Priority as a Contractual Development against Bankruptcy*

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In a three-period debt contract model, in which some investors can reschedule (namely, “banks”) and others cannot (namely, “the public debtholders”), the following results are established. If none of the debtholders can reschedule, priority structure is irrelevant to the firm’s equity value. When the entrepreneur is in a situation in which his project must be financed by both a bank and the public debtholders, the equity value is maximized by placing the bank in subordinate position to the public debtholders. Hence, this paper concludes that contract priority is relevant to the firm’s equity value in case the entrepreneur has to finance his project from a group of investors heterogeneous in terms of their ability to reschedule. This paper also addresses the issue of underinvestment and broadly presents an economic rationale for widely adopted socio-economic arrangements as the practices conducted to enhance the viability of beneficial economic relations.

I. Introduction

Priority is a debt structure that governs the order in which the firm’s value is distributed in financial distress to debtholders. The

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choice of priority must take account of the case in which the contract cannot be fulfilled as promised. This paper investigates how the decision is made on the distribution of the firm's value in financial distress, and explores the role of priority in debt contracts.

Bankruptcy is an institution an economy relies on for attainment of long run efficiency by its selection function for profitable businesses against unprofitable ones. More often than not, however, this supposed role is not adequately played. The reason is that under uncertainty ex ante efficiency does not necessarily imply ex post efficiency. Suppose that borrowers have private information about their types. If an equilibrium does not separate borrowers based on types, then lenders will suffer from being locked in the contractual relationship when it turns out that they contracted with bad borrowers. In this circumstance, as suggested by Dewatripont and Maskin (1990), investors decentralize the credit markets as a way of committing not to refinance, thereby screening out bad projects. Unfortunately, this institutional arrangement passes up long-term projects which are eventually valuable and yet yield inadequate short-term profits. If an equilibrium separates borrowers' types, signaling or screening costs must be borne by some economic agents. All in all, in both cases, underinvestment arises. In the case of symmetric information, the cause for underinvestment is the cost as associated with rescheduling: contracts may not be rescheduled even when the continuation value is greater than liquidation value. This paper follows this strand of symmetric information. Specifically, the model specification set up by Bulow and Shoven (1978) is adopted.

There are two types of investors in the present model depending on the costs incurred by actions taken after the initial contract is drawn up. In this paper, an "action" refers to one of the following two behaviors: (i) rescheduling when the firm is in financial distress, or (ii) profits verification when the entrepreneur does not truthfully report his profits. For the first type of investors, which are called "banks", costs incurred by these actions are small, and for the second type, which are called "the public debtholders", they are large. This paper assumes the cost differentials in a pronounced way: for banks, the cost incurred in actions (i) and (ii) are assumed to be zero, and for the public debtholders, they are assumed to be prohibitively high. The public

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1 It should be noted that a bank in this paper does not play a role of financial intermediation and therefore, is not subject to regulation. See Kareken (1986) or Pierce (1991) for intermediation. The issue of monitoring is not involved in this paper. Refer to Diamond (1984) and Williamson (1986) for this problem.

2 In reality, it is too costly for the public debtholders even to discuss bringing the case to courts for liquidation or reorganization. Since the bankruptcy court, as an institution for handling
debtholders can be envisioned as a large group of small investors, whose group action is virtually impossible.\textsuperscript{3} A conceivable case for the prohibitively\textsuperscript{3} high costs is the well-known "free-ride problem", which is by and large expected to happen if a great many people attempt to make a collective action. None of the debtholders will likely volunteer costly rescheduling efforts for the benefit that must be shared by a lot of people. All each debtholder does is to check if she is paid back as promised. Thus, if any debtholder is not repaid as promised, then the firm is not able to stay in business. In this paper, there are several banks in the economy who compete with one another for making their loans. Therefore, a bank cannot enjoy any rent on account of their ability to reschedule contracts. It is assumed that the public debtholders have the following characteristic. In financing his project, the entrepreneur can raise any unsatisfied portion of project funds from the public debtholders: that is, if the entrepreneur cannot get the full financing of his project from banks, the rest of the needed funds can be collected from the public debtholders.\textsuperscript{4} In this study, the entrepreneur's objective is to maximize the firm's equity value. Debt arrangement is determined by the entrepreneur, and investors accept the arrangement preferred by the entrepreneur in so far as the compliance with the entrepreneur's preference makes no difference to their expected return.\textsuperscript{5} Therefore, a contract is said to be optimal if it is a bankrupt firm, is an option that comes up after the parties put some efforts to rescheduling to no avail, the public debtholders can innocuously assume to be unable to take this action. This paper also implicitly assumes that the cost associated with the application of Chapter 7 of the U.S. Bankruptcy Code (for liquidation) is zero. Hence, when it comes to liquidation, the claimants do not have to make their choices whether they enforce it among themselves or through the court. As banks in this paper can verify profits with incurring no cost, there is no reason for the entrepreneur to lie to them for the purpose of taking some fraction of the firm's value to which he is not entitled in the process of liquidation. This implies that the claimants do not face a choice problem between the two routes (private and legal) to liquidation as long as contracts include a bank as a debtholder. In contracts with the public debtholders only, as shall be seen in section 4, the bankruptcy decision and the choice of priority are irrelevant to this problem. In models with costly legal and administrative procedures for liquidation, bankruptcy court costs can be thought of as the excess of the reorganization costs over the liquidation costs. In this case, however, the parties should make a choice between two types of liquidation. Nonetheless, this does not affect the qualitative results of this paper.

