Agricultural Price Policy
in a Less Developed Economy
with Endogenous Urban Wages

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In less developed countries with endogenous urban wages, agricultural price liberalization raises rural incomes, but has an ambiguous effect on urban wages. Nevertheless, we show that the urban-rural labor income ratio unambiguously falls. Hence, price liberalization promotes rural development and diminishes labor income inequality. However, the price liberalization also discourages expenditures by urban firms on worker training and reduces urban employment and profits. Since urban employers and workers are politically powerful in poor countries, the latter findings help to explain the resistance to price liberalization policies often observed in these countries.

I. Introduction

Liberalization of agricultural prices in less developed countries has long been advocated by economists, but this policy recommendation has met with considerable resistance in many countries. The advice of the economists can be easily understood within the framework of the Harris-Todaro (1970) dualistic model when urban wages are exogenous. In this model, an agricultural price liberalization increases labor's value in the rural sector, which increases rural employment by siphoning workers out of urban unemployment. Moreover, urban employment is unaffected, and urban-rural labor income inequality diminishes. ¹

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¹ Basu (1984) gives a clear exposition of the Harris-Todaro model. The implications of agricultural price liberalization can readily be inferred from his Figure 6.1 (p. 73).
However, as this paper shows, the implication that an agricultural price liberalization leads to unambiguous benefits is contingent on the exogeneity of the urban wage. Due in large part to the work of Stiglitz (1974, 1976, 1982), considerable progress has been made beyond the Harris-Todaro (1970) assumption of an exogenous urban wage. Today, two major theories of wage determination, labor turnover theory and efficiency wage theory, have replaced the exogenous wage assumption. But the newer generation of models have not been used to re-examine the implications of agricultural price liberalization.

In this paper, we present a dualistic model of a less developed country which incorporates both costly labor turnover and efficiency wages in urban firms. In this model a higher agricultural price, which increases rural incomes, has an ambiguous effect on urban wages. Nevertheless, we can show that the urban-rural labor income ratio unambiguously falls. Hence, as in models with exogenous urban wages, agricultural price liberalization promotes rural development and diminishes urban-rural labor income inequality.

However, in contrast to models with exogenous urban wages, our model predicts that price liberalization will reduce urban employment. In addition, the policy discourages expenditures by urban firms on worker training, and reduces urban profits. Since urban employers and workers have substantial political power in many poor countries, these implications help to explain the resistance to agricultural price liberalization.

The organization of the paper is as follows. Section II explains the structure of our model. The policy implications of the model are derived in Section III and the Appendix. In the final section we offer some concluding remarks.

II. The Model

Since we wish to identify a group with strong economic incentives to resist agricultural price liberalization, we focus on urban firms in a dualistic, less-developed economy. Suppose urban firms produce a manufactured good, while the rural firms in the economy produce an agricultural good. The manufactured good is numeraire and \( p \) denotes the price of the agricultural good. The urban firms are wage setters, rather than wage takers (as in the Harris-Todaro model), and pay a wage \( w \). For

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2 Current versions of labor turnover and efficiency wage theories in less developed countries can be found in Batra and Lahiri (1988) and Isfahani and Selhei-Isfahani (1989).
simplicity, workers in the rural sector are treated as peasant farmers with incomes \((r)\) proportional to the price of the agricultural good, \(r = p\beta\), where \(\beta > 0\) and \(w > r\).

To avoid needless complications arising from sectoral differences in the composition of consumption, assume that the elasticity of substitution between agricultural and manufactured goods equals one in the utility function of each worker. Then the income shares spent on agricultural and manufactured goods, \(c_a\) and \(c_m\), respectively, will not vary by income or sector.\(^3\) Hence, real wage rates in the urban and rural sectors are given by \(w/(c_a p + c_m)\) and \(r/(c_a p + c_m)\), respectively, which implies that \(\Omega = w/r\) is the nominal and real labor income ratio between the urban and rural sectors.

