A Basic Model Incorporating Exchange Rate Risk in the Foreign Direct Investment Decision*

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This paper extends the discussion of exchange rate risk and its impact on a risk averse multinational undertaking foreign direct investment. Drawing on the economics and finance literatures, a basic model is presented which incorporates exchange rate risk into the firm’s objective function. The model highlights the problems created if exchange rate risk is not considered when determining the optimum level of capital and the effect of exchange rate movements on the firm’s cash flows, as well as how the standard responses to these movements impact the firm. Tax and host country policies are analyzed showing additional ways accounting for exchange rate risk affects the risk averse firm.

I. Introduction

Issues relating to Foreign Direct Investment (FDI) during the past twenty years have generated a large literature. In an attempt to explain the existence of FDI, market imperfections of some sort have been assumed to exist (Ray (1977), Lunn (1980, 1983), Scaperlandra and Balough (1983), Scaperlandra and Mauer (1969), Aliber (1970, 1971), and Ragazzi (1973)). Some explain FDI as the result of a portfolio diversification process (Aggarwal (1977), Hartman (1977), and Rugman (1977)). In addition there have been studies concerned with why a foreign firm investing in a host country would have an advantage over local firms (Buckley and Dunning (1976), Buckley (1979)), why firms produce in the foreign market rather than service it through exports (Buckley and Dunning (1976), Buckley and Mathew (1979, 1980), and Lall (1980)), and why FDI exists rather than just licensing a local firm to produce and/or distribute the product (Contractor (1984))? With the collapse of the Bretton Woods system and it’s fixed exchange rate regime in the 1970’s, a further complication has been added to the FDI decision, namely “exchange rate risk”. Drawing on both the economics and finance literatures as a background, this paper presents a model of FDI which includes exchange rate risk explicitly in the investing firm’s objective function. In addition the model provides an analysis of the way both exchange rate movements and the generally recommended responses to these movements impact the firm. Finally, the model shows interesting effects form various tax and host country policies

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on the desired level of investment.

An earlier version of this model was used in successfully testing the aversion of U.S. multinationals to exchange rate risk when undertaking FDI (Clare (1992)).

II. Literature Review

In the economics literature one finds studies testing the hypothesis that firms are averse to exchange rate risk when undertaking FDI (Kelly and Philippatos (1982), Igawa (1983), Clare (1992), and Goldberg and Kolstad (1995)). In attempting to deal with exchange rate risk when undertaking FDI, the recommendation that firms borrow in the host country results in a two step process (Stevens (1972), Hartman (1977), and Kwack (1972)). First, the Optimum amount of capital is first determined. The second, risk is addressed with the prescription often being to borrow in the foreign market.

Yet there is always a cost to borrowing and the firm may find that credit is rationed (either by government regulation or bank policy) or available only at increasing cost. In addition this method seems to treat exchange rate risk as an after the fact consideration (after deciding on the level of investment). Yet if the firm is averse to such risk then one would expect it to be taken into consideration when deciding whether or not to invest, the level of investment to undertake, and whether to expand or contract its current position.

In the finance literature one finds a great deal of discussion given to the various ways exchange rate movements impact the firm, the difference between exchange rate risk and exposure, and how the risk should be handled. These methods of dealing with the risk are of great value, but again they are taken after the fact (which in this case is not only after the investment has occurred but also after movement in the exchange rate has impacted the firm).1

1. Movements in Exchange Rates and the Firm

Glaum (1990) classifies the ways exchange rate movements impact the firm into two broad categories: accounting exposure and cash flow exposure.

**Accounting exposure** (translation exposure) occurs when the firm consolidates the financial statements at year end and now finds the exchange rate different from the time when the foreign denominated assets and liabilities were first entered in the books. This in turn can result in foreign currency gains and losses. The magnitude of these gains and losses depends on the method of translation used which in turn is determined by accounting standards. Glaum (1990) points out that this view of exchange rate risk is static (since it is based on historical values) and that the real effect which exchange rate risk has on the firm is through its impact on the firm’s cash flows.

**Cash Flow exposure** is subdivided into two groups: transaction exposure and economic exposure. **Transaction exposure** occurs when the firm enters into a transaction where a specified

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payment will either be made or received at some future date in terms of a foreign currency. In this case any change in the exchange rate will result in a change in the home currency value of the transaction. In a case such as this, the firm may enter into a forward contract thereby locking in a home currency value to the transaction.

Economic exposure (operating exposure) is concerned with the impact which exchange rate changes have on the firm’s operating cash flows which are generated in terms of the foreign currency and then converted back to the home country currency. It is the home currency value of these future flows which are discounted to arrive at the firms’ value. Thus, the home currency value of these cash flows depend on revenues minus and costs incurred in terms of the foreign currency and the exchange rate. There are two ways the home currency value of these operating cash flows are affected by movements in the exchange rate.

