The Effects of Off-farm Employment on Inequality: Theories and an Empirical Application to the Taiwan Case

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The expansion of off-farm employment has been shown to be an important factor in bringing down or stabilizing the income inequality for farm households and for the economy as a whole in both Japan and Taiwan, and also to a lesser degree in South Korea, in the 1950s and 1960s. The basic reason was that income from such employment benefitted the land-poorer farmers more than the land-richer ones. In this paper, a model is constructed to show that with the expansion of off-farm non-agricultural employment, the inequality of earned income among households will decline so long as households have equal access to industrial employment, that the wage rate keeps rising, and that agricultural employment declines absolutely, irrespective of whether or not the total population increases, relative prices change, or agricultural productivity rises. It also shows that as an economy grows, the ratio of capital income to earned income can fall if there is an increase in agricultural productivity or a rise in the price of agricultural products, when the elasticity of substitution is less than unity and when industrial expansion is strong enough to ensure non-increasing agricultural employment. Together, these propositions entail that the inequality does not have to rise even in the initial stage of economic growth. The data of Taiwan are then analyzed and used to illustrate the validity of the propositions.

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I. Introduction

The expansion of off-farm employment has been shown to be important in bringing down or stabilizing the income inequality for farm households and for the economy as a whole in both Japan and Taiwan, and to a lesser degree also in South Korea, in the 1950s and 1960s. The basic reason was that income from such employment benefitted the land-poorer farmers more than the land-richer ones. In this paper, models will be constructed to show that with the expansion of off-farm non-agricultural employment, inequality of earned income among households will decline as long as households have equal access to industrial employment, that the wage rate keeps rising, and that agricultural employment declines absolutely. Disparity between capital income and earned income can also go down, under a set of reasonable conditions. As a result, national inequality does not have to rise even in the initial stage of economic growth.

Oshima (1993, pp. 166-71) illustrates the importance of the rise in income from off-farm employment in bringing down or stabilizing farm household income inequality and perhaps also national income inequality in Japan, Taiwan and South Korea from the 1950s to the 1970s. This rise in off-farm-employment income helped to reduce farm household inequality because poorer households which had less land to cultivate resorted more to off-farm employment as a source of income, so when such income rose, the inequality between households was brought down. This also helped to reduce national inequality because the rise in such income often narrowed the income disparity between agricultural and non-agricultural households. A more detailed analysis of the case of Japan can be found in Mizoguchi and Terasaki (1992) and Mizoguchi, Takayama and Terasaki (1981). The case of Taiwan has been investigated in details by Fei, Ranis and Kuo (1979) and Chu (1995).1

1. Both in Japan Taiwan, the expansion of off-farm employment was related to the rise of rural industries and/or the extension of manufacturing activities from the urban areas to the country side. Dehghan (1995) makes a cross-country study of the relationship between income distribution and overurbanization, and found a negative relationship for the high-income countries (with per capita income of more than US$3,000 in 1979), although the results for the other groups of countries were inconclusive. It is worth noting that due to the lack of data at the time, Taiwan was not included in the sample.
Few attempts have been made to account for this important phenomenon from a theoretical point of view. Most of the existing theories adopt the viewpoint that inequality has to rise in the initial stages of economic development, prior to the ultimate reduction in the later stages. This is the so-called “inverted-U hypothesis” which was originally presented by Kuznets (1955). He attributes the initial rise to the emergence of a high-productivity sector from the traditional (agricultural) sector the income from which had been fairly equally distributed. He suggests that the inequality declines in the later stages of growth because the state adopts more egalitarian taxation and welfare policies. In Robinson (1976), knight (1976) and Fields (1979), the inverted-U pattern is described as the statistical inevitability of moving employment from a low to a high income level, both exogenously given. In the neoclassical part of his models, Bell (1979) shows that the ratio of capital income to the sum of labor and agricultural income will rise continuously with economic development, if the industrial elasticity of substitution is greater than unity. An inverted-U pattern will be observed if the elasticity of substitution is less than unity, but based on reasonable assumptions of the values of the parameters considered in his model, such a turning point will come only very late in the development process.

In Rauch’s (1993) model, which extends the Harris-Todaro model of internal migration, the size of the urban unemployed first rises then falls, as does economy-wide inequality. The Bourguignon (1990) model considers three population subgroups (the traditional sector and the labor and capital owners in the modern sector) separately, and takes into account the demand factors and the corresponding changes in the terms of trade (relative prices between agricultural and industrial products). Inequality could not be reduced unless the share of the traditional sector in GDP rises, when the wage in the modern sector is indexed by the modern good. When the modern wage rate is indexed by both the modern and traditional goods, a falling GDP share of the traditional sector together with elastic capital-labor substitution in the modern sector is still sufficient to unambiguously rule out egalitarian growth.

None of these models take into account the role of off-farm
employment in determining rural and national income inequality, which is the focus of this paper. It will be shown that once this factor is highlighted, the picture of income inequality will become much more optimistic. In what follows, Section II shows how the rise in off-farm income can reduce rural income inequality under a variety of sets of conditions; Section III shows the conditions under which the national functional distribution of income can drop as the economy grows. The data of Taiwan will then be analyzed to illustrate the validity of the model in Section IV. Finally, Section V will give the concluding remarks.

II. Growth and the Inequality of Earned Income

Let there be two agricultural households to begin with; one is endowed with more land than the other. Agricultural productions of the two households are given by

\[ x_i = a_i l_i^\beta \quad a_i > 0, \quad i = 1, 2, \quad 1 > \beta > 0 \quad (1) \]

where \( x_i \) is agricultural production and \( l_i \) is the labor input of the \( i \)-th household. It is assumed \( a_1 > a_2 \), indicating that the first household is the richer one, as it is endowed with more land. For any given real wage (in terms of agricultural products), \( w \), both households maximize agricultural income by demanding agricultural labor inputs according to

\[ l_i = \left( \frac{\beta}{w} \right)^{1/(1-\beta)} a_i^{1/(1-\beta)}, \quad i = 1, 2. \quad (2) \]

Let \( \alpha \) and \( 1-\alpha \) be the shares of total amount of labor employed by households 1 and 2. If labor is homogeneous, and if labor is taken as a synonym for population, then to make sense of comparing the incomes between the two households requires specifying that \( \alpha = 1 - \alpha = 1/2 \). Empirically this means sorting all households according to the size of their income, then grouping them into two “aggregate” households of equal size. When labor is heterogeneous, and when \( l \) is in terms of efficient units of labor, a may be different from 1/2, even
when the two aggregate households have the same physical number of members (labor). It is assumed here that \( a > 1 - a \). Such an assumption is of course not favorable to the expected results.

The assumptions involved in the current model can now be itemized as follows:

(A1) Total population is set at unity and remains unchanged.

(A2) There is no agricultural technological progress, meaning that \( a \) in the agricultural production does not change over time.

(A3) The terms of trade between agriculture and industry are unity and remain unchanged, so \( w \) is the real wage in terms of either agricultural or industrial products. This assumption will be true if the economy in question is a small open one, with all relative prices given exogenously in the world market.