\textsuperscript{3} If the public debtholders are regarded as bondholders, the bond trustee could represent the bondholders to some extent, in which case rescheduling may not be entirely impossible. Still rescheduling would be costlier with the public debtholders than with banks. As long as this difference in rescheduling cost prevails, the qualitative results remain valid.

\textsuperscript{4} This flexible feature of the public debtholders makes arguments easier in the following.

\textsuperscript{5} The entrepreneur obtains loans from banks on a one-to-one basis, and makes an announcement to the public debtholders on how much it needs to borrow from them for the (partial or full) satisfaction of the investment fund.
the contract that maximizes the firm's equity value. It is shown in this paper that priority differentiates debtholders with respect to their ability to reschedule and enhances the probability of the firm working out of financial distress, thereby increasing the firm's equity value. This fact rests on the real-world observation that the firm's value as a going concern is greater than its liquidation value.\(^6\) Two kinds of priority structure are considered. If some debtholders are placed senior to others, the contract is said to have "absolute priority" structure. If no debtholder is placed senior or subordinate to any other debtholder, the contract is said to have "equal priority" structure.

This paper establishes the following results. If none of the debtholders are able to reschedule, then priority is irrelevant to the equity value of the firm. However, if the entrepreneur's project must be financed by both a bank and the public debtholders, it is optimal to place the bank in subordinate position to the public debtholders. This paper also addresses the issue of underinvestment. If all debtholders are incapable of rescheduling contracts, underinvestment happens because of the entrepreneur's inability to commit to make the promised repayments every time he affords to do so. Underinvestment still occurs when the entrepreneur is bound to finance his project from both a bank and the public debtholders. The reason in this case is that the public debtholders free ride on the benefit that arises from the rescheduling struck between the entrepreneur and the bank. This leakage of gains from rescheduling out of the rescheduling parties into the nonrescheduling ones discourages the former's willingness to reschedule. Therefore, rescheduling does not take place every time the firm is in financial distress and thus not all the investment opportunities with positive net present value are undertaken.

This paper is organized as follows. In section \(\text{II}\), an example is presented to clarify the subject of the paper. Section \(\text{III}\) introduces the model. In section \(\text{IV}\), contracts between the entrepreneur and the public debtholders are explored. Section \(\text{V}\) is devoted to the case in which the entrepreneur contracts with both a bank and the public debtholders. In section \(\text{VI}\), the related literature is briefly reviewed, an exposition of real-world practices is given to establish the general implication of the paper. Section \(\text{VII}\) offers concluding remarks.

\textbf{II. A Motivation Example}

Suppose that an entrepreneur has a project with the profits profile \((\bar{\pi}_1, \bar{\pi}_2)\) where \(\bar{\pi}_1\) and \(\bar{\pi}_2\) are independently distributed. Assume that

\(^6\) See Assumption 3(i) in the text.
\( \bar{x}_1 = 150 \) or \( 50 \); \( \bar{x}_2 = 100 \) or \( 90 \), each with the equal probability. The liquidation value of the firm is assumed to be 70% of the period 2 profits; that is, \( \bar{L}_3 = 0.7 \bar{x}_2 \). So the liquidation value of the firm at the end of period 1 is expected to be 70; i.e., \( \bar{E}L_2 = 70 \). In period 0, \( I \) is financed from two risk-neutral investors \( b \) and \( p \) where \( I \) is the fund needed to be borrowed for the project. Let \( R_i^t \) be the contract repayment to debtholder \( i \) in period \( t \) and \( \gamma^t \) be the total repayment which are actually made to \( i \).

**Case 1.** Suppose that both \( b \) and \( p \) do not reschedule. Let \( b \) and \( p \) have equal priority, and \((R_b^0, R_p^0) = (80, 0)\) and \((R_b^1, R_p^1) = (70, 90)\). Then it is easy to obtain that \( E\gamma^p = \frac{1}{2} \left[ \frac{80}{120} (50 + 70) \right] + \frac{3}{4}[80] = 60 \) and \( E\gamma^b = \frac{1}{2} \left[ \frac{100}{200} (50+70) \right] + \frac{3}{4}[70+90] = 120 \). Hence, if \( I \leq 180 \), then this contract can be implemented. An equivalent absolute priority contract can be established as follows. Assume that \( p \) takes the senior position. Let \((R_b^0, R_p^0) = (60, 0)\) and \((R_b^1, R_p^1) = (90, 90)\). Then it is still true that \( E\gamma^p = 60 \) and \( E\gamma^b = 120 \). An example showing the converse can also be easily constructed. In general, when investors cannot reschedule, there exists the equivalent absolute (equal, resp.) contract for any equal (absolute, resp.) priority contract.

**Case 2.** Suppose that \( b \) can reschedule and \( p \) cannot. Consider the same equal priority contract considered in case 1. Let \( \gamma^b_i \) be the liquidation payoff to debtholder \( i \). Similarly, \( \gamma^p_i \) is the rescheduling payoff to \( i \). Then \( \gamma^b_i = \frac{100}{200} (50+70) = 80 \) and \( \gamma^p_i = (−30)+90 = 60 \). Then there will be no rescheduling and the firm will be liquidated. Therefore, \( E\gamma^b \) and \( E\gamma^p \) are the same as in case 1. With this same contract repayment, let's put \( p \) in senior position. Then \( \gamma^b_i = (−30)+70 = 40 \) and \( \gamma^p_i = (−30)+90 = 60 \). Hence, rescheduling will take place and \( E\gamma^b = 110 \) and \( E\gamma^p = 80 \). Therefore, if \( 180 < I \leq 190 \), then an absolute priority contract can be implemented, but not an equal priority one. Notice also that \( E\gamma^b \) is less with absolute priority. This suggests that even when there is some limit in borrowing from the rescheduling lender, absolute priority contracts are feasible whenever equal priority contracts with the same contract repayments are. This point will be made clear in section V.