First, given the volume of these foreign currency denominated cash flows any movement in the exchange rate will change their home currency value. This Glaum (1990) refers to as the “conversion effect”.

Second, depending on the degree of competition which exists in the foreign market for the final good, along with whether the competitors are local or foreign firms themselves, the firm may observe a change in the foreign price and quantity of its sales. The same holds true in the foreign input market. Thus the change in the exchange rate itself may also cause a change in the magnitude of the foreign currency denominated cash flows before they are even converted at the new rate. This Glaum (1990) refers to as the “competitive effect”. The firm cannot rely on purchasing power parity to protect it because P.P.P. is based on national flows whereas the firm is dealing with specific prices of specific inputs and outputs and there is no reason to assume they will move exactly as the aggregate (Grant and Soenen (1991)). In attempting to deal with this problem the firm can try changing the price of the good in the foreign market but this may well have an impact on its market share. Thus the success of such steps will depend on the price elasticity of demand for the good, the degree of competition along with whether the competitors are local or foreign firms. Certainly the firm can try altering its’ input mix between foreign and domestic (home country) inputs, factor substitution or even shifting part of its’ production to an alternative location. But such steps may not be feasible or even possible. To shift location may require a great deal of time, inputs from the home or other foreign countries may be poor substitutes for those already in use, and the ability to substitute factors will be constrained by the production function. Finally it should be kept in mind that once again this response is coming after the fact.

Hedging is of doubtful remedy since we are talking about a continuous stream which theoretically extends to infinity. Grant and Soenen (1991) point out that hedging a continuous flow against a currency which is continuously appreciating or depreciating will not maintain a given home currency value to the flow especially if the forward rate is truly an unbiased estimator of the future spot rate. In addition Shapiro and Rosenberg (1976) along with Giddy (1976) have shown that the forward as well as spot rates are variable. This means that firm does not know what the home currency value will be of the future cash flows. This in turn means the firm does not really know what the true value is of the asset generating this flow. The greater the variation in these rates the less certain the firm that the value of the asset will be within a given range.
2. Risk and Exposure

Risk is generally referred to in terms of dispersion of outcomes (as measured by variance) around some anticipated value or as increased variability (Jacque (1981), Adler and Dumas (1984) and Glaum (1990)). Therefore in this paper exchange rate risk will refer to the variation in the exchange rate around its anticipated value.

Exposure on the other hand is generally viewed as what is at risk. In the case of operating exposure it is the home currency value of operating cash flows earned in the foreign currency which are at risk due to the variation in the exchange rate. Therefore in this paper the term exposure will refer to the foreign currency denominated cash flows, and the volume of these cash flows will be referred to as the magnitude of the firm’s exposure. This is done to keep the discussion as straightforward as possible. Additionally the simplifying assumption is made that movements in the exchange rate have no impact on sales and prices in the home or any other country. This focusing on the conversion aspects of economic exposure will enable the development of a model which will highlight: some of the ways exchange rate movements impact the firm, the impact of steps firms often take to counter exchange rate movements, and an additional way exchange rate risk impacts the firm when taxes are present.

III. Model

When viewing exchange rate risk from the position of a multinational, consideration should be given not only to the effect it may have on the firm’s balance sheet (translation exposure), but also the impact it has on the home currency value of its projected foreign sales, costs and profits (operating exposure). Quite simply the greater the variation in the exchange rate, the greater the variation in the home currency value of its projected foreign sales and foreign costs and the more care the firm should take in deciding whether or not to invest, the level of investment, and whether to expand or contract its current position. Thus one would expect the risk averse firm to take the variation in the exchange rate into consideration when determining its optimum level of investment. The model we use is a modification of the neo-classical investment model with exchange rate risk incorporated directly into the firm’s objective function. There are two sources of capital: the home country and host country.

Begin with a U.S. multinational which has a foreign subsidiary and the sole source of randomness in the value of the firm is due to the randomness of the exchange rate. If management is risk neutral, then maximizing expected utility of the firm’s value is equivalent to maximizing expected value of profits. If management is risk averse, then a different outcome can be obtained. The goal of the firm is to maximize its expected utility of the market value in terms of the home currency, in the presence of exchange rate risk. It is assumed that any transaction occurring in the foreign currency is subject to this risk and the firm’s measure

2. The term firm is used to refer to the decision maker that is acting on behalf of the owners. The assumption is that the individual is acting in accordance with a utility function which exhibit’s risk aversity.
of this risk is variation in the exchange rate. So as to focus on the role of exchange rate risk, it is assumed that the exchange rate is the only random variable. Therefore the price of outputs and inputs in all markets are held constant.