(A4) The labor market is competitive and freely accessible, and the institutional arrangement and locational distribution of industrial establishment are such that any household can be involved in both agricultural production and industrial employment at the same time. This is the “off-farm employment” proper considered in this paper.\(^2\)

(A5) Both households optimize the deployment of labor by maximizing the sum of labor and agricultural income (or “earned” income).

(A6) \( a_1 > a_2 \) and \( a > 1 - a \). In addition, at the beginning when there is not yet an industrial sector and all labor is employed by agriculture, the first household is a net employer, i.e., \( l_1 > a \). This basically means that while the first (richer) household is the richer in both land and (efficient) labor, it is comparatively richer in land than in labor. When the two households demand labor to maximize output according to (2), such an assumption implies that

\[
b > a/(1-a) \tag{3}
\]

where

\[
b = (a_1/a_2)^{1/(1-\beta)}
\]

2. Migration is not an issue. This is in line with the setting adopted in the neoclassical part of Bell’s model, but in sharp contrast with models that are based on the dichotomy of incomes.
meaning that \( a \) is not large enough and \( a_1 \) (relative to \( a_2 \)) is not small enough to make household 2 a net employer.

(A7) Some unspecified force and an infusion of capital start industrial production, which keeps expanding given the continuous flow of investment form again unspecified sources. Industry and agriculture compete for labor in the free market, making the real wage, \( w \), rise higher and higher from \( t=0 \) on ward.

At a given \( w \), household 1's earned income is

\[
y_1 = a_1 l_1^\beta - w l_1 + a w
\]

where \( y_1 \) is real income in terms of either the agricultural or industrial products, given (A4). Maximizing \( y_1 \) implies maximizing household 1's agricultural profit, which in turn implies a demand for 1 according to equation (2).

Similarly, household 2's earned income is

\[
y_2 = a_2 l_2^\beta - w l_2 + (1 - a) w.
\]

Its demand for 1 is also given by equation (2).

The following proposition can now be derived:

**Proposition 1**

If (A1)-(A7) are true, then over time the inequality of earned income between household 1 and 2 declines.

**Proof**

Differentiation \( y_1/y_2 \) with respect to \( w \) gives

\[
\frac{d(y_1/y_2)}{dw} = \left[ y_2 (dy_1/dw) - y_1 (dy_2/dw) \right]/y_2^2.
\]

From equation (2), (4) and (5)
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\[ \frac{dy_1}{dw} = \frac{d(a_1l_1^\beta - w_1 + \alpha w)}{dw} \]

\[ = (dx_1/dl_1)(dl_1/dw) - w(dl_1/dw) - l_1 + \alpha \]

\[ = (dx_1/dl_1 - w)(dl_1/dw) - l_1 + \alpha \]

\[ = \alpha - l_1 \]

as \( dx_1/dl_1 = w \), i.e., the agricultural marginal product of labor equals the real wage. Similarly,

\[ \frac{dy_2}{dw} = 1 - \alpha - l_2. \] (8)

Substituting (7) and (8) into (6), one obtains

\[ \frac{d(y_1/y_2)}{dw} = A/y_2^2 \] (9)

where \( A = y_2(\alpha - l_1) - y_1(1 - \alpha - l_2) \).

It is convenient to divide the increases in \( w \) into three stages, as shown in Figure 1. At \( t=0 \), the equilibrium real wage, \( w_0 \), is determined by the intersection of \( D_1 \), which is household 1’s demand for labor given by equation (2), and \( D_2 \), which is household 2’s demand for labor from the same equation. As \( \alpha = O_1E \) and \( 1 - \alpha = O_2E \), household 1 is a net employer, employing \( EF \) of labor from household 2.

It is apparent that at the point of intersection, \( K \), \( O_1F = l_1 > \alpha \) and \( O_2F = l_2 < 1 - \alpha \).

In the first stage, \( w \) increases from \( w_0 \) to \( w_1 \), and household 1 (2) remains a net employer (employee), while household 2’s industrial employment increases from zero to \( JM \). In the second stage, \( w_1 < w < w_2 \), where \( w_2 \) is the real wage at which the amounts of industrial employment of the two households become equal, i.e., \( RS = ST \). So during stage 2, while household 2’s industrial employment rises continuously from \( JM \) to \( ST \), household 1’s industrial employment rises from zero to \( RS \). At \( w_2 \), total industrial employment amounts to \( RT \). Stage 3 begins when \( w \) rises beyond \( w_2 \).
In stage 1, \( \alpha - l_1 < 0 \) and \( 1 - \alpha - l_2 > 0 \), \( A \) is negative as \( y_1 > y_2 > 0 \), so expression (9) is negative. In stage 2, it is obvious from Figure 1 that \( 1 - \alpha - l_2 > \alpha - l_1 > 0 \). Given \( y_1 > y_2 > 0 \), this again implies that \( A \) is negative, as is expression (9). For stage 3, it is hard to tell the sign of \( A \) from Figure 1; algebraic derivation is needed.

From the definition of \( A \) in (9), and form (2), (4) and (5),

\[
A = [a_2 l_2^\beta + w(1 - \alpha - l_2)](\alpha - l_1)
- [a_1 l_1^\beta + w(\alpha - l_1)](1 - \alpha - l_2)
= a_2 l_2^\beta (\alpha - l_1) - a_1 l_1^\beta (1 - \alpha - l_2)
= (\beta / w)^{\beta/(1 - \beta)} [a_2^{1/(1 - \beta)} (\alpha - l_1) - a_1^{1/(1 - \beta)} (1 - \alpha - l_2)]
= (\beta / w)^{\beta/(1 - \beta)} \Omega
\]

where \( \Omega = a_2^{1/(1 - \beta)} (\alpha - l_1) - a_1^{1/(1 - \beta)} (1 - \alpha - l_2) \).

From (2)

\[
dl_i/dw = a_i^{1/(1 - \beta)} d[(\beta / w)^{1/(1 - \beta)}] / dw, \quad i = 1, 2
\]

\[
(dl_1 / dw) / (dl_2 / dw) = a_1^{1/(1 - \beta)} / a_2^{1/(1 - \beta)}.
\]

Therefore

\[
dl \Omega / dw = -a_2^{1/(1 - \beta)} (dl_1 / dw) + a_1^{1/(1 - \beta)} (dl_2 / dw) = 0
\]

meaning that \( \Omega \) does not change with \( w \). In stage 1, \( 1 - \alpha - l_2 > 0 > \alpha - l_1 \), \( \Omega \) is negative. As \( w \) rises, \( \Omega \) will remain negative. Given \( (\beta / w)^{\beta/(1 - \beta)} > 0 \), this implies that \( A \) will also remain negative through all stages of increases in \( w \), and therefore expression (9) is always negative. Q.E.D.
Proposition 1 says that the relative inequality of earned income between the two households will definitely decline in the process of continuous industrialization. The reason is intuitive. Household 1 is initially richer in both land \((a_1)\) and labor \((\alpha)\), but \((A6)\) dictates that land is more unequally distributed than labor. With industrialization and the accompanying rising demand for labor, the relative price of labor \((w)\) rises while that of land falls correspondingly. Hence the value of the asset that is less unequally distributed rises relative to that of the asset that is more unequally distributed, and the relative inequality falls as a result.