To introduce labor turnover, we follow Basu (1984) by assuming that the quit rate at the representative urban firm is given by \(q = q(\Omega)\). Of course, the firm has no control over rural incomes \((r)\). But, given \(r\), each firm chooses \(w\) and hence \(\Omega = w/r\), which in turn determines the quit rate. Let the quit rate be decreasing and strictly convex with respect to the wage ratio, such that \(q_\Omega(\Omega) = \partial q/\partial \Omega < 0\) and \(q_{\Omega\Omega}(\Omega) = \partial^2 q/\partial \Omega^2 > 0\). Note that these restrictions on the quit rate, and the co-existence of a positive quit rate with \(w > r\), have been shown to be consistent with rational behavior by migrant workers who maximize utility rather than income, and derive utility from time spent in the rural sector, where most of their relatives are located in poor countries (Gruver and Zeager, 1990).

To incorporate the labor turnover and efficiency wage theories of wage determination into the model, we assume that output per laborer \((x)\) depends on three variable factors: the urban/rural labor income ratio \((\Omega)\), the level of training expenditures \((t)\) per laborer, and the number of laborers \((L)\) employed by the firm. That is, the firm's production function can be written as \(xL = x(\Omega, t, L)\).

Including \(\Omega\) in the production function captures the essential idea of efficiency wage theory — that an increase in the firm's wage relative to the alternative wage raises the productivity of workers at the firm. This might occur because increases in the wage ratio \((\Omega)\) raise the potential cost of shirking or stealing to workers, or improve the quality of job applicants.\(^4\)

\(^3\) One objection to this assumption is that it is inconsistent with Engel's law, but the assumption allows us to avoid some issues that would complicate the model to a great degree and contribute little toward the objective of this paper.

\(^4\) For a discussion of alternative motivations for efficiency wage models, see Weiss (1990), Carmichael (1990), and Lang and Shulamit (1990). Note that the nutrition version of efficiency wages, where labor productivity depends on the urban wage level \((w)\), rather than the
Assume that \( x \) increases at a decreasing rate with respect to \( \Omega \), i.e., \( x_\Omega = \frac{\partial x}{\partial \Omega} > 0 \) and \( x_{\Omega\Omega} = \frac{\partial^2 x}{\partial \Omega^2} < 0 \).

Since urban firms produce the manufactured good, which presumably requires more capital intensive production techniques than the agricultural good, worker training is a more important issue in urban firms. Thus, the presence of training per worker (\( t \)) in our production function helps to capture part of the labor turnover theory. We assume that training increases output per worker at a diminishing rate, that is, \( x_t = \frac{\partial x}{\partial t} > 0 \) and \( x_{tt} = \frac{\partial^2 x}{\partial t^2} < 0 \).

Of course, \( x \) decreases with respect to the number of laborers and total output (\( L_x \)) is concave with respect to the number of laborers, which implies that \( x_L < 0 \) and \( 2x_L + Lx_{LL} < 0 \). Since incentive effects of efficiency wages are unrelated to the training level chosen by the firm, we assume that \( x_{LL} = x_{t\Omega} = 0 \). Likewise, since the efficiency wage and labor turnover arguments are unrelated to the employment level, \( x_{L\Omega} = x_{QL} = 0 \) and \( x_{Lt} = x_{tL} = 0 \).

Because our primary interest in this paper is in the incentives for urban employers to oppose an agricultural price liberalization, we focus mainly on the behavior of these firms in our formal model. However, we do wish to point out that our model, along with an agricultural price fixed by the government, is consistent with a full general equilibrium system under the following assumptions.

Assume that a tariff on imports of the agricultural good accounts for the difference between the world price and the government-established domestic price. Then the product markets will clear, with international trade accounting for any difference between production and domestic uses. With regard to the labor markets, the Harris-Todaro migration equilibrium condition would ensure that equilibrium exists despite the existence of positive urban unemployment. Furthermore, none of our assumptions are inconsistent with equilibrium in the other factor markets.