Since the firm has both sales (\( S_f \)) and costs (\( C_f \)) in the foreign market then its foreign profits (operating cash flows) must be converted to dollars at the existing exchange rate (\( \varepsilon \)). Thus \( \bar{R}_f = \varepsilon R_f \) where \( R_f = (S_f - C_f) \) which is the foreign currency denominated operating cash flows. Since the exchange rate (\( \varepsilon \)) is a random variable, so is the dollar value of the foreign flows (\( \bar{R}_f \)).

Assuming the exchange rate (\( \varepsilon \)) is the only random variable, then:

\[
\text{VAR} (\bar{R}_f) = \sigma^2 R_f^2,
\]

where \( \sigma^2 \) is the variance of the exchange rate. Thus the variance of \( \bar{R}_f \) depends only on the variation in (\( \varepsilon \)). Therefore, \( \sigma^2 \) is the exchange rate risk, \( R_f \) is the exposure to the risk and the size of \( R_f \) is the magnitude of the exposure.

The more averse the firm to risk the greater the impact this variable will have on the investment decision. If one assumes that the firm’s Von Neuman Morgenstern utility function is \( U(V) = -e^{-\gamma V} \) then \( 2\gamma \) is the measure of absolute risk aversion for the firm.

It is assumed that the capital comes from two sources: the U.S. (\( K \)) and the foreign location (\( F \)) and that it is purchased and financed in terms of the respective currency. The capital financed in dollars is the U.S. parent’s contribution and its implicit rental cost is what could have been earned from the capital if it had been used in the U.S. As such, the U.S. parent discounts at the rental price of capital which exists in the U.S. (\( r \)). The rental price consists of the rate of interest (\( i \)) plus the rate of depreciation (\( D \)) minus the percentage change in the price of capital goods (\( \%P_k \)) all of which are multiplied by the price of capital goods (\( P_k \)) in the U.S.: \( P_k (i + D - \%P_k) \). By the same token, (\( r_f \)) is the rental price of capital in the foreign market. To keep the model simple and without loss of generality, it is also assumed that all labor employed is in the foreign country (\( L \)) and paid in terms of the foreign currency (\( w_f \)). The price of final goods in all markets are assumed given, along with the cost of capital goods and labor in the foreign market.

The firm’s objective is to maximize expected utility of profits subject to the constraints imposed by the production function. Since there are two sources of capital it is a three factor production function \( \dot{Q} = g(F, L, K) \), which is homogeneous of degree 1 and of the form:

\[ g = F^aL^bK^c. \]

Hence the firm’s problem is to maximize \( E(U(h + R_f\varepsilon)) \)
If \( \varepsilon \) is normal with mean \( \mu \) and \( \sigma^2 \), then this expected utility maximization problem is equivalent to maximizing:

\[
U = \left\{ Rg - \mu K \right\} + \left\{ \alpha P_f \left( g - \mu L - \sigma(F) \right) - \gamma \sigma^2(R_f)^2 \right\}
\]

s.t. the production constraint,

where:

\[
U = \left\{ \text{cash flows generated in } \$$ \right\} + \left\{ \varepsilon(R_f) \right\} - \left\{ \text{cost of the risk} \right\}.
\]

Therefore \( \sigma^2(R_f)^2 \) is the cost of exchange rate risk to the firm which is included in the firm's objective function, where:

- \( \alpha \) is the proportion of output \( g \) which is sold to the U.S. market or negotiated in terms of dollars. \( 0 < \alpha < 1 \).
- \( \alpha' \) is the proportion of final output \( g \) which is sold in the foreign market or in terms of the foreign currency. \( \alpha' = (1 - \alpha) \).
- \( P \) is the price of final goods sold in terms of dollars which is assumed constant regardless of destination.
- \( P_f \) is the price of final goods in terms of the foreign currency which is assumed constant regardless of destination.

and:

\[
R_f = P_f(\alpha')g - \mu L - \nu F
\]

which is the foreign currency denominated cash flows.

Taking the first order conditions of the objective function with respect to the factors and solving for the optimum level of U.S. parent's participation \( K^* \) yields:

\[
K^* = \left[ \alpha' P_f(\alpha')g \varepsilon + R(\alpha)g \varepsilon \right] \gamma^{-1},
\]

where \( \varepsilon' \) is the risk adjusted exchange rate \( \varepsilon' = (\varepsilon - 2\sigma^2(R_f)) \) and \( \gamma \) is the elasticity of output with respect to \( K \). Thus, the optimum level of U.S. capital \( K^* \) is not only a function of sales (which have always been considered the primary determinant), but also depends on exchange rate risk. Exchange rate risk is now taken into consideration when determining the optimum level of capital.
To investigate the impact which a change in exchange rate risk has on the optimum participation of the U.S. parent, \((K^*)\) is differentiated with respect to \(\sigma^2\) which yields:

\[
dK^*/d\sigma = (-2\gamma (R_f) P_f' g \sigma (\nu)^{-1} + (K^*) (g^*)
\]

which for \(R_f > 0\) is negative along with \(g^*)^3\)