### III. The Model

This paper is proceeded with a three-period debt contract model. In period 1, the initial contract is drawn up: that is, the debt repayments \( R_i^1 \) are promised to be made in period \( t \) (\( t = 1, 2 \)) to debtholder \( i \) in return for her investment \( I^i \) in the entrepreneur's project.
i is \( b \) or \( b_j \) \( (j=1,2,\cdots,m) \) if she is a bank, and 1,\( \cdots \),\( n \) if she is one of the \( n \) public debtholders. For the public debtholders as a whole, superscript \( p \) shall be used. The entrepreneur's project yields profits \( \bar{\pi}_i \) in period \( t \) \( (t=1,2) \), which follow a distribution function \( F(\bar{\pi}_1, \bar{\pi}_2) \) on the support \( \mathbb{R}^2 \). \( F(\bar{\pi}_1, \bar{\pi}_2) \) is assumed to be common knowledge in period 1. The profits shall be denoted by \( \bar{\pi}_i \) when they are known, whereas \( \bar{\pi}_i \) are random variables. All investors are assumed to be risk neutral and the rate of interest is zero. By competition among investors, the entrepreneur can finance his project as long as the expected value of each investor's payoff is the same as her contribution to the firm.

The uncertainty is resolved in period 1.\(^7\) That is, \( \bar{\pi}_1 \) and \( \bar{\pi}_2 \) are known (to the financial markets as well as to the parties to the initial contract) at the end of period 1. Depending on the types of debtholders that the entrepreneur contracts with, and on the profits and the scheduled repayments, one of the following three outcomes shall take place in period 1: liquidation, rescheduling, and payback in full out of \( \pi_1 \). Unless the firm is liquidated, the firm proceeds on to period 2. Let \( \gamma_t \) denote the repayments that are actually made by the entrepreneur to debtholder \( i \) in period \( t \). \( \delta_t \) is the entrepreneur's equity return in period \( t \). \( R_t \) is the scheduled repayments due period \( t \) totaled over debtholders. \( \gamma_t \) is similarly defined. That is, \( \gamma_t = \sum_i \gamma_t^i \) and \( R_t = \sum_i R_t^i \). In addition, the following notational conventions are made. \( R' = R_1 + R_2 \), \( R = R_1 + R_b \), \( \gamma' = \gamma_1 + \gamma_2 \), \( \gamma = \gamma_1 + \gamma_2 \), and \( \delta = \delta_1 + \delta_2 \). In this model, the promised repayments to banks must be made if the entrepreneur makes enough profits, because they can costlessly verify the profits. The entrepreneur, however, is allowed to default on the promised repayments to the public debtholders, taking advantage of their inability to take profits verification actions. The following assumptions are made.

**Assumption 1. (Diversion and Residual Claim of Equity)**

(i) \( \delta_t > 0 \) only if \( \gamma_t = R_t \) \( (t=1,2) \)

(ii) \( \gamma_2 \leq \pi_2 \)

Assumption 1 says that if there are any profits remaining after the repayments are made in period 1, it will belong to the entrepreneur. It will not be saved for repayment in period 2. However, the entrepreneur

\(^7\) This information structure enables the rescheduling in period 1 to be a bargaining with complete information. That the period 2 profit is known in period 1 is not essential to the main result.
can get something only after debtholders are paid back in full. For what follows, let $\gamma_i^l$ denote the liquidation payoffs to debtholder $i$.

**Assumption 2.**

(i) (Priority)

(a) With absolute priority in initial contract, $\gamma_i^l > 0$ only if $\gamma_j^l = R'$ for all debtholders $j$ placed senior to $i$.

(b) With equal priority in the initial contract, $\gamma_i^l > 0$ for all $i$ with $R'/R$ being debtholder $i$'s claim fraction in liquidation.

(ii) (Debt Covenant)

Debtholders who make a refinancing loan in period 1 are ranked below the initial debtholders.

**Assumption 3. (Liquidation)**

(i) The liquidation value is less than the value as a going concern: that is, the liquidation value of the firm in period 1 is the fraction $\alpha \in (0,1)$ of the firm's value in period 2 as a going concern$^8$: that is, the liquidation value $L_2$ is $\alpha \pi_2$.

(ii) The firm cannot be partially liquidated.

(iii) Any debtholder, if not repaid in full, is able to enforce liquidation.

Assumption 2(i) states that when the firm under an absolute priority contract is liquidated, the proceeds are distributed to some debtholder only if all the debtholders senior to her are paid back in full for the promised reayments totaled over periods. In case of equal priority, $R'/R$ is the fraction for debtholder $i$. Assumption 2(ii) is a standard by which the old debtholders prevent the entrepreneur from diluting their claims through additional debt raising. Assumption 3(i) captures a usual circumstance firms encountered when they default on debt repayment. Assumption 3(ii) describes that the firm is valuable only in its entirety. In this model, the entrepreneur is the only part of the firm that can be separated from it. This happens in liquidation, which lowers its value from $\pi_2$ to $L_2$. Assumption 3(iii) describes that the firm

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$^8$ $\alpha$ is exogenously given.
is stopped from operation if it does not make the promised repayments and is at last liquidated in case there is no rescheduling.