Consider then the problem facing the representative urban firm, which is to choose the values of \( w, t, \) and \( L \) that maximize the expected present value of total profits (\( \pi L \)) over all future periods. This can be

urban-rural wage ratio (\( w/r \)), is not consistent with our model. For example, if \( w \) increases but \( r \) increases by a larger proportion, productivity would diminish in our model, but not in the nutrition version of efficiency wages. However, one can question whether the nutrition version is applicable to the urban sector (the high-wage sector), while the shirking version has been applied even to high-income countries (Akerlof and Yellen, 1986).

\(^5\) Note that under our formulation of the production function, the marginal product of labor is \( \frac{\partial L_x}{\partial L} = x + Lx_L \).
characterized as a dynamic programming problem. Let all future time from $T_0$ to infinity be divided into two portions: a small increment of time from $T_0$ to $T_0 + \tau$, and all remaining future time. Suppose all positions at the firm are filled with trained workers at time $T_0$.

During the time between $T_0$ and $T_0 + \tau$, a worker either quits (with probability $q\tau$) or remains with the firm (with probability $1-q\tau$). If the worker quits, the firm produces no output at that position from $T_0$ to $T_0 + \tau$, and must incur an expenditure of $\tau$ to train a new worker. Because the time horizon of the firm is infinite, the expected present value of profit subsequent to time $T_0 + \tau$ is $\delta\pi$, where $\delta = e^{-\mu\tau}$, and $\mu$ is the discount rate. However, if the worker stays, the firm receives a profit of $(x-w)\tau$ at that position between time $T_0$ and $T_0 + \tau$, and the expected present value of profit subsequent to $T_0 + \tau$ is again $\delta\pi$.

Applying the principle of optimality in dynamic programming, the problem for the firm can be written

$$\max_{w,\tau,L} \pi(w,\tau,L),$$

where

$$\pi(w,\tau,L) = (1-q\tau)((x-w)\tau + \delta\pi(w,\tau,L)L + q\tau[\delta\pi(w,\tau,L)-1]L).$$

Solving (2) for $\piL$, and taking the limit as $\tau \to 0$, we obtain\(^6\)

$$\pi(w,\tau,L)L = (x-w-qt)L/\mu.$$

The representation of $\pi$ in (3) has a simple interpretation. The expected present value of profit per period ($\piL$) equals the value of an infinite annuity, discounted at rate $\mu$. The annual payment from the annuity is revenue net of labor costs, all expressed in per period terms.

Substituting $\Omega p\beta$ for $w$ in (3), the problem for the representative urban firm becomes

$$\max_{\Omega,\tau,L} \piL = [1/\mu][x(\Omega,\tau,L)-\Omega p\beta-q(\Omega)\tau]L.$$

The first-order conditions for a solution to (4) are given by

\[^6\] Solving (2) for $\piL$ yields $\pi(w,\tau,L) = (1-q\tau)(x-w-\tau)eL/[1-\delta]$. Substituting $e^{-\mu\tau}$ for $\delta$ yields, $\pi(w,\tau,L)L = (1-q\tau)(x-w-\tau)eL/[1-e^{-\mu\tau}]$. Applying L'Hopital's rule and taking the limit as $\tau \to 0$, we obtain, $\lim_{\tau \to 0} \{[(x-w)-1]eL/[\mu e^{-\mu\tau}] = (x-w-qt)L/\mu$.
(5) \[ \frac{\partial (\pi L)}{\partial \Omega} = \frac{1}{\mu} \left[ x_{\Omega} p \beta - q_{\Omega t} \right] L = 0, \]

(6) \[ \frac{\partial (\pi L)}{\partial t} = \frac{1}{\mu} \left[ x_{t} - q(\Omega) \right] L = 0, \text{ and} \]

(7) \[ \frac{\partial (\pi L)}{\partial L} = \frac{1}{\mu} \left[ L x_{L} + \pi \right] = 0. \]

Rewriting first-order conditions (5), (6), and (7) as

(8) \[ x_{\Omega} / p \beta - q_{\Omega t} / p \beta = 1, \]

(9) \[ x_{t} = q(\Omega), \text{ and} \]

(10) \[ x + L x_{L} = w + q t, \]

their intuitive interpretation becomes clear.