In fact one can go a step further to see what happens as the proportion of the subsidiaries’ output sold in terms of dollars increases relative to that sold in terms of the foreign currency. Referring to Equation (2),

\[
dK^*/d\sigma = (-2\gamma (R_f) P_f' \sigma (\nu)^{-1} + (K^*) (g^*)
\]

where:

\[
g_\sigma = [g_F (2\gamma (R_f) \nu P_f g \sigma (\nu)^{-2}) + [g_L (2\gamma (R_f) \nu P_f g \sigma (\nu)^{-2})]
\]

\[
+ [g_R (-2\gamma (R_f) P_f \sigma (\nu)^{-1})],
\]

which for sake of simplicity is written as:

\[
g_\sigma = [g_F F^*_\sigma] + [g_L L^*_\sigma] + [g_R K^*_\sigma].
\]

It is noted that for a given level of risk aversion \(\gamma\), as the firm increases the proportion of output being sold in dollars, \((\sigma)\) increases and \((\sigma')\) decreases. Thus the first two terms in brackets \([g_F F^*_\sigma]\) and \([g_L L^*_\sigma]\) are increasing while the third \([g_R K^*_\sigma]\) diminishes. Remembering that:

\[
R_f = P_f (\sigma') g - \nu L - \nu F,
\]

so as \((\sigma')\) decreases \(R_f\) decreases which means the size of the firm’s exposure is reduced. Thus \(dK^*/d\sigma\) is still less than \(0\) but smaller in magnitude. As the proportion of the firms output which is sold in dollars increases \(\sigma \to 0\). However before \(\sigma' = 0\), \(R_f\) reaches zero and \(R_f = 0\) when \(P_f (\sigma') g = \nu L + \nu F\). When \(R_f = 0\) both \(K^*_\sigma\) and \(g^*_\sigma = 0\) which means \(dK^*/d\sigma = 0\) and the firm is completely hedged.

The question now is: What happens when the firm continues to sell more of its output in dollars so that \(R_f < 0\)? Notice as \((\sigma)\) continues to increase, the magnitude of \(F^*_\sigma\) and \(L^*_\sigma\) increases while the magnitude of \(K^*_\sigma\) decreases. Thus \([g_F F^*_\sigma]\) and \([g_L L^*_\sigma]\) begin to

3. See appendix A for a complete derivation of \(K^*_\sigma\) and \(g^*_\sigma\).
dominate $\varepsilon_0$. With a negative $R_f$, the signs of $K_0^\alpha$, $F_0^\ast$, $L_0^\ast$ change so $F_0^\ast$ and $L_0^\ast < 0$. Thus with $[\varepsilon_0 F_0^\ast]$ and $[\varepsilon_0 L_0^\ast]$ having gained in magnitude relative to $[\varepsilon_0 K_0^\alpha]$ one would expect $\varepsilon_0 < 0$. Take the extreme case where $\alpha = 1$ and $d' = 0$. That is all sales are in dollars. In this case $\varepsilon_0$ consists only of $[\varepsilon_0 F_0^\ast]$ and $[\varepsilon_0 L_0^\ast]$ both of which are $< 0$. Therefore $\varepsilon_0 < 0$. Now the optimum $K$ given by Equation (1) becomes:

$$K^\ast = [P(\varphi) \varepsilon f](\sigma)^{-1}.$$  \hspace{1cm} (1-a)

We can see that $K^\ast$ is not affected directly by $\sigma^2$ but it is by $\varepsilon$ which in turn is affected by $\sigma^2$ through its impact on $F^\ast$ and $L^\ast$. Thus $dK^\ast/d\varepsilon = K^\ast g(\varepsilon^\prime)$. Noting that:

$$K^\ast = [P(\varphi) \varepsilon f](\sigma)^{-1}$$

which is positive and $\varepsilon_0 < 0$, then $dK^\ast/d\varepsilon < 0$.

What has happened is that the increase in $\sigma^2$ has increased the variation in foreign costs. Since there are no sales in the foreign market there is nothing to offset the increased variation in costs, so to maximize profits the firm reduces $F^\ast$ and $L^\ast$. Thus even when all sales are in dollars an increase in exchange rate risk has a negative impact on foreign investment ($K^\ast$) through its impact on the foreign costs.

It can be seen from the above discussion the crucial role which $R_f$ plays in the risk variable. In general the recommended methods of responding to adverse movements in exchange rates (which one finds in the finance literature) is to attempt to reverse or to lessen the impact which such movements had on $R_f$, the foreign denominated cash flows. The methods include: altering the price in the foreign market, altering the input mix between foreign and home or other country, and even shifting production to another location. These methods are attempts to restore or even expand the subsidiaries operating cash flows.