IV. The Public Debtholders

This section examines how the bankruptcy decision is made and shows that priority is irrelevant to the firm's equity value if rescheduling is not possible. There are $n$ public debtholders and the following notations shall be used: $\gamma_i^r = \gamma_i^1 + \cdots + \gamma_i^n$ and $R_i^r = R_i^1 + \cdots + R_i^n$, and $I_i^r = I_i^1 + \cdots + I_i^n$. In this section, $R = R^r$ and $I = I^r$.

It should be noted that a debt contract with the public debtholders is "incomplete" in the sense of Grossman and Hart (1986). If a contract is incomplete in their sense, variables that the parties contract upon are not verifiable to the third party. As in this paper the profits verification cost is prohibitively high to the public debtholders, they cannot verify the truth even when the entrepreneur attempts to renege on the contract by on purpose defaulting on the contract repayments though it makes profits enough to fulfill the contract. In this circumstance, it is obvious that the entrepreneur will default on the contract repayments due the last period. The entrepreneur fulfills the contract only because he would be better off by having the continuous relations with these debtholders than by taking the money presently available at the expense of future benefit. Hence, there is no reason for the entrepreneur to make the promised repayments in the last period because there is no future that the entrepreneur has to be concerned about. In a three-period debt contract, in which debt repayments are scheduled in the last two periods (which are called period 1 and period 2), the only term of contract that is feasible between the entrepreneur and the public debtholders is short-term: that is, the promised debt repayments in the last period (period 2) is set to be zero. Hence, short-term contracts are the only feasible from: i.e., $R_i^r = R_i^1 > 0$ and $R_i^2 = 0$, where $i = 1, \cdots, n$.

The irrelevance of priority to the equity value is immediate since default on any debtholder's claim will lead the firm into liquidation regardless of her priority. Therefore, what is important to the entrepreneur is just $R_i^r$, the total repayment promised to the public debtholders as a whole. Lemma 1 shows how the decision is made in period 1.

**Lemma 1.** Suppose that the entrepreneur draws up a contract with the public debtholders. Then the following hold in period 1.
(i) The firm is liquidated if \( \min(\phi, \pi_2) < R^p \).

(ii) The entrepreneur makes the promised repayments in full if \( \min(\phi, \pi_2) \geq R^p \).

Furthermore,
\[
(\delta, \gamma^e) = \begin{cases} 
(\pi_1, L_2) & \text{if } \min(\phi, \pi_2) < R^p \\
(\pi_1 + \pi_2 - R^p, R^p) & \text{otherwise},
\end{cases}
\]

where \( \phi \) (or, \( \tilde{\phi} \), respectively) is a variable whose value is \( \infty \) if the entrepreneur has access to banks in period 1 for new debt raising, and \( \pi_1 \) (or, \( \tilde{\pi}_1 \), respectively) otherwise.

**Proof.** If the firm is liquidated, its equity is \( \pi_1 \). If the entrepreneur pays back in full or raises new debt, the equity is \( \pi_1 + \pi_2 - R^p \). Notice that \( \pi_1 + \pi_2 - R^p < \pi_1 \) if and only if \( \pi_2 < R^p \). If \( \pi_2 < R^p \) either with or without the ability to go to banks for additional loans, then the entrepreneur will choose liquidation and \( \delta = \pi_1 \). With absolute priority, there exists \( k < n \) such that
\[
\gamma^e = \begin{cases} 
R^i & \text{if } i \leq k-1 \\
L_2 - \sum_{i=1}^{k-1} R^i & \text{if } i = k \\
0 & \text{if } i \geq k+1
\end{cases}
\]

Then \( \gamma^e = \sum_{i=1}^{k-1} \gamma^i = L_2 \). With equal priority, \( \gamma^i = \frac{R^i}{k} L_2 \) for all \( i \). Then
\[
\gamma^e = \sum_{i=1}^{k-1} \gamma^i = L_2.
\]
Suppose that \( \pi_2 \geq R^p \). If \( \pi_1 \geq R^p \), then the entrepreneur affords the full repayments, which yields a greater equity than liquidation does. If \( \pi_1 < R^p \), then the entrepreneur can raise new debt from banks since \( \pi_2 \geq R^p - \pi_1 \). Hence if \( \pi_2 \geq R^p \) and the entrepreneur has access to banks for new debt raising, then the debt will be serviced and thus \( \delta = \pi_1 + \pi_2 - R^p \) and \( \gamma^i = R^i \) for \( i = 1, \ldots, n \). That is, \( \gamma^e = R^p \).

The implication that Lemma 1 carries is straightforward. If the future profits are large, then entrepreneur will try to fulfill the promise in order to capture them. If they are small, then the entrepreneur will choose to default. Understanding the inability of the public edbtholders to take a collective action against default, the entrepreneur has only to consider the total scheduled debt repayment \( R^p \). Hence, priority does not matter as will be stated in the ensuing Proposition 1.