The availability of off-farm industrial employment is therefore the key equalizing factor. This factor will not really show up in models that are based on the dichotomy of industrial and agricultural incomes, and does not show up in models that consider only the functional distribution of income between the sum of labor and land income on the one hand and capital income on the other. Proposition 1 reveals the importance of the effects of such employment because it looks at the relative inequality between households within the category of earned income.

The conclusion of Proposition 1 does not change when the increase in population is allowed. Specifically, one can replace \((A1)\) with the following:

\((A1)'\) Total population rises over time, but industrial expansion is strong enough to absorb the additional labor supply, so the real wage remains on a rising trend.

Corollary 1 then follows:

**Corollary 1**

If \((A1)'\) and \((A2)-(A7)\) are true, relative inequality of earned income between the two households declines over time.

**Proof** (See Appendix I)

It is worth noting here that while \((A1)'\) allows the total population to increase, the percentages of the population belonging to the two households, \(\alpha\) and \(1 - \alpha\), remain unchanged. In the case of
homogeneous labor, such an assumption in addition does not allow any
difference in the rate of improvement in the quality of labor between
the two households.

Another assumption that can be relaxed is the lack of agricultural
technological progress or changes in the relative price. In the former
case, $a$ can be redefined to incorporate improvements in technology, so
it becomes a strictly increasing function of $t$. In the latter case, $a$
can be redefined to incorporate relative prices, that is, $a_1^{it}$ becomes
agricultural output in terms of industrial products, while $w$ becomes the
real wage in terms of the same products.

Let us now consider the case of a fall in $a_1$ and $a_2$ by the same
proportion, $\gamma$, caused by a fall in the relative price of agricultural
products, which, in turn, is caused by changes in the world relative
prices (if the economy in question is a small open one), or by the
evolving demand factors (if the economy in question is large or closed),
i.e.,

(A8) When $a_1$ and $a_2$ fall, the newly released labor from
agriculture is fully absorbed by the continuously expanding industry, so
$w$ keeps rising.

It is shown in Appendix II that the corollary below is true:

Corollary 2

If assumptions (A1)-(A8) are true, the relative inequality of
earned income between the two households declines over time.

It is evident from the proof in Appendix II that if $a_i$, $i = 1, 2$, rises
while $w$ remains unchanged, relative inequality of earned income will rise.
However, rising $a_1$ and $a_2$ along with an unchanged $w$ implies an
increase in agricultural employment, which is inconsistent with rapid
industrialization. So when $a_1$ and $a_2$ rise, a better assumption than that
of an unchanged real wage would be that of unchanged or falling
agricultural employment, i.e.,

(A8)' $a_1$ and $a_2$ rise, but the corresponding increases in demand for
agricultural labor is more than offset by the increases in demand for
labor from the rapidly expanding industry, so agricultural employment
continues its falling trend.
It can also be shown here that inequality falls as the economy grows, i.e.,

Corollary 3

If (A1)-(A7) and (A8)' are true, relative inequality of earned income between the two household declines over time.

Proof (See Appendix III)

It is worth adding that the rise in $a_1$ and $a_2$ could be caused by an increase in the relative price of agricultural products, or by agricultural technological progress. In the latter case, (A8)' requires that the same rate of improvement in agricultural technology is enjoyed by the two households.

Corollaries 1-3 reinforce the idea presented in Proposition 1. The idea is robust with respect to changes in total population as long as such increases are symmetric between the two households and the real wage keeps rising. It is robust with respect to a fall in the relative price of agricultural products as long as the real wage rate keeps rising. Finally, it is robust with respect to a rise in the relative price of agricultural products, or to agricultural technological progress as long as agricultural employment keeps falling, and in the case of agricultural technological progress, the increase in productivity is enjoyed by the two households symmetrically.

Also worth noting is that while the above propositions deal only with households that are farmer households to begin with, they also have implicit implication for nation-wide inequality. Although the model starts with two farmer households and zero non-agricultural activity, over time the involvement in off-farm employment becomes intensive. In government surveys, such as those in Taiwan, farmer households are classified as such if they fulfill a given criteria of minimum agricultural production. As off-farm employment grows, many farmer households will simply give up farming and will no longer be classified as “farmer households” in officially released statistics. That is, they “graduated,” and their non-agricultural activities are no longer called “off-farm” employment in official statistics.
However, this does not affect the implications of the current model. In the model, the "farmer households" are the only households the nation has, and all non-agricultural employment are "off-farm" employment. In other words, while in name the model deals with only farmer households and their off-farm employment, in reality it deals with the nation as a whole.

III. Functional Distribution of Income: The Capital Income Enters the Picture.

The last section is concerned with earned income only. To complete the analysis, the final category of income, namely, capital income or profits has to be introduced.

There are at least two ways of doing this. One way is to define a third household, a capitalist, who owns the entire capital stock and does not consume but only invests. On the other hand, households 1 and 2 referred to above can be assumed to have zero savings, so their earned income is their entire income. In this case, the economy-wide income distribution is then the distribution of income among the three households. If it is assumed that the income of the capitalist household is larger than either of households 1 and 2, and if the latter two households are grouped together under the title of "worker/farmers," overall inequality can be neatly decomposed into two parts as follows: (i) inequality between the capitalist group and the worker/farmers group, and (ii) inequality within the group of worker/farmers. The last section centers around point (ii), and an analysis is necessary for point (i).

The other way to introduce capital income is to assume that there are no households in the economy other than households 1 and 2, and that these two households save part of their income. If so, their total income becomes

\[ z_i = y_i + c_i, \quad i = 1, 2 \]  

(11)

3. In this case, the capitalist should be assumed to have foreign economic assistance to initially start the industrialization, and to maintain rapid industrialization which is needed for the rising wage and industrial employment whenever his own resource is not sufficient.
where $c_i$ is capital income of the $i$th household. If it is also assumed that household 1 always saves more than household 2, so that $c_1 > c_2$, it follows that $z_1 > z_2$ always. In this case, the distribution of $z$ can be decomposed into three parts as follows: (i)' the inequality between $y_1$ and $y_2$ (inequality of earned income), (ii)' the inequality between $c_1$ and $c_2$ (inequality of capital income), and (iii)' the functional distribution of income between $y_1 + y_2$ (earned income) and $c_1 + c_2$ (capital income). Part (i)' is the subject of the last section, and (ii)' will be discussed at the end of this section. What needs to be analyzed here is (iii)'. Analytically, this part (iii)' is exactly the same as point (i) previously referred to. It is the functional distribution of income. So no matter which of the two approaches described above one prefers, an analysis is necessary for the changes in functional distribution of income as an economy grows.

Bell (1979) has given such an analysis. His model will be introduced and then his assumptions will be generalized.

To facilitate the analysis, let us define some additional notation. Total worker/farmer income or $y_1 + y_2$ (earned income) can be written as

$$Y_L = wL_1 + pV_2$$ (12)

where $w$ (as before) is the real wage in terms of industrial products, $L_1 = N - L_1 - L_2$ or total industrial employment, and $p$ is the relative price of agricultural products in terms of industrial products, a variable not explicitly defined in the last section. Finally, $V_2 = x_1 + x_2$ or total agricultural production (ignoring intermediate consumption). The expression in (12) is the same as Bell's equation (3) (Bell, 1979, p.51) except that it is in terms of total rather than per capita units. Meanwhile the capital income, which Bell assumes to be zero, is absent.