Assume an expected exchange rate less than $\varepsilon$ emerges. Call this $\varepsilon_0$. Thus, $R_f$ now translates into fewer dollars ($\uparrow R_f \downarrow$). This can be viewed in terms of Equation (1) written in expanded form:

$$K^\ast = [e P_f(\varphi') F - 2 \sigma^2 P_f(\varphi') F - P(\varphi) \varepsilon f](\sigma)^{-1}$$
$$= [e_0 S_\varepsilon - 2 \sigma^2 (S_f - C_f) S_\varepsilon + S_\varepsilon^2](\sigma)^{-1}, \hspace{1cm} (1-b)$$

where $e_0 < e$ and $S_\varepsilon$ is sales in terms of dollars. The decrease in $e$ to $e_0$ reduces the dollar value of the foreign cash flows and consequently the optimal level of $K$ to $e_0 K^\ast$. 

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Thus the firm is no longer in the optimal position. In response, the firm might try to increase the price of the final good in the foreign market. This would increase $S_f$ and therefore offset some of the impact of the drop in $\varepsilon$. However, it can also be seen that this increases the magnitude of the firm’s exposure. If the firm also uses inputs from the home country which are paid in dollars then it may try to substitute the use of foreign for home country inputs. Given the foreign price for these inputs and the lower rate $\varepsilon_0$, this may reduce the overall dollar costs of production. In addition it is noted that such a substitution will increase $C_f$ and reduce the magnitude of the exposure $R_f = (S_f - C_f)$.

Since the firm has sales in dollars as well as in terms of the foreign currency, it may even try to increase the proportion of its sales in dollars ($\alpha$). This would decrease $S_f$, increase $S_d$ and reduce the magnitude of the firm’s exposure.

The effectiveness of any of these actions depends on many factors which are beyond the firm’s control. The firm’s ability to change the price in the foreign market depends on the degree of competition, whether the competitors are local or foreign firms and the price elasticity of demand for the final good. The success from substituting home country inputs for foreign depends on the degree to which they can be substituted for one another, their relative prices in terms of dollars and whether or not such substitute inputs exist. Trying to re-negotiate sales in terms of dollars will depend heavily on the firm’s negotiating ability. It may be extremely difficult to convince buyers to pay a higher price in dollars directly than if they took the indirect route through the foreign currency.

Shifting production to another location is of course a possibility but such a decision may require a great deal of time, especially if there are no alternative facilities available. Even if they are available, the shift in production may require more time than is feasible and in the interim the exchange rate may move again but in the opposite direction. The general prescription of borrowing in the foreign market impacts the cash flow through the foreign interest payments and this reduces the magnitude of the exposure by increasing the $C_f$ component of $R_f$.

All of these suggested methods of dealing with the adverse effect of exchange rate movements occur after the fact. Even if the firm has tried to plan ahead for their implementation, it is not sure what the actual change in the exchange rate will be. Since the firm does not know what the actual change will be, it cannot be certain of the exact mix or the extent to which the above steps should be carried out. The greater the variation in the exchange rates, the less certain the firm will be regarding the correct mix and extent of the steps it will have to take. Therefore, the greater the need for the risk averse firm to take exchange rate risk into consideration when determining it’s optimal combination of factors and investment.

This does not mean the above mentioned steps are of no value. Quite the contrary, they can be of tremendous value in dealing with events which have occurred while the firm is re-evaluating what it’s optimal position should be and in the process of moving towards it’s new $K^*$. Throughout this paper it has been assumed that movements in the exchange rate had
no impact on the prices of inputs and outputs. To the extent that movements in the exchange rate does affect these price, then such movements may have a greater impact on the exchange rate since the changes in input and output prices will change the size of the exchange rate through changes in input and output markets in which the firm is a participant. Thus the exact impact which changes in exchange rates have on the firm will vary from industry to industry, firm to firm and on a case by case basis. The volatility in exchange rates in recent years make it even more crucial to include exchange rate risk when determining the optimal combination of factors and level of investment.

IV. Policy Implications

The model has some interesting implications for trade, tax and remittance policies. Examples of trade and remittance policies would be: the imposition of a minimum level of exports for the firm, requiring a minimum level of domestic content in production, and limiting remittances to the foreign exchange earnings of the firm. Tax policies, such as a general tax on profits and a tax on the foreign ownership is also an issue to be considered.