In their seminal paper, Bulow and Shoven (1978) show that the bankruptcy decision may not be efficiently made as guided by the firm's liquidation and continuation values. This paper confirms part of their claims. That is, the firm can be led into bankruptcy even if its value as a going concern exceeds its liquidation value. This is shown in case (i)
of Lemma 1. Bulow and Shoven (1978) also claim that a financially distressed firm can remain in business even if its liquidation value is greater than its continuation value. Due to Assumption 3(i), this case is not examined in detail here. Nonetheless, it is easily shown that the bankruptcy decision, where the liquidation value is greater than the continuation value, depends on how the excess of the liquidation value over the contract repayments, \( L_2 - R^p \), is distributed (if \( L_2 > R^p \)): if this excess belongs to the entrepreneur, the firm is liquidated, and if this excess belongs to the debtholders, (i.e., if the debtholders hold the total value of the liquidated firm,) then the firm stays in business provided that the total promised repayments (\( R^p \)) are less than its continuation value (\( \pi_3 \)). Bulow and Shoven's argument, however, does not depend on who has claims on \( L_2 - R^p \). This loss of generality comes about from the assumption of this paper that the period 2 profit is known for certain in period 1. Consider the following example. Suppose that \( \pi_1 = 0 \), \( L_2 = 100 \) and \( R^p = 60 \). In the scenario of this paper, \( \pi_3 \) can be set to be a number known with certainty, say 80, at the time of the bankruptcy decision (period 1). In this case, the entrepreneur will choose liquidation, because the firm's equity value is 40 (=100 -60) in liquidation, whereas it is 20 (=80 -60) in continuance. Now suppose that \( \pi_2 \) is still stochastic in period 1, and assume that it is 160 or 0 with the same probability, so that the expected profit in period 2 is 80. This is in the same spirit as Example A of Bulow and Shoven (1978). Then the entrepreneur will choose continuation, because the expected equity value in continuation is 50 (= (160 -60)/2 +0/2). That is, if period 2 profit is stochastic in period 1, it is possible for the entrepreneur to invoke his limited liability by exploiting the case in which things go bad. All in all, this paper confirms Bulow and Shoven (1978): the bankruptcy decision cannot be efficient because it is made by only some (and not all) claimants\(^9\) to the firm's value.

**Proposition 1.** In a contract with the public debtholders in period 0, the firm's equity value does not depend on priority structure.

**Proof.** 

\[
\max_{R^p, \ldots, R^p} \int \delta(\bar{\pi}_1, \bar{\pi}_2) \, dF(\bar{\pi}_1, \bar{\pi}_2)
\]

subject to 

\[
\int \gamma(\bar{\pi}_1, \bar{\pi}_2) \, dF(\bar{\pi}_1, \bar{\pi}_2) = I,
\]

where by Lemma 1 \( \gamma(\pi_1, \pi_2) = \begin{cases} 
R^p & \text{if } \min(\phi, \pi_2) \geq R^p \\
L_2 & \text{otherwise}
\end{cases} \) and \( \delta(\bar{\pi}_1, \bar{\pi}_2) = \max(\bar{\pi}_1 + \bar{\pi}_2 - R^p, \bar{\pi}_1) \) with either absolute or equal priority. The

\(^9\) These claimants are usually referred to as "the equity-bank coalition".
entreprenuer chooses the same \( R^e \) regardless of priority and divides it into \( R^k \)'s in such a way that 
\[
\int \gamma^e(\tilde{\pi}_1, \tilde{\pi}_2) \ dF(\tilde{\pi}_1, \tilde{\pi}_2) = I^k \text{ for } k=1, 2, \ldots, n^{10}.
\]
Since the equity value is 
\[
E(\tilde{\pi}_1 + \tilde{\pi}_2) + \alpha \int_{\min(\tilde{\pi}_1, \tilde{\pi}_2) \leq R^e} \tilde{\pi}_2 \ dF(\tilde{\pi}_1, \tilde{\pi}_2) + \int_{\min(\tilde{\pi}_1, \tilde{\pi}_2) > R^e} \tilde{\pi}_2 \ dF(\tilde{\pi}_1, \tilde{\pi}_2) - I,
\]
the solution \( R^e \) is the smallest \( R^e \) that satisfies the constraint.

The non-renegotiability of contracts with the public debtholders costs the economy the problem of underinvestment as long as the firm's liquidation value is smaller than its value as a going concern because the equity value is less than \( E(\tilde{\pi}_1 + \tilde{\pi}_2) - I \). The source of this inefficiency is the lack of the entrepreneur's ability to commit not to make a fraudulent default on the contract repayments. This cost is borne entirely by the entrepreneur in this partial equilibrium model. That is, if \( \alpha \) is so small that
\[
E(\tilde{\pi}_1 + \tilde{\pi}_2) - I \text{ is negative and } E(\tilde{\pi}_1 + \tilde{\pi}_2) - I \text{ is positive, then the public debtholders would have made their loans elsewhere in the economy and still have earned the identical return. However, the entrepreneur loses the investment opportunity which is socially profitable. Therefore, in the partial equilibrium framework, the public debtholders successfully inflict the social costs on the entrepreneur which emanate from both their inability to reschedule contracts and his inability to commit not to make a fraudulent default.}

\section{A Bank and the Public Debtholders}

This section examines the case in which the entrepreneur is bound to raise debt from both a bank and the public debtholders. Let \( b \) be bank from which the entrepreneur obtains the loan \( \tilde{I} \). \( \tilde{I} \) cannot exceed \( I \), the maximum loan \( b \) makes to the entrepreneur.\(^{11}\) In this section, \( \tilde{I} \) is strictly less than \( I \): the entrepreneur finances at least \( I - \tilde{I} \) from investors who are unable to reschedule. It is clear from section 4 that the entrepreneur will not give up his access to \( b \) because the entrepreneur is able to increase the firm's equity value by obtaining refinancing from \( b \). The contracts only with the public debtholders allow the firm less equity value than the contracts with

\(^{10}\) The number of debtholders \( n \) and the contract repayment to each of them may be different depending on whether the contract has absolute priority structure or equal priority structure. In either priority structure, the entrepreneur finances his project under the constraint of each debtholders' lending ability.