If one assumes, as Bell does (p.50), that the industrial production function is linearly homogeneous, and defines the rental price of capital as $r$, it is obvious that total industrial production (ignoring intermediate consumption), defined as $V_1$, equals $wL + rK$, and capitalist income becomes

$$Y_K = rK$$ (13)
which is again basically the same as Bell’s definition (Bell, 1979, pp.52).

All of the assumptions used in Bell’s analysis of income distribution under the neoclassical mode can now be itemized as follows:

(BA1) \( V_1(K, L_1) \) is linearly homogeneous and well-behave (see Bell, pp.53m for details).

(BA2) \( V_2(H, L_2) \), where \( H \) is the exogenously fixed land input, is Cobb-Douglas (Bell, pp.56).

(BA3) Total population (labor) remains unchanged.

(BA4) There is no agricultural technological progress, nor are there changes in the terms of trade.

(BA5) The labor market is competitive.

(BA6) The industrial sector keeps expanding, \( w \) keeps rising.

Bell then establishes the following:

**Bell’s Theorem**

*If (BA1)-(BA6) hold, the ratio of worker/farmer income to capitalist income will decline, if the industrial elasticity of substitution, denoted by \( \sigma \) and defined as the absolute value of the percentage change in factor intensity \( (K/L) \) given a one-percentage change in the marginal rate of technical substitution between capital and labor, is greater than unity, and that ratio will first decline and then rise if \( \sigma \) is less than unity.*

Using a set of reasonable values of the parameters, he then shows that even when there is a turning point due to \( \sigma < 1 \), it will come late in the development process. So the theorem itself is at odds with the phenomenon described in the last section. However, it is obvious that many of the assumptions of the theorem are quite stringent and can be revised. One can show that when agricultural technological progress is allowed, as it should be if the model is to reflect reality in many developing economies, and if the demand for labor is still strong enough to warrant a non-declining agricultural employment, the picture instantly becomes more optimistic:

(BA4)' There is agricultural technological progress (or equivalently,
the agricultural relative price increases), the agricultural demand for labor rises; however, the rapidly expanding industry competes for labor and in the end \( w \) rises while \( L_1 \) and \( L_2 \) remain unchanged.

**Proposition 2**

If \((BA1) - (BA3), (BA4)'\), and \((BA5)\) are true, \( Y_L/Y_K \) declines if \( \sigma > 1 \), it remains unchanged if \( \sigma = 1 \), and it rises if \( \sigma < 1 \).

**Proof**

Assume that \( p \) (or agricultural productivity) rises by \( m \) times; a constant \( L_2 \) implies that \( w \) also rises by \( m \) times. Meanwhile, a constant \( L_1 \) implies that \( wL_1 \) rises by \( m \) times, so if \( \sigma > (<=) 1 \), \( rK \) rises by more (less) than \( m \) times, while if \( \sigma = 1 \), then \( rK \) rises by exactly \( m \) times.

Before \( p \) rises,

\[
Y_L/Y_K = (wL_1 + pV_2)/rK = (wL_1/rK) + (pV_2/rK) \tag{14}
\]

After it rises, it becomes

\[
Y_L'/Y_K' = (mpV_2/nrK) + (mwL_1/nrK) \tag{15}
\]

where \( n \) is the multiplier of \( rK \). It is clear that \( Y_K/Y_L \) falls (rises) or remains unchanged depending on whether \( n \) is greater (less) than or equal to \( m \), or equivalently, whether \( \sigma \) is greater (less) than or equal to unity. Q.E.D.

By introducing agricultural technological progress or a rise in the relative price of agricultural products here\(^4\), one immediately sees a possible reason for optimism: if \( \sigma < 1 \), the functional distribution of income will become less unequal. Imagine a situation where industry expands and \( w \) rises continuously. Meanwhile, agriculture enjoys technological progress or its relative price rises, while \( L_1 \) (\( L_2 \)) keeps

\(^4\) This would happen if the demand for industrial workers is strong enough, and if there is not enough foreign exchange to import food. Both scenarios are not unlikely for a fast growing developing economy.
rising (falling). One can regard the rise in \( w \) as comprised of two steps. In the first step, Proposition 2 is in order, \( L_1 \) and \( L_2 \) are unchanged, and the inequality of the functional distribution of income is reduced when \( \sigma < 1 \). In the second step, Bell's theorem is in order, and the inequality of the functional distribution of income rises at least in the initial stage of development. It is possible that the decrease in inequality in the first step is less than fully offset by its increase in the second step, so the overall effect is that it falls, even in the initial stage of economic development.

At the beginning of this section it is noted that there are two ways to introduce capital income. If the first way described there is adopted, the task of introducing possible optimism for changes in income distribution in the initial stage of development has now already been completed. From Proposition 1 and Corollaries 1–3, inequality between households 1 and 2 will be reduced throughout the development process under a variety of conditions. From Proposition 2, inequality between the capitalist group and the worker/farmers group may be reduced even in the initial stage of development if \( \sigma \) is less than unity. Taken together, these Propositions point to the possibility of a falling economy-wide inequality in the initial stage of development, under reasonable conditions.

If the second way of introducing profits is adopted, one still has to worry about the distribution of profits between households 1 and 2. This paper will not examine this problem; the issue has already been discussed elsewhere in similar settings. Chu (1986) describes the changes in the distribution of income of two households both of which are employed by the same (a single industrial) sector as well as save part of their income. It is shown there that given the constant savings rates of the two households, there will be a steady-state distribution of capital ownership, and the distribution of the capital income of the two households will converge to that steady-state configuration. The trend of changes in profit distribution over time then depends on the initial point. The trend would be for equalizing (unequalizing) if the initial distribution of capital ownership is more (less) unequal than the one at the steady-state.

In this paper, there are two sectors instead of one. So Chu's
model has to be revised to be directly applicable. However, since the result there is already ambiguous, it is unlikely that the ambiguity will vanish when an additional sector is introduced. Moreover, in the current model, as \( w \) rises to very high levels, asymptotically the agricultural sector vanishes, and the economy approaches the single-sector state described in Chu (1986). Given this, and all the possible configurations of the initial distribution of capital, it is even more likely that the trend of the distribution between \( c_1 \) and \( c_2 \) in (11) above will be ambiguous over time.

So again, the possibility of optimism is retained when the second way of introducing profits is adopted. Of the three parts of the decomposition of (11), part (i) sees a falling trend of inequality according to Proposition 1 and Corollaries 1–3 if industrial expansion is rapid enough to warrant a continuously rising real wage and industrial employment. The inequality in part (iii) may be reduced if there is agricultural technological progress or a rise in the relative price of agricultural products, and if the industrial elasticity of substitution is less than unity. Finally, the inequality in part (ii) will most likely follow an ambiguous trend. Together, their directions of change do not rule out a falling overall inequality, even in the initial stage of economic development.