1. Trade and Remittance Policies

An example of trade policies is for host governments to impose a minimum level on the exports of foreign firms. If these exports generate dollar revenues for the U.S. owned firm, then in fact such requirements will aid the firm in reducing the magnitude of its exposure to exchange rate risk. Thus, what at first may appear to be a restriction may turn out to yield a side benefit. This can be seen more clearly by expanding Equation (1). Recalling that $\hat{e}$ is the risk adjusted exchange rate ($e - 2\gamma \hat{e}^2(R_f)$) and $\gamma$ is the elasticity of output with respect to $K$, Equation (1) may be rewritten as:

$$K^* = \left[ e^P(\hat{e}^{P}(K^*) - 2\gamma \hat{e}^2(R_f) e^P(\hat{e}^{P}(K^*) e^P + \gamma^2 e^P(\hat{e}^{P}(K^*) e^P) (\gamma)^{-1} \right],$$

(1-c)

where the middle term inside the brackets contains the risk variable which may be written as:

$$X = (2\gamma (S_f - C_f) S_f \gamma)^{-1},$$

(3)

4. When applying the NPV or APV approach the level of investment is pre-determined as is the combination of factors which give rise to the cash flow. Then the usual recommendation is to adjust this cash flow for the risk. This is responding after the fact. In the model presented, the risk is taken into consideration when determining the combination of factors (which are components in the cash flow) and the level of investment. Now the firm can proceed with NPV or APV.
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where: \( S_f = P_f (\alpha') \).

Recall that \( \alpha \) is the proportion of total output sold in terms of dollars and \( \alpha' \) is the proportion of total output sold in terms of the foreign currency. Since \( \alpha' = (1 - \alpha) \), then any increase in the proportion of total output which is sold in terms of dollars \( \alpha \) will directly reduce \( \alpha' \). Differentiating (3) with respect to \( \alpha' \) we have:

\[
X_{\alpha'} = 2\beta'(2S_f - C_f)(P_f g r) (\gamma)^{-1}
\]

which will be positive unless foreign costs are more than twice the sales in the foreign market.

To the extent that the firm negotiates sales in terms of dollars, \( S_f \) decreases which in turn decreases the magnitude of its exposure to risk. Given that \( K_C < 0 \), \( -\gamma \) then any reduction in exposure will tend to increase foreign investment. This should be taken into consideration along with any other benefits and costs resulting from such a policy.

Another policy is for less developed countries to limit remittances of dividends or interest to the foreign parent by limiting them to the foreign exchange earnings of the firm. To the extent that this would encourage the U.S. parent to increase sales in terms of dollars (which is foreign exchange from the perspective of the host country) it will reduce the magnitude of the firm’s exposure and again yield a side benefit.

At the same time developing countries have required that production consist of some minimum share of local contribution. This would have the effect of increasing \( C_f \) which again tends to reduce the magnitude of the exposure to risk through \( R_f \).

Differentiating (3) with respect to \( C_f \) yields:

\[
X_{C_f} = (-2\beta' 2S_f)(\gamma)^{-1}
\]

which is negative.

Since \( K_C < 0 \), \( -\gamma \) then \( K_{C_f} > 0 \) which means such a program would tend to have a positive impact on foreign investment which will partially offset the negative effect of such an overall increase in costs.

2. Tax Policies

Consider the effect an overall tax on profits will have on foreign investment. Assume that the tax allows full deduction of costs and would, therefore, be considered neutral in the sense that it treats both costs and revenues equally. Since exchange rate risk would be considered a cost only to the risk averse firm then from the government’s viewpoint this would not be included as a deduction in the calculation of taxable profits.

Therefore the objective function which the firm now maximizes is:
where:
\[ t = \text{tax rate}, \]
\[ t' = (1 - t). \]

Evaluating the first order conditions and solving for \( K^* \) yields:
\[
K^* = (eP\alpha g + P_{\alpha g}e - ewL - e(r_f F) - (\gamma K)) - \gamma a^2(R_f)^2. \tag{4}
\]

With the exception of the risk term, taxes cancel out. To find the impact which taxes have on \( K^* \) we differentiate (4) with respect to \( t \).

Given: \( f = (1 - t) \) then,
\[
K_t^* = [( -f) (2\gamma (R_f) P_{\alpha g}g)] [(1 - t)(r)]^{-2}, \tag{4-a}
\]

which is negative.

Thus even the imposition of a general tax which permits full deduction of costs reduces investment through the risk variable. Hence, unless the firm never pays any income taxes on the profits of the foreign affiliate, then the risk will have an impact on the firm via taxes. This can be viewed as another reason for firms to seek out tax havens.

A tax on foreign ownership would occur if the government (U.S. or foreign) imposed a tax on any financial flow from the U.S. parent. This would generally be imposed from the U.S. side in an attempt to stop investment outflows. Such a tax may be viewed as an increase in the rental price of capital by the U.S. parent. Thus the objective function now becomes:
\[
U = eP\alpha g + P_{\alpha g}e - ewL - e(r_f F) - (r + t)K - \gamma a^2(R_f)^2. \]

Evaluating the first order conditions and solving for \( K^* \) yields:
\[
K^* = [e'P_f(\alpha')g + P(\partial g/\partial z)(r + t)]^{-1}. \tag{5}
\]

Differentiating with respect to \( t \) yields:
\[
K_t^* = -[e'P_f(\alpha')g + P(\partial g/\partial z)(r + t)]^{-2} \tag{5-a}
\]
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which is negative.