\(^{11}\) For instance, the bank limits her investment in the entrepreneur's project to \( I \) in her effort to diversify from risks, considering the possibility of refinancing in financial distress.
both a bank and the public debtholders.

The following Lemma 2 states that if contract repayments, \( R^e \) and \( R^b \), are given, then it is Pareto optimal from period 1's viewpoint to place \( b \) in subordinate position to the public debtholders in the initial contract. This hinges on the fact rescheduling becomes easier with this priority structure. Rescheduling shall take place only if all parties to the contract are better off with it than without. The rescheduling debtholder's payoff in liquidation is smaller in this priority structure than in others. Then the party (as the only debtholder who is able to reschedule) is willing to reschedule for the future profits for which she would not if she were placed in senior or equal position to other debtholders. This makes greater the chance that firm operates in period 2.

**Lemma 2.** Suppose that the same \( R^e \) and \( R^b \) prevail regardless of the priority structure of the initial contract. If the bank is ranked subordinate to the public debtholders, then

(i) the probability of rescheduling is strictly higher, and

(ii) the equity value and the expected value of total repayments that will be made to the public debtholders (i.e. \( E\delta \) and \( E(\gamma^b + \gamma^e) \)) are both strictly higher than if the bank is in senior or equal position to the public debtholders.

**Proof.** Consider the following four cases: (i) \( \pi_1 \geq R \), (ii) \( R \leq \pi_1 + L_2 < R + L_2 \), (iii) \( R^e \leq \pi_1 + L_2 < R \), and (iv) \( \pi_1 + L_2 < R^e \). In case (i), the contract is enforced as promised. In case (ii), \( R < \pi_1 + \pi_2 \) and \( \pi_1 < R \). Then a new debt of \( R - \pi_1 \) is raised. Now consider case (iii). If \( \pi_1 + \pi_2 \geq R \), then a new debt of \( R - \pi_1 \) is raised. If \( \pi_1 + \pi_2 < R \), then the rescheduling parties to the contract, without having recourse to the financial markets, must decide on rescheduling or liquidation. Let \( \gamma^* \) (1), \( \gamma^* \) (2), and \( \gamma^* \) (e), be the liquidation payoffs to \( b \), if she is in senior, subordinate and equal position to the public debtholders, respectively. Then \( \gamma^* \) (1) = \( \min(L_2 + \pi_1, R^e) \), \( \gamma^* \) (2) = \( L_2 + \pi_1 - R^e \) and \( \gamma^* \) (e) = \( \frac{R^e}{\pi}(L_2 + \pi) \) by Assumption 2(i). In this case, \( \gamma^* \) (1) > \( \gamma^* \) (e) > \( \gamma^* \) (2). In rescheduling, \( b \)'s payoff would be \( \beta^b \pi_2 + (\pi_1 - R^e) \beta^b \) is the solution to the following two-person Nash Bargaining game between \( b \) and the entrepreneur:

\[
\max_{0 < \beta < 1} [\beta \pi_2 - L_2][(1-\beta)\pi_2 - 0].
\]

Then \( \beta^b = \frac{1}{2}(1 + \omega) \). The second term of \( b \)'s rescheduling payoff, \( \pi_1 \).
- $R^b$, is $b$'s payoff in period 1 if $\pi_1 \geq R^e$. If $\pi_1 < R^e$, then $R^e - \pi_1$ is the refinancing from $b$ necessary to pay back the public debtholders (and keeping the firm going). Rescheduling takes place if and only if $\frac{1}{3}(1 + \alpha)\tilde{\pi}_2 + (\pi_1 - R^e) > \gamma^b L(x)$, where $x = 1, 2, \text{ and } e$. Hence, rescheduling is the most probable if $b$ is placed in subordinate position. This implies that $E \delta(\tilde{\pi}_1, \tilde{\pi}_2)$ and $E[\gamma^b + \gamma^e](\tilde{\pi}_1, \tilde{\pi}_2)$ are both strictly larger with $b$ in subordinate position because $(1 - \beta^b)\pi_2 = \frac{1}{3}(1 - \alpha)\pi_2 > 0$ and $R^e + [\beta^e \pi_2 + (\pi_1 - R^e)] > L_2 + \pi_1$; i.e., both $\delta$ and $\gamma^b + \gamma^e$ are higher with rescheduling than in liquidation. Finally, consider case (iv). If $\pi_2 + \pi_3 \geq R$, then again the entrepreneur has access to the financial markets and can work out of financial distress. Suppose that $\pi_1 + \pi_2 < R$. Being locked in, the rescheduling parties find rescheduling to be the only way that keeps the firm going. In this case, $\gamma^b(1) = \min(R^b, L_2 + \pi_1)$, $\gamma^e(2) = 0$ and $\gamma^e(2) = \frac{R^b}{L_2 + \pi_1}$. Hence, $\min(\gamma^b(1), \gamma^e(2)) > \gamma^e(2)$. Analogous to case (ii), with $\beta^b = \frac{1}{3}(1 + \alpha)$ again, the probability of rescheduling, $E \delta(\tilde{\pi}_1, \tilde{\pi}_2)$ and $E[\gamma^b + \gamma^e](\tilde{\pi}_1, \tilde{\pi}_2)$ all attain the highest values if the bank is placed subordinate to the public debtholders.