IV. The Case of Taiwan: An Illustration

For the purpose of empirical application, of the two ways to introduce capital income into the analysis discussed earlier, the second way is obviously more useful. In reality when all of the households in an economy are grouped into two aggregate households of equal size, one rich and one poor, in all likelihood both households will have capital income. So in this section, attention is focused on the second way of dealing with capital income, i.e., equation (11).

Assuming \( c_1 < c_2 \) so that \( z_1 > z_2 \), that is, the household richer in earned income (agricultural and labor income combined) is also richer in capital income, so its total income is higher. Decomposition of the Gini index of inequality as described in Fei, Ranis and Kuo (1979) can be applied to (11):

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\[ G = \phi G_y + (1-\phi)G_c \]  

where \( G \) is the Gini of total income, \( \phi \) is the share of earned income in total income (the two households combined), \( G_y \) is the Gini of earned income, and \( G_c \) is the Gini of capital income.

This equation simply means that the inequality of total income measured by the Gini coefficient can be decomposed into the weighted average of the Gini coefficients of earned income and of capital income.

Differentiation (16) with respect to time gives

\[ \frac{dG}{dt} = \phi(\frac{dG_y}{dt}) + (1-\phi)(\frac{dG_c}{dt}) + (G_y - G_c)(\frac{d\phi}{dt}). \]  

The first two are changes in the Gini coefficients of earned and capital income respectively. The third term captures the effect of changes in the relative share of the two sources of income in total income. Arguments in the previous sections predict that if industrial expansion is strong enough, the first term on the right hand side of (17) will be negative, and the second and third terms are ambiguous in sign.

Let us now apply the above analysis to the case of Taiwan to see if the prediction of a falling inequality of earned income is borne out, and to see how much that term accounts for the decrease in overall inequality observed in Taiwan in the 1960s and 70s.

The source of the data is based on Tables 3.2 and 10.2 in Fei, Ranis and Kuo (1979). In those tables, overall inequality as measured

5. Actually only the data in the upper one-third of those tables are used. The lower two-thirds there refer to income distributions among the rural and urban households respectively. The reasons for such a selection are as follows: (i) This paper deals with aggregate households that are “rural” and “urban” at the same time; as such they correspond to the “all households” category or the upper one-third of the tables. (ii) The “urban” and “rural” households referred to in the lower two-thirds of the tables are not actually households that belong exclusively to a sector, as commonly defined in dualistic development models. According to the Survey of Personal Income Distribution in Taiwan Area, Republic of China, compiled by the Directorate General of Budget, Accounting and Statistics of that country, households are classified as “rural” if their agricultural activities fulfill a certain set of minimum requirements which are defined in terms of the land area they cultivate, the number of hogs they keep, etc., as noted earlier in the text. In reality, most of the “rural” households obtain part of their income from non-agricultural employment, and some of the “urban” households still derive income from agricultural

The data in Fei, Ranis and Kuo’s Tables 3.2 and 10.2 actually have four sources of income: agricultural (corresponding to our agricultural income), wage (corresponding to our labor income), property (corresponding to our capital income or profits) and miscellaneous income. In addition, the data are not based only on two aggregate households, so the ranking among the incomes of a given source may be different from that among total incomes. These two considerations imply that (16) should be written as

$$G = \Phi_aR_aG_a + \Phi_wR_wG_w + \Phi_eR_eG_e + \Phi_mR_mG_m$$  \hspace{1cm} (18)

where a, w, e and m refer to the four sources of income respectively, and R is the “rank correlation coefficient.” Again, the Gini coefficient of activities. The distinction is then a matter of degree. This is interesting statistically, but perhaps less so theoretically. (iii) As the non-agricultural sectors expanded rapidly in Taiwan, each year many “rural” households no longer met the above-mentioned minimum requirements and were reclassified as “urban” households. So the “urban” (and “urban”) households” in the Fei, Ranis and Kuo’s tables actually refers to different households groups between different years. That is, even if there is no natural increase in the number households groups between different years. That is, even if there is no natural increase in the number of households, so the “name tags” of all households remained unchanged between the years, the households that belong to the category of “rural households” in one year would not be the same as those belonging to the same category in another year. The number of households included in the “rural” group fell over the years. As a result, if one applies (17) to either of the two groups, it is unclear how to interpret the results.

6. When there are n households, define their total incomes as $z_1, z_2, \ldots, z_n$ such that $z_1 \leq z_2 \leq \ldots \leq z_n$. The ranking among incomes of a given source, say $y$, is said to be the same as that among total incomes if $y_i \geq y_j$ whenever $i > j$.

7. In footnote 6, suppose the ranking among incomes of a given source, $y$, is different from that among total incomes. Let the ranking (in ascending order) of $y$ be presented by a
total income is the weighted sum of the Gini coefficients of the different sources of income. In this case there are four sources: agricultural (a), labor (or wage, w), capital (c), and miscellaneous (m).

The Ginis in equation (18) above can be found in Fei–Ranis–Kuo’s Tables 3.2 and 10.2. All of the shares and the rank correlation coefficients are given in Table 10.2. Variable $G_m$ (the Gini of miscellaneous income) is not given directly but can be computed from the two tables as can its rank correlation coefficient, $R_m$.

To bring equation (18) closer to (16), one can combine the agricultural and labor incomes into one single category of earned income, $y$, as follows:

$$G = \phi_y R_y G_y + \phi_c R_c G_c + \phi_m R_m G_m$$ (19)

where

$$\phi_y R_y G_y = \phi_a R_a G_a + \phi_w R_w G_w.$$ (20)

One has to determine the values of $G_y$ (the Gini of earned income) for the two years under consideration, namely 1968 and 1972, to know how the argument in Section II fares in the case of Taiwan. The share parameter of earned income ($\phi_y$) is simply the sum of the shares of agricultural and wage income ($\phi_a + \phi_w$). On the other hand, there is no way to know $R_y$, the rank correlation coefficient, and $G_y$ (the Gini of earned income), when all one knows from Fei–Ranis–Kuo’s book is the multiplicative sum of the three terms on the left-hand-side of (20). However, it is known from Fei–Ranis–Kuo’s Table 10.2 that the rank correlation coefficient between wage income and total income ($R_w$) equals unity for both 1968 and 1972. In addition, the following observation shows that the rank correlation coefficient between earned income and total income ($R_y$) necessarily lies between $R_a$ (rank correlation coefficient between agricultural income and total income) and

vector $(r_1, r_2, \ldots, r_n)$, which is a permutation of $(1, 2, \ldots, n)$, for households $1, 2, \ldots, n$. Then the rank correlation coefficient is the correlation coefficient between $(r_1, r_2, \ldots, r_n, \ldots, r_n)$ and $(1, 2, \ldots, n)$.

8. $R_m = 1 - R_a - R_w - R_c$. In addition, "the estimated total Gini" = $\sum \phi_i G_i$, $i = a, w, c, m$. Given the estimated total Gini, $\phi_i$, $i = a, w, c, m$ and $G_i$, $i = a, w, c$ in the two tables, $G_m$ can be computed.
unity:

Observation

If the ranking among incomes from a given source is the same as that of total income, when one adds that income source to any other source of income, the rank correlation coefficient between total income and the sum of these two sources of income will be less than (or equal to) unity and greater than (or equal to) that between total income and the latter source of income.