Thus, as expected, a tax on the U.S. component will have a negative impact on foreign investment.

V. Summary and Discussion

It has been pointed out that the real impact which exchange rate risk has on the firm is through its impact on the firm’s cash flows. The greater the dispersion in exchange rates the greater the dispersion in the home currency value of foreign cash flows. This in turn means the less certain the firm is that the value of these cash flows will fall within a given range of the anticipated value. Therefore, the greater the risk. If the firm is indeed risk averse then this should be taken into consideration when determining the optimal combination of factors and level of investment.

Generally the discussion regarding how to deal with adverse effects of exchange rate movements has centered on steps to be taken in response to the exchange rate movements. Thus the firm is recognizing the problem after it has occurred and certainly after determining (what it thinks is) the optimal position. However, if the firm is risk averse, then the exchange rate risk is a cost and by not including it in the original objective function the firm has not determined the optimal level of capital. Quite simply the firm has overestimated the value of the cash flows and in turn overestimated the optimum level of capital.

The steps generally recommended in the past are attempts to offset the impact exchange rate movements have on the cash flow. In essence the firm is trying to bring the cash flows back to a level appropriate with the level of capital. However this level of capital is not optimum (the level of capital is greater than the optimum). The model presented in this paper incorporates exchange rate risk explicitly in the firm’s objective function. Thus based on the degree to which the firm is risk averse it can determine the optimum level of capital and accept the level of risk with which it is comfortable. Now when it responds to movements in the exchange rate it will be trying to bring sales up to the appropriate level for the optimum level of capital. In addition the model enables the firm to analyze the way in which the various responses affect not only sales and costs but also the magnitude of the firm’s exposure. Should the firm consider the new situation to be permanent, then it has a framework for determining the new optimum level of capital and the generally recommended steps provide a good short run response.

It can also be seen from the model that as long as the firm is risk averse there is no neutral tax unless it is allowed to deduct for the exchange rate risk. The model also shows that host government policies have a more complex impact on the firm which is risk averse. Whereas many policies of host governments may tend to increase the firms costs, some of this may be partially offset by a reduction in the firms exposure.
Appendix

Mathematical Derivations

Derivation of $K^*g$

To find the impact which a change in $\sigma^2$ has on $K^*g$ is needed. Differentiating Equation (1) with respect to $g$ and nothing that $\epsilon'$ contains $R_f$ which is a function of $g$ yields:

$$K^*g = (P_f \partial x/v)[e + 2\sigma^2(C_f - 2S_f)] + R_f/v,$$

where $S_f$ and $C_f$ are sales ($P_f \partial x$) and costs ($wL + \nu F$) in the foreign market respectively.

Since $(P_f \partial x/v)$ and $(Pax/v)$ are both positive, then $K^*g > 0$ if $[e + 2\sigma^2(C_f - 2S_f)] > 0$.

Now if $[e + 2\sigma^2(C_f - 2S_f)] > 0$, then $e/2\sigma^2 > (C_f - 2S_f)$.

Noting that $e' = e - 2\sigma^2R_f$, then if $e' > 0$, then $e/2\sigma^2 > R_f$. Therefore if $R_f > (C_f - 2S_f)$, then by transitivity $e/2\sigma^2 > (C_f - 2S_f)$ and $K^*g > 0$. Now if $R_f > (C_f - 2S_f)$ then $S_f > (C_f - 2S_f)$ or $S_f > 2(C_f - S_f)$. Solving for $S_f$ yields: $S_f > 2/3 C_f$. Thus if $S_f > 2/3 C_f$, then $R_f > (C_f - 2S_f)$. Since $e/2\sigma^2 > R_f$, then $e/2\sigma^2 > (C_f - 2S_f)$ and $K^*g > 0$.

Derivation of $g'_o$

Assuming $K^*g > 0$, only $g'_o$ is needed to complete the investigation into the impact a change $\sigma^2$ has on $K^*$. Since $g$ is a function of the three factors $F$, $L$, and $K$ then:

$$g'_o = [g'_o F'_o + g'_o L'_o + g'_o K'_o],$$

where $g'_o$, $g'_L$, and $g'_K$ are the marginal products of the respective factors. Thus to obtain $g'_o$, $K^*_o$,$L^*_o$ and $F^*_o$ must first be derived.

Derivation of $K^*_o$

Partially differentiating Equation (1) with respect to $\sigma^2$ yields:

$$K^*_o = (-2\gamma(R_f)P_f \partial x \nu)(\nu)^{-1}$$

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which is negative (for \( R_f > 0 \)).

**Derivation of \( L_o^* \)**

To obtain \( L_o^* \), the first order condition for \( L \) must be solved for \( L^* \) and then differentiated with respect to \( \sigma^2 \).