An interesting point to note is that the term of contract between the entrepreneur and the bank does not matter under Assumption 2(i). By the incompleteness of contracts between the entrepreneur and the public debtholders, these parties will draw up a short-term contract, which means that the public debtholders have no claims in period 2. Hence, the bank's decision on rescheduling can be made on the basis of $R^b$ and $R^e$ only: the bank is not concerned with how $R^b$ is divided into $R^b_1$ and $R^b_2$. In other words, in case of rescheduling, the bank is the only debtholder remaining in the contract and her concerns are whether it can receive $R^b$ or not, and if not, how much valuable the firm would be. The next proposition establishes that the feasibility and optimality of an absolute priority contract in which the rescheduling debtholder is placed in subordinate position whenever contracts of other priority structures are feasible.

**Proposition 2.** In an optimal debt contract with both a bank and the public debtholders, the bank is placed in subordinate position to the public debtholders.

**Proof.** Suppose that there is an optimal debt contract with both a bank and the public debtholders, in which the bank in ranked either senior or equal to the public debtholders. It will be shown that there always exists a debt contract with the bank in subordinate position, which yields a higher equity value. In particular, a simple lowering of the bank's priority position
while preserving the scheduled debt repayments $R^b$ and $R^e$ as in the supposed contract suffices for this purpose. Notice that the supposed debt contract satisfies the constraint of limited access to rescheduling. Now lower the bank's priority as suggested above. Because $R$ is fixed, there is no change in repayment behavior for cases in which $\pi_1 \geq R$ or $\pi_1 + L_2 \geq R$. Thus investigation can be confined to cases in which $\pi_1 + L_2 < R$. (This is corresponding to cases (iii) and (iv) in the proof of Lemma 2.) Let $\gamma^i(x)$ be debtholder $i$'s payoff if $i$ is in position $x$, where $x = 1$ if $i$ is placed senior, 2 if $i$ is placed subordinate, and $e$ if $i$ is placed equal to $j$. ($i \neq j$ and $i, j = b, p$).

Lemma 2 states that $E\delta(\widehat{\pi}_1, \widehat{\pi}_2)$ becomes larger with this reshuffling of priority positions. Also, this reshuffling is feasible as shown in the following. ($\widehat{\pi}_1$ and $\widehat{\pi}_2$ of integrands are suppressed.)

(1) $I = \int [\gamma^b(1) + \gamma^e(2)]dF(\widehat{\pi}_1, \widehat{\pi}_2) < \int [\gamma^b(2) + \gamma^e(1)]dF(\widehat{\pi}_1, \widehat{\pi}_2)$ and

(2) $\int [\gamma^b(2)dF(\widehat{\pi}_1, \widehat{\pi}_2) \leq \int [\gamma^b(1)dF(\widehat{\pi}_1, \widehat{\pi}_2) = I^b \leq I$

if the bank is in senior position in the initial contract;

(3) $I = \int [\gamma^b(e) + \gamma^e(e)]dF(\widehat{\pi}_1, \widehat{\pi}_2) < \int [\gamma^b(2) + \gamma^e(1)]dF(\widehat{\pi}_1, \widehat{\pi}_2)$ and

(4) $\int \gamma^b(2)dF(\widehat{\pi}_1, \widehat{\pi}_2) \leq \int \gamma^b(e)dF(\widehat{\pi}_1, \widehat{\pi}_2) = I^b \leq I$

if the bank and the public debtholders are in equal position in the initial contract.

(1) and (3) directly follow from Lemma 2. (2) and (4) follow from three facts: (i) the bank in subordinate position reschedules for all values of $\pi_1$ and $\pi_2$ for which she would reschedule and for some values of $\pi_1$ and $\pi_2$ for which she would not if she were in senior or equal position, (ii) the supposed contract satisfies the constraint of limited access to rescheduling, and (iii) the rescheduling payoff to the bank, $\frac{1}{2}(1+\alpha)\pi_2 + \left[\pi - R^n\right]$, is identical in both priority structures. Hence, the contract which places the bank in subordinate position is feasible and maximizes the firm's equity value.

Proposition 2 establishes that the entrepreneur is able to work out of financial distress more easily by placing the rescheduling party in subordinate position and thereby, linking the firm's future operation more closely with rescheduling debtholder's future payoff.

In case some but all debtholders are able to reschedule, underinvestment takes place. The public debtholders, who are incapable of rescheduling contracts, free ride on the benefit that comes from the rescheduling struck by the entrepreneur and the bank. The benefit leakage
from the parties who reschedule to those who do not discourages the rescheduling parties from rescheduling to the extent that they would in absence of such nonrescheduling parties at the time of initial contract. Consider case (iv) in the proof of Lemma 2. Rescheduling condition is $\frac{1}{2}(1 + \alpha) \pi_2 > R^p - \pi_1$. Notice that $R^p - \pi_1 > 0$ in (iv). If $\pi_2$ is very small, rescheduling cannot take place. This drives the equity value of the firm down below $E(\tilde{\pi}_1 + \tilde{\pi}_2) - I$ because the firm dose not operate with probability 1 in period 2.