The left-hand side of (8) reflects the two benefits to the firm of increasing \( \Omega \). First, labor productivity rises as the wage of workers increases relative to their alternative wage. Second, the rate of labor turnover declines as \( \Omega \) increases, which in turn reduces the turnover costs of the firm. Condition (8) says that to maximize profit the firm must balance these two marginal benefits against the marginal cost of increasing \( \Omega \). Condition (9) says that the firm must also balance the marginal benefit and marginal cost of additional training. Condition (10) simply states that the marginal product of labor \( (\partial L x / \partial L) = x + L x_{L} \) must equal the expected per-period cost per laborer, which is the sum of the wage and the expected training cost per period.

First-order conditions (5)-(7) are necessary and sufficient for an interior optimal solution to the problem of urban firms if the profit function is strictly concave in \( \Omega, t, \) and \( L \). This in turn requires that

(11) \[ \frac{\partial^2 (\pi L)}{\partial \Omega^2} = \mu^{-1} \left[ x_{\Omega \Omega} - q_{\Omega t} \right] L < 0, \]

(12) \[ \frac{\partial^2 (\pi L)}{\partial t^2} = \mu^{-1} x_{tt} L < 0, \]

(13) \[ \frac{\partial^2 (\pi L)}{\partial L^2} = \mu^{-1} \left[ 2 x_{L} + L x_{LL} \right] L < 0, \]

(14) \[ D = \left[ L / \mu \right]^2 \left\{ [x_{\Omega \Omega} - q_{\Omega t}] x_{tt} - (q_{\Omega})^2 \right\} > 0, \text{ and} \]

(15) \[ H = \mu^{-3} L^2 \left\{ [x_{\Omega \Omega} - q_{\Omega t}] x_{tt} - (q_{\Omega})^2 \right\} \left[ 2 x_{L} + L x_{LL} \right] < 0, \]

where \( D \) is the determinant of the Hessian matrix of second-order partial derivatives of the profit function with respect to \( \Omega \) and \( t \), and \( H \) is the
determinant of the Hessian matrix of second-order partial derivatives of the profit function with respect to \( \Omega, t \) and \( L \). The assumption imposed above ensure that (11) (12) and (13) hold. If we assume that (14) holds, then (15) also follows. Under these conditions the first order conditions are sufficient for an optimal solution to (4).

III. Policy Analysis

Individual urban firms have no power to influence labor incomes in the rural sector, but governments of less developed countries to intervene extensively in agricultural markets, which allows them to alter the distribution of income. In this model, the government alters rural incomes \( (r = p\beta) \) by exercising its influence on the price of the agricultural good \( (p) \). For example, many African governments use marketing boards to alter the prices received by food producers, and elsewhere price ceilings on food products are common.

Consider the consequences of government intervention which allows \( p \), and hence \( r \), to rise. Since urban wages are endogenous in our model, the increase in rural incomes leads to reactions by urban workers and firms: the workers' quit rates increase, while the firms adjust their wages, training, and employment to the new optimal levels. The directions of change necessary for these endogenous variables to attain the new equilibrium are derived in the Appendix.