Solving the first order condition for \( L^* \) yields:

\[
L^* = (P \cdot \epsilon \cdot \gamma)(\gamma_o)^{-1} + (\text{Reg}(\epsilon \cdot \gamma_o))^{-1}.
\]  

Taking the partial derivative of \( L^* \) with respect to \( \sigma^2 \) yields:

\[
L_o^* = (2\gamma(R_f)wP \cdot \gamma)(\epsilon \cdot \gamma_o)^{-2}
\]  

which is positive (when \( R_f > 0 \)).

**Derivation of \( F_o^* \)**

Solving the first order condition for \( F^* \) we obtain.

\[
F^* = (P \cdot \epsilon \cdot \gamma)(\gamma_f)^{-1} + (\text{Reg}(\epsilon \cdot \gamma_f))^{-1}.
\]  

Partially differentiating \( F^* \) with respect to \( \sigma^2 \) yields:

\[
F_o^* = (2\gamma(R_f)\epsilon \cdot \gamma)(\epsilon \cdot \gamma_f)^{-2}
\]  

which is positive (when \( R_f > 0 \)).

**Derivation of \( g_o \)**

Thus: \( g_o = [g_F F_o^*] + [g_L L_o^*] + [g_R K_o^*] \), where all values are \( \neq 0 \).

Substituting for \( F_o^* \), \( L_o^* \) and \( K_o^* \) respectively yields:

\[
g_o = [g_F(2\gamma(R_f)\epsilon \cdot \gamma)(\epsilon \cdot \gamma_f)^{-2}] + [g_L(2\gamma(R_f)wP \cdot \gamma)(\epsilon \cdot \gamma_o)^{-2}] + [g_R(-2\gamma(R_f)P \cdot \epsilon \cdot \gamma)(\epsilon \cdot \gamma_f)^{-1}].
\]  

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where the sign is unclear since $F^*_{\sigma}$ and $L^*_\sigma \geq 0$ whereas $K^*_\sigma < 0$. However, if the negative effect of $K^*_\sigma$ on $g_\sigma$ is of sufficient magnitude then the net effect will still be negative.

It is noted that:

a. $(e^r_r)$, $(e^w_w)$ and $(e^r)$ are the prices of the respective factors in terms of dollars,
b. $(g_\sigma)$, $(g_1)$ and $(g_\mu)$ are the respective marginal products; and
c. The profit maximizing combination of factors is:

$$(g_\sigma/e^r_r) = (g_1/e^w) = (g_\mu/e^r) = k,$$ where $k$ is some constant value.

Simplifying Equation (viii) by moving $2\gamma(R_j\delta R)$ outside the brackets yields:

$$2\gamma R_k [(r e^r r)^{-1} + w e^w w)^{-1} - P f g].$$

Cancelling out $(r_\sigma)$ and $(w_\sigma)$ in their respective terms leaves:

$$2\gamma R_k [(r e^r r)^{-1} + w e^w w)^{-1} - P f g].$$

In order for $g_\sigma < 0$; then if $R_j < 0$.

$$[(P f e^r)^{-1} + (P f e^r)^{-1} - P f g] < 0.$$

Given $(\alpha + \sigma + \tau) = 1$, then by employing Euler’s Theorem the payments going to the foreign factors may be written as:

$$wL = (P f e^r g + (P f e^r)) \alpha,$$

$$\nu F = (P f e^r g + (P f e^r)) \alpha \varepsilon$$

Given $R_j = S_j - wL - \nu F$, then substituting for $wL$ and $\nu F$ yields:

$$R_j = S_j - (P f e^r g + (P f e^r) \alpha) \alpha - (P f e^r g + (P f e^r) \alpha) \varepsilon,$$

where $S_j$ represents foreign sales.

Foreign sales may also be written as $P f e^r g (\alpha + \varepsilon + \tau)$. Therefore $R_j$ can be written as:

$$R_j = P f e^r g (\alpha + \varepsilon + \tau) - (P f e^r g + (P f e^r) \alpha) \alpha - (P f e^r g + (P f e^r) \alpha) \varepsilon.$$
By combining like terms $P \cdot d_{\text{gr}}$ and $P \cdot d_{\text{ge}}$, drop out leaving:

$$R_f = P \cdot d_{\text{gr}} - ((P \cdot e^r)_{\text{age}} + (P \cdot e^r)_{\text{agr}}).$$  \hspace{1cm} (ix)

Thus when $R_f > 0$, $[((P \cdot e^r)_{\text{age}} + (P \cdot e^r)_{\text{agr}} - P \cdot d_{\text{gr}}) \times 0$

and therefore $e_o < 0$. when $R_f < 0$, then $[((P \cdot e^r)_{\text{age}} + (P \cdot e^r)_{\text{agr}} - P \cdot d_{\text{gr}}) > 0$. So again $e_o < 0$. 

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