VI. Applications and Discussion

Bulow and Shoven (1978) find out that the bankruptcy decision may be socially inefficient because it is made by the participants in the rescheduling meeting (usually known as "the equity–bank coalition") because the rescheduling parties are concerned only with their own payoffs. Their paper, taking a firm's priority as exogenously given, does not consider how the bankruptcy decision affects the initial contract with regard to the firm's choice of priority structure. Hart and Moore (1990) argue that priority has a role in financial contracts in a circumstance that the manager wishes to enlarge the firm and thus undertake every investment opportunity regardless of its profitability. For the case in which the return from a new investment is deterministic and its cost is stochastic, they show that it is optimal to have a debt contract with priority. 13 The present study attempts to highlight the role of priority by eliminating the possibility of a sequence of fund raisings over time and in this respect, it differs from Hart and Moore (1990), which rests on the presence of future investment opportunities. 13

Fama (1985) notes that banks are distinguished from other financial intermediaries in the respect that they are required to hold non-interest-bearing reserves against deposits. This feature makes banks unique and bank loans are thus useful in reducing information costs. To make this feature more pronounced, bank loans are placed last or close to last in the line of priority among debtholders. The present paper provides a different kind of reason why banks should be placed below other debtholders, which is the reduction of the social cost emanating from inadequate rescheduling.

In line with both the information story of Fama (1985) and the rescheduling story of this paper, Hoshi, Kashyap and Scharfstein (1990)


13 In a situation which calls for a sequence of fund raisings, the debtholders will likely be assigned priority according to the order of the time they made their loans to the firm. See Fama and Miller (1972) and White (1980).
study the role of the socio-economic institution for firms in financial distress. Their empirical evidence is from Japan. They claim that firms with close financial relationships to banks and their trading partners can more effectively avoid the problems associated with financial distress.\footnote{In particular, they investigate the investment behavior of financially distressed firms.} Many firms in Japan has a financial arrangement in the industrial structure known as the "keiretsu", a group of firms pivoted around affiliated banks and financial institutions. Firms in the industrial groups also have strong product-market ties to each other which are strengthened by mutual share ownership. In this environment, it is easy for firms to work out of financial distress because typical free-rider problem is of less concern since the firm's securities are held by a few claimants, close relationship makes the claimants well-informed about the firm and its prospects, and financial distress is less likely since customers and suppliers of a firm, typically owning some of its equity, are more likely to continue their product market ties. Rather than focusing on the institutional arrangement, presented in this paper is contractual arrangement which makes financial distress easier to be overcome by firms. In particular, the arrangement of priority is explored in light of its role in providing the debtholders with an incentive to reschedule the initial contract, working the firm out of financial distress, and increasing its equity value.

Many people recognize that the distinction between debt and equity is getting more blurred and some even argue that in Japan debt and equity seem to switch their roles.\footnote{On one hand, the equitable subordination among debtholders are tantamount to metamorphosing debt into equity; and on the other hand, equity which does not utilize flexibility in financial payback as dividends are found to be fairly insensitive to contingencies.} In this respect, the absolute priority structure constructed in reverse order of rescheduling ability is an effective way of "equitifying" some portion of corporate debt. Sheard (1991) asserts that the Japanese Main Bank System seems to sustain by the implicit arrangement that makes the main bank mostly responsible for helping its firm out of financial distress. In this rescue operation, other banks remain fairly inactive. He suggests that this delegation system among banks can be made by placing the main bank in junior position. An implicit arrangement of "equitable subordination" among banks (with the main bank placed last in the line of priority) is believed to increase the viability of firms in financial crisis and the profitability of banks in a long-term relationship between firms and banks. With this arrangement, a firm virtually faces two types of debtholders: i.e., rescheduling ("banks") and nonrescheduling ("the public debtholders").

There are many practices which are based on the same spirit as the
voluntary subordination among the Japanese Main Banks. In actual businesses, for instance, a variety of guarantee programs provided by governments and bank are intended to reduce the anxiety that one party of transactions might hold against the other for default on the transactions. These practices lessen the likelihood of disruption in their relations and thus enlarge the scope for mutually beneficial transactions that otherwise might not take place. The substance lies in the presence of economic agents who are capable of capital injection as appropriate for economic damages and financial losses. Governments and banks which assume this role can be deemed to subordinate themselves of their own volition to any other stakeholders to prevent economically beneficial transactions from being passed up. This need may well be acuter for LDCs' firms whose credibility have not yet been established in the international markets. Governments and banks of LDCs that stand behind their firms in transactions with foreign firms can be thought of as a kind of equityholders who ensure to assume responsibility for default on contracts. Therefore, by the same logic, the guarantee programs preclude the occurrence of economically desirable transactions from remaining at the inadequate level.

VII. Concluding Remarks

This paper established absolute priority as a debt structure that maximizes equity value when debtholders are differentiated by their ability to reschedule. If none of the debtholders can reschedule, priority structure is irrelevant to the firm's equity value. If, however, the entrepreneur is constrained to finance his project from both a bank and the public debtholders, it is in the entrepreneur's interest to have an absolute priority contract, with the bank being placed in subordinate position to the public debtholders.

Although the arguments were presented concerning bankruptcy, the implication is fundamentally broad. In many economic circumstances, the economic viability can be enhanced by flexible agents being placed subordinate to those who are not. This paper lends itself to explaining the equitable and voluntary subordination among the Japanese Main Banks. This study also renders substance to many practices such as loan guarantee and letters of credit provided by banks and governments of the LDCs as devices to reduce the likelihood of disruption in transaction with firms of the LDCs.

References