In the Appendix we show (expression (A.10)) that the increase in the price of the agricultural good results in the following effect on the urban wage rate:

\[
(16) \quad \frac{dw}{dp} = \{ (\mu^{-1}\beta^2 px, L) / D \} + \Omega \beta.
\]

The first term of (16) is negative because of the strict concavity of output per person with respect to training, but the second term is positive. Thus, the increase in agricultural prices can lead to either an increase or a decrease in the firm's wage. On one hand, the firm may find the previous wage less effective with respect to worker efficiency and labor turnover, and respond by decreasing \( w \), which further reduces \( \Omega \). But, on the other hand, the firm may find it optimal to defend its wage premium by increasing \( w \), thus bringing the ratio \( \Omega \) back up to maintain output efficiency and hold down the training costs arising from turnover.\(^7\)

\(^7\) A simultaneous increase in urban wages and rural incomes may appear to be inconsistent with the balance of supply and demand in a general equilibrium context not made explicit.
However, even if the representative urban firm finds it optimal to increase its wage, the optimal percentage increase would be less than the increase in the rural wage. That is, \( \Omega = w/r \) will decrease for as we show in the Appendix (expression (A.6)),

\[
(17) \quad \frac{d\Omega}{dp} = \mu^{-1}\beta x_{\mu}L/D < 0.
\]

Recall that in labor turnover and efficiency wage models, each firm has the power to set its own wage optimally. But with identical urban firms, the wage paid by the representative urban firm can also be interpreted as the wage rate for the urban sector. Thus, equation (17) implies that a development policy which increases the agricultural price always results in a reduction in urban/rural labor income inequality.\(^8\) It is also important to note that this result holds with the labor turnover and efficiency wage mechanisms operating simultaneously.

Many economists would consider the increase in rural incomes, both absolutely and relative to urban wages, a desirable consequence of agricultural price liberalization, because rural poverty diminishes and labor income inequality in the economy declines. To show some potential reasons for resistance to price liberalization, we turn now to the predictions of the model with respect to the other choice variables of the firm, which are also derived in the Appendix. There, we show (expression (A.8)) that the agricultural price increase results in a lower optimal level of worker training by urban employers,

\[
(18) \quad \frac{dt}{dp} = \mu^{-1}q_{\Omega}\beta L/D < 0.
\]

Given that the optimal wage ratio declines, the quit rate will increase. Hence, it is not surprising that the optimal response by urban firms is to cut expected per-period training costs by decreasing the level of training per-laborer.

The urban firm's demand for labor will also decline in response to the increase in agricultural prices, as derived in the Appendix expression (A.9),

\[\text{here. However, there will also be losers as a consequence of an agricultural price increase.}\]

We show below that both profits and the demand for urban labor decline for each firm. Laborers shifting from urban to rural employment will experience a decrease in income if the new higher rural income is still below the old urban wage.

\(^8\) If a Harris-Todaro migration equilibrium condition, that is \( \Omega = w/r = \phi'(1/(1-u)) \), where \( u \) is the urban unemployment rate and \( \phi' > 0 \), as in Stiglitz (1974), were added to the model, then under most if not all versions of this hypothesis the decrease in the wage ratio would also lead to a reduced equilibrium urban unemployment rate.
(19) \[
d\ell / dp = -[\partial \pi / \partial p] / \{[1/\mu][2x_L + Lx_{LL}]\} < 0.
\]

Figure 1 is instructive in understanding (19). Note that first-order condition (10) can be rewritten as

(20) \[
\pi = -\mu^{-1}Lx_L.
\]

The curves labeled \(\pi^0\) and \(\pi^1\) in Figure 1 represent the level of per-laborer profit \((\pi = x - w - qt)\) at the firm for the optimal \(w\) and \(t\), before and after the increase in \(p\), respectively. The curve \(-\mu^{-1}x_L L\) is unaffected by the change in \(p\) because of the two independence assumptions, \(x_{LL} = x_{Lp} = 0\). The initial level of labor demand \((L^0)\) is set where \(\pi^0 = -\mu^{-1}Lx_L\), satisfying (20). To the left of \(L^0\), each additional worker will increase total profit, because the individual contribution of a new employee to total profits \((\pi)\) exceeds the reduction in total output (and profits) from existing employees \((-\mu^{-1}x_L L)\) caused by diminishing returns to labor.