Proof (See Appendix IV)

Table 1 gives the values of variables in (18) for the two years. The above observation implies that for 1968, 0.979 \leq R_y \leq 1, and for 1972, 0.958 \leq R_y \leq 1. Given (20) and the fact that a fall in \( R_y \) is favorable to the theme of this paper, the assumption that in 1968 \( R_y = 0.979 \) (1) and in 1972 \( R_y = 1 \) (0.958) would be most (least) favorable to the argument. From (20) and Table 1, for 1968 \( \phi_y R_y G_y = 0.176 \) and for 1972 \( \phi_y R_y G_y = 0.164 \). Given that \( \phi_y \) equals 0.659 and 0.692 respectively for 1968 and 1972, \( R_y G_y \) equals 0.267 and 0.237 for the two years respectively. If \( R_y \) equals 0.979 (1) and 1 (0.958) respectively for 1968 and 1972, \( G_y \) would be equal to 0.273 (0.267) and 0.237 (0.247) for the two years. The above two cases can be called Scenarios 1 and 2 respectively, with the former (latter) being the most (least) favorable to the argument. In both cases, as the Propositions in Section II above predict, the Gini of earned income falls as the economy grows.

One would also be interested in knowing how much the fall in the Gini of earned income contributes to the fall in the inequality of total income. It is obvious that equation (19) can be differentiated in the same way as (16) (see equation (17)). The details are given in Table 2. Here it suffices to say that difference in the Gini coefficient of total income between the two years can be decomposed into the effects of changes in (1) the Gini of earned income, (2) the Gini of capital income, (3) the Gini of miscellaneous income, (4) the rank correlation coefficient between earned income and total income, (5) the rank correlation
coefficient between capital income and total income, (6) the rank correlation coefficient between miscellaneous income and total income, (7) the share of earned income in total income, (8) the share of miscellaneous income in total income, and (9) a residual term.

Table 2 reports the actual decomposition of the Gini of total income for the two scenarios previously described. The residual term is less than 0.3 percent in both scenarios. The contribution of changes in the Gini of earned income to changes in the Gini of total income is 34.00 percent at the lowest (Scenario 2) and 64.07 percent at the highest (Scenario 1). The contribution of changes in the share of earned income in total income ranges form 17.32 percent to 17.35 percent. These results are rather favorable to the arguments presented in the above sections. The Gini of earned income falls as predicted in Section II, and it explains an important portion of changes in the inequality of total income. The share of earned income rises, bearing out the possibility of optimism given in Proposition 2 in Section III. It explains about 17 percent of the changes in the inequality of total income.

Several other observations give additional support to our argument:

(i) Employment in the primary sector in Taiwan generally followed a falling trend between 1964 and 1980, during which period income inequality was also falling, according to the 1989 Report on Survey of Personal Income Distribution in Taiwan Area, Republic of China. According to the 1990 Taiwan Statistical Data Book, primary employment was 1,810,000 persons in 1964 and 1,277,000 persons in 1980. Such an absolute fall in primary (mainly agricultural) employment is consistent with the assumptions made in Sections II and III above.

(ii) According to Galenson (1979, pp.415) and to the 1990 Taiwan Statistical Data Book, real monthly earnings in the manufacturing sector in Taiwan rose almost continuously between 1964 and 1980. Such a rise in the non-agricultural wage rate is also consistent with the assumptions needed for our argument.

(iii) According to the Report on Survey of Personal Income Distribution referred to above, in Taiwan the share of employees' compensation (mainly non-agricultural labor income) in total income rose sharply since 1964, and peaked off after 1976-81. Meanwhile the
share of agricultural proprietors’ income (the main source of agricultural income) fell sharply since 1961 but bottomed out after 1980. The contrast between the two trends is consistent with the picture presented in Sections II and III above. In addition, as the contrast has disappeared since 1980, so has Taiwan’s trend of falling inequality.

(iv) The existence of a freely competitive labor market is an important assumption behind our argument, and such an assumption is consistent with the description of Taiwan’s labor market as given in, e.g., Wu (1990).

V. Concluding Remarks

Using a model of economic development and income distribution and revisions of the neoclassical part of Bell’s model of income distribution, this paper tries to show that a fall in income inequality is possible even in the initial stage of economic development under reasonable conditions, given the expansion of off-farm employment.

The key assumptions behind our argument are a freely competitive labor market and rapid industrialization. The former eliminates the possibility of urban unemployment. It also breaks the dichotomy of sectors for which migration is an important issue. The households considered in this paper are “rural” and “urban” at the same time, as they have free access to employment in all sectors at all times. Households are distinguished not by the sector (or sectors) from which they derive their income, but by the initial ownership of land and labor with which they are endowed.

The assumption of rapid industrialization implies that the non-agricultural wage rate keeps rising and that agricultural employment falls absolutely. When these conditions are met, Section II above establishes an argument for falling inequality, under conditions such as a rise in total population, changes in the terms of trade, and improvement in agricultural technology.

The mechanism of falling inequality between the two households considered in Section II is actually very simple. The first household is richer than the second in both land and labor endowment. But the degree to which the first household is richer than the second in land
ownership is greater than that to which it is richer than the other household in labor ownership. During industrialization, with the expansion of off-farm employment, the scarcity of labor rises relative to land, and hence the relative inequality between the two households falls.

Such a mechanism explains why our conclusions are different from those of Bell’s neoclassical model, which is based on a similar set of assumptions about the labor market. The latter considers only the functional distribution of income. The models in section II consider instead the distribution of income between households. Furthermore, it is shown in Section III above that when Bell’s model is revised to take into account the possibility of agricultural technological progress (or a rise in the relative price of agricultural products), the possibility of falling inequality between the functional incomes (labor/agricultural income and capital income) cannot be ruled out.

When the labor market does not function in a neoclassical fashion, the optimistic picture painted in this paper will in general no longer hold. This explains the contrast between the situation of falling inequality presented here and the high probability of rising inequality (at least in the early stage of development) found in, e.g., Rauch’s and Bourguignon’s models, both of which are based, implicitly or explicitly, on an imperfect, segmented labor market.

The decomposition of Taiwan’s income inequality between 1968 and 1972 in Section IV above substantiates the argument emphasized in the earlier sections. Changes in the inequality of earned (the sum of agricultural and labor) income explain a good portion of the changes in the inequality of total income. Moreover, the share of earned income rose, bearing out the possibility of optimism expressed in our revisions of Bell’s theorem.

