alpha n/4b(n-1)$ and get $\Pi_i^d = \alpha^2 n^2/16b(n-1)^2$. The one period gain by cheating is $\Pi_i^d - \Pi_i^c = \alpha(n-2)^2/16b(n-1)^2$ (=A). The present discount value of the future loss by defecting, when a discount factor is δ, is $\delta\alpha^2(n-2)^2/4(1-\delta)b(n-1)n^2$ (=B). A cartel is stable if the loss (B) is greater than the gain (A). Therefore, the range of δ in which a cartel is stable is $\delta > (n+1)^2/(n^2+6n+1) = \delta_c(n)$.

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