The application of the Propositions developed in this paper to the case of Taiwan is of course far from a complete explanation of what actually happened in the 1960s and 1970s in Taiwan. Even without considering the social and political factors, one still has to take into account the land reform carried out there in the 1950s, the predominance of small and medium-sized enterprises in manufacturing, and the improvements in infrastructure and in education, et. al., in addition to the factors considered here. These are the different aspects of a
The Effects of Off-farm Employment on Inequality

complicated picture, of which the argument presented in this paper is only one part. It is hoped that it is an important part of the complete picture.
Table 1

<table>
<thead>
<tr>
<th></th>
<th>1968</th>
<th>1972</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>0.3260</td>
<td>0.2897</td>
</tr>
<tr>
<td>(\phi_a)</td>
<td>0.1523</td>
<td>0.1027</td>
</tr>
<tr>
<td>(\phi_w)</td>
<td>0.5066</td>
<td>0.5895</td>
</tr>
<tr>
<td>(\phi_c)</td>
<td>0.2777</td>
<td>0.2577</td>
</tr>
<tr>
<td>(\phi_m)</td>
<td>0.0634</td>
<td>0.0501</td>
</tr>
<tr>
<td>G_a</td>
<td>0.1817</td>
<td>0.1105</td>
</tr>
<tr>
<td>G_w</td>
<td>0.2932</td>
<td>0.2604</td>
</tr>
<tr>
<td>G_c</td>
<td>0.4598</td>
<td>0.4235</td>
</tr>
<tr>
<td>G_m</td>
<td>0.3771</td>
<td>0.3335</td>
</tr>
<tr>
<td>R_a</td>
<td>0.9790</td>
<td>0.9580</td>
</tr>
<tr>
<td>R_w</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>R_c</td>
<td>0.9970</td>
<td>1.0000</td>
</tr>
<tr>
<td>R_m</td>
<td>0.9630</td>
<td>0.9710</td>
</tr>
</tbody>
</table>

Sources: (i) \(\phi_m\) is computed: \(\phi_m = 1 - \phi_a - \phi_w - \phi_c\) is also computed; the estimated total Gini in Fei, Ranis and Kuo (1979)'s Table 3.2 equals \(\Sigma \phi_i F_i, i = a, w, c, m\), and one can solve that equation for \(G_m\).
(ii) \(R_i, i = a, w, c, m\) are from ibid., Table 10.2 (pp. 354-5), where accuracy is up to three digits after the decimal point.
(iii) The rest are from ibid., Table 3.2 (pp. 92-3).
### Table 2

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th></th>
<th>Scenario 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>%</td>
<td>Value</td>
<td>%</td>
</tr>
<tr>
<td>( \Delta G )</td>
<td>-0.0363</td>
<td>100.00*</td>
<td>-0.0363</td>
<td>100.00*</td>
</tr>
<tr>
<td>(1)( \phi_y R_y \Delta G_y )</td>
<td>-0.0233</td>
<td>64.07</td>
<td>-0.0123</td>
<td>34.00</td>
</tr>
<tr>
<td>(2)( \phi_c R_c \Delta G_c )</td>
<td>-0.0097</td>
<td>26.73</td>
<td>-0.0097</td>
<td>26.73</td>
</tr>
<tr>
<td>(3)( \phi_m R_m \Delta G_m )</td>
<td>-0.0024</td>
<td>6.58</td>
<td>-0.0024</td>
<td>6.58</td>
</tr>
<tr>
<td>(4)( \phi_y G_y \Delta R_y )</td>
<td>0.0036</td>
<td>-9.96</td>
<td>-0.0073</td>
<td>20.10</td>
</tr>
<tr>
<td>(5)( \phi_c G_c \Delta R_c )</td>
<td>0.0004</td>
<td>-0.98</td>
<td>0.0004</td>
<td>-0.98</td>
</tr>
<tr>
<td>(6)( \phi_m R_m \Delta G_m )</td>
<td>0.0002</td>
<td>-0.48</td>
<td>0.0002</td>
<td>-0.48</td>
</tr>
<tr>
<td>(7)(R_y G_y - R_c G_c) \Delta \phi_y</td>
<td>-0.0013</td>
<td>17.32</td>
<td>-0.0063</td>
<td>17.35</td>
</tr>
<tr>
<td>(8)(R_m G_m - R_m G_m) \Delta \phi_m</td>
<td>0.0013</td>
<td>-3.57</td>
<td>0.0013</td>
<td>-3.57</td>
</tr>
<tr>
<td>(9)e</td>
<td>-0.0001</td>
<td>0.29</td>
<td>-1.0001</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Note: * Actual sum may not be 100.00 because of rounding.
Appendix 1

Let the total population (labor) be \( N(t) \), where \( t \) is time and \( N(0) = 1 \). Earned incomes of the two households become respectively

\[
y_1 = a_1 l_1^\beta - w l_1 + N \alpha w \tag{a1.1}
\]

and
\[
y_2 = a_2 l_2^\beta + N(1 - \alpha)w \tag{a1.2}
\]

where both \( N \) and \( w \) are strictly increasing functions of time.

Over time

\[
dy_1/\!\!dt = (N \alpha - l_1)(dw/\!\!dt)
+ \alpha w(dN/\!\!dt) + (\partial y/\!\!\partial l_1)(dl_1/\!\!dw)(dw/\!\!dt). \tag{a1.3}
\]

\[
dy_2/\!\!dt = [N(1 - \alpha) - l_2](dw/\!\!dt) + (1 - \alpha)w(dN/\!\!dt)
+ (\partial y_2/\!\!\partial l_2)(dl_2/\!\!dw)(dw/\!\!dt). \tag{a1.4}
\]

Since \( \partial y_i/\!\!\partial l_i = \partial(x_i - wl_i)/\!\!\partial l_i = dx_i/\!\!dl_i - w = 0 \), for reasons explained before in the proof of Proposition 1, (a1.3) and (a1.4) can be rewritten as

\[
dy_1/\!\!dt = (N \alpha - l_1)(dw/\!\!dt) + \alpha w(dN/\!\!dt). \tag{a1.5}
\]

\[
dy_2/\!\!dt = [N(1 - \alpha) - l_2](dw/\!\!dt) + (1 - \alpha)w(dN/\!\!dt). \tag{a1.6}
\]

Using (a1.5), (a1.6) and differentiating \( y_1/y_2 \) with respect to \( t \), one obtains

\[
d(y_1/y_2)/\!\!dt = A'/y_2^2 \tag{a1.7}
\]
where $A' = B_1 \frac{dw}{dt} + B_2 \frac{dN}{dt}$, $B_1 = y_2(N(1-a)-l_1) - y_1[N(1-a)-l_1]$, and $B_2 = [y_2^w w - y_1(1-a)w]$.  

From (1) and (2),

$$B_2 = [\alpha a_2^{1/(1-\beta)} - (1-a) a_1^{1/(1-\beta)}] \left( \frac{\beta}{w} \right)^{\beta(1-\beta)}(1-\beta)w. \tag{a1.8}$$

From (A6) $\alpha a_2^{1/(1-\beta)} - (1-a) a_1^{1/(1-\beta)} < 0$ so $B_2 < 0$, implying that if the additional labor supply is absorbed by industrial expansion without causing any change in $w$, the inequality falls.

Expression $B_1$ is the same as $A$ in (9) except for the presence of $N$. It can be shown that its presence does not affect the sign of $A$, so that $B_2$ is still negative. As $B_1$ and $B_2$ are both negative, so is $A'$, and so is expression (a1.7). Q.E.D.
Appendix II

When \( a_i \) (\( i=1,2 \)) changes but \( w \) remains constant, from (1), (2), (a1.1) and (a1.2),

\[
d(y_1/y_2)/dt = A''/y_2^2
\]  

(a2.1)

where \( A'' = \left[ y_2 (\beta/w)^{\beta(1-\beta)} a_{1}^{1/(1-\beta)} - y_1 (\beta/w)^{\beta(1-\beta)} a_{2}^{1/(1-\beta)} \right] \gamma \)
and \( \gamma = (da_1/dt)/a_1 = (da_2/dt)/a_2 < 0. \)

Again from (1), (2), (9) and (a1.1),

\[
A'' = \left[ (1 - \alpha) a_{1}^{1/(1-\beta)} - \alpha a_{2}^{1/(1-\beta)} \right] N w^{(1-2\beta)/(1-\beta)} \beta^{\beta(1-\beta)} \gamma.
\]

From (A6), \( (1 - \alpha) a_{1}^{1/(1-\beta)} - \alpha a_{2}^{1/(1-\beta)} > 0 \), so \( A'' < 0 \), and therefore expression (a2.1) is negative.

If \( w \) actually rises, the proof of Corollary 1 already indicates that a rise in \( w \) by itself reduces the inequality, so the equalizing force would be reinforced, and relative inequality has to fall. Q.E.D.
Appendix III

When \( w \) and \( a_i, i=1, 2, \) change, from (1), (2), (a1.1) and (a1.2),

\[
\frac{d(y_1/y_2)}{dt} = A'''/y_2^2 \tag{a3.1}
\]

where

\[
A''' = y_2(N\alpha - l_1)w[\frac{d\theta}{dw}/w] + y_2(\beta/w)^{2/(1-\beta)}a_1(1-\beta)\gamma
\]

\[
- y_1[N(1-a)-l_2]w[\frac{d\theta}{dt}/w]
\]

\[
- y_1(\beta/w)^{2/(1-\beta)}a_2^{1/(1-\beta)}\gamma.
\]

If \([d\theta/dw]/w = \gamma\), from (1), (2), (a1.1) and (a1.2), \(A''' = 0\). Meanwhile for any \( l_i^*, i=1, 2 \), if \( w = a_i\beta l_i^{*\beta - 1}, i=1, 2, \) \( w = we^{rt} \) and \( a^* = a_i e^{rt}, i=1, 2, \) then \( w^* = a_i^{*\beta} l_i^{*\beta - 1}, i=1, 2, \) meaning that agricultural employment remains unchanged if \( w \) and \( a_i, i=1, 2, \) change by the same proportion. So if \( w \) and \( a_i, i=1, 2, \) rise by the same proportion \( \gamma \), both agricultural employment and relative inequality remain unchanged. But \((A8)^*\) dictates that agricultural employment falls, so \( w \) has to rise by more than \( \gamma \). From the proof of Corollary 1, the inequality falls when this happens. Q.E.D.
Appendix IV

Let \( a = (a_1, a_2, \ldots, a_i, \ldots, a_n) \) be a certain type of income source, such as agricultural income, and that \( a_1 \leq a_2 \leq \ldots \leq a_i \leq \ldots \leq a_n \), where \( i \) refers to the \( i \)-th household. The ranking order of the \( n \) families in terms of total income is not necessarily the same as \( a \)'s. That ranking can be expressed in the order of the \( n \) households defined above as \( r = (r_1, r_2, \ldots, r_i, \ldots, r_n) \), which is a permutation of \( \lambda = (1, 2, \ldots, n) \). So while the \( i \)-th family's \( a_i \) is the \( i \)-th smallest, its total income is the \( r_i \)-th smallest among all total incomes.

A supporting gap is defined as \( a_i - a_j \geq 0 \) where \( i > j \) and \( r_i \geq r_j \) and a contradicting gap is defined as \( a_i - a_j \geq 0 \) where \( i > j \) and \( r_i < r_j \).

Total gaps are

\[
S_a^+ = \sum (a_i - a_j) \geq 0 \text{ for all } i > j \text{ and } r_i > r_j \tag{a4.1}
\]

\[
S_a^- = \sum (a_i - a_j) \geq 0 \text{ for all } i > j \text{ and } r_i < r_j \tag{a4.2}
\]

Let another source of income be \( b \). It is already assumed that the ranking among \( b \)'s is the same as that among total income. Suppose further that the sum of \( a \) and \( b \), call \( d \), adopts the same ranking as \( a \)'s, that is, \( d_i = a_i + b_i \geq d_j = a_j + b_j \) for all \( i > j \). Now

\[
S_d^+ = \sum (d_i - d_j) \geq 0 \text{ for all } i > j \text{ and } r_i > r_j \tag{a4.3}
\]

\[
S_d^- = \sum (d_i - d_j) \geq 0 \text{ for all } i > j \text{ and } r_i < r_j \tag{a4.4}
\]

It is obvious that if an element \((a_i - a_j)\) is in \( (a4.1) \), \( d_i - d_j \) or \((a_i - a_j)\) must be in \( (a4.3) \). Analogously, if \((a_i - a_j)\) is in \( (a4.2) \), \((d_i - d_j)\) must be in \( (a4.4) \).

For all elements \((a_i - a_j) + (b_i - b_j)\) in \( (a4.3) \), \( r_i > r_j \). Since the ranking among \( b \)'s is the same as that among total income, \( b_i > b_j \). This implies \((a_i - a_j) + (b_i - b_j) > a_i - a_j\) for all elements in
(a4.3). So \( S_d^+ > S_a^- \). Analogously, for all elements \( (a_i - a_j) + (b_i - b_j) \) in (a4.4), \( r_i > r_j \), and \( b_i > b_j \), and as a result \( S_d^- < S_a^- \).

\[
G_a^+/G_a = (S_a^+/n - S_a^-/n)/(1 + S_a^-/S_a^+)
\]

\[
= (1 - S_a^-/S_a^+)(1 + S_a^-/S_a^+).
\]

\[
G_d^+/G_d = (1 - S_a^-/S_a^+)(1 + S_a^-/S_a^+).
\]

(a4.5)

\[
G_d^+/G_d > G_a^+/G_a \text{ because } S_d^- > S_a^+ \text{ and } S_d^- < S_a^-.
\]

Suppose now that the addition of \( b \) to \( a \) changes the ranking among \( a \), instead of keeping it intact. Since the ranking among \( b \) is the same as that among total income, a change in the ranking necessarily involves some elements that are originally in (a4.2) being moved to (a4.3). For elements that are originally in (a4.1), that is, \( a_i > a_j \) and \( r_i > r_j \), \( a_i + b_i > a_j + b_j \) because \( b_i > b_j \), so they remain in (a4.3); they only become larger. With the original elements getting larger and with the entry of new elements, \( S^+ \) necessarily becomes larger, that is, \( S_d^+ > S_a^+ \).

For \( S^- \), the opposite is true. An element is in (a4.2) if \( a_i > a_j \) and \( r_i < r_j \). The latter implies \( b_i < b_j \). Therefore for all elements in (a4.4), \( 0 \leq d_i - d_j = (a_i - a_j) + (b_i - b_j) < (a_i - a_j) \). \( S^- \) necessarily becomes smaller because it loses some elements and the remaining ones all diminish. That is, \( S_d^- < S_a^- \).

So \( G_d^+/G_d > G_a^+/G_a \) whether or not the addition of \( b \) changes the ranking among \( a \).
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References