Figure 1
The Effect of an Agricultural Price Increase on Urban Labor Demand

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Note that the slope of \(-\mu^{-1}Lx_L\), which equals \(-\mu^{-1}[x_L + Lx_{LL}]\), may be negative since \(x_{LL}\) may be positive; however, the production function strict concavity condition \([2x_L + Lx_{LL} < 0]\) ensures the slope of \(-\mu^{-1}x_L L\) will always be less negative than the slope of \(\pi\), which is \(\mu^{-1}x_L\). That is, the graph of \(\pi\) will always cut the graph of \(-\mu^{-1}Lx_L\) from above. It follows that even when the slope of \(-\mu^{-1}Lx_L\) is negative, \(L\) will fall with an increase in \(P\).
Now consider the implications in Figure 1 of raising \( p \), which increases \( r \). The per-laborer profit curve will shift down to \( \pi^1 \) because labor productivity declines and labor turnover costs increase. The new level of labor demand \( (L^1) \) is to the left of \( L^0 \), where \( \pi^1 = -\mu^{-1}Lx_L \) again satisfies (20). Therefore, Figure 1 provides a geometric depiction of condition (20) as well as an illustration of the way an increase in the agricultural price translates into a decrease in the firm’s demand for labor.

Finally, the agricultural price increase will also have a detrimental effect on the profit levels of urban firms. To prove this result, we need to use

\[
(21) \quad \pi(\Omega^1, t^1, L^1; p^1)L^1 < \pi(\Omega^1, t^1, L^1; p^0)L^1
\]

where the superscripts 0 and 1 indicate the optimal variable values associated with the initial and new \( p \), respectively, and \( p^1 > p^0 \). Given that the price of the agricultural good is lower on the right-hand side of (21), it must be true that the absolute rural income \( (r = p\beta) \), where \( \beta > 0 \) is also lower on the right-hand side. However, since the urban-rural labor income ratio \( (\Omega) \) is the same on both sides of (21), the urban wage must be lower on the right-hand side. Thus, since training \( (t) \) and employment \( (L) \) levels are the same on both sides, profits per laborer \( (\pi) \) must be higher on the right-hand side of (21).

Now, from the observation that the same set of decision variables \( (w, t, \text{and} L) \) are feasible both before and after the increase in \( p \) (i.e., all non-negative values are feasible) and the definition of a maximum, it follows that

\[
(22) \quad \pi(\Omega^1, t^1, L^1; p^0)L \leq \pi(\Omega^0, t^0, L^0; p^0)L^0,
\]

for all \( \Omega, t, \text{and} L \geq 0 \). In particular,

\[
(23) \quad \pi(\Omega^1, t^1, L^1; p^0)L^1 \leq \pi(\Omega^0, t^0, L^0; p^0)L^0.
\]

From (21) and (23) it follows that,

\[
(24) \quad \pi(\Omega^1, t^1, L^1; p^1)L^1 < \pi(\Omega^0, t^0, L^0; p^0)L^0.
\]

Therefore, optimal total per-firm profit must decrease as a result of the increase in the agricultural price. Furthermore, combining the results in obtained (18), (19), and (21) with the observation that urban employers and workers often have substantial political power in less developed countries, we see reasons why resistance to agricultural price liberalization arises in many countries.
IV. Conclusions

We have examined agricultural price liberalization in a dualistic, less developed economy with endogenous urban wages. A comparative static analysis demonstrates that, even though price liberalization may lead to an increase in urban wages, the urban-rural labor income ratio will unambiguously decline, as in models with exogenous urban wages.

However, in contrast to models with exogenous urban wages, our model predicts that an agricultural price liberalization will reduce urban employment. Moreover, the policy also discourages expenditures by urban firms on worker training, and reduces urban profits. Combining these results with the fact that urban employers and workers have substantial political power in many less developed countries helps to explain the resistance to agricultural price liberalization often observed in these countries.

Appendix

The Appendix contains the derivations of the comparative static results reported in Section III of the paper. Taking the total differential of first order conditions (5)-(7) and of the equation relating $\Omega$, $p$ and $w$, i.e.

$$(A.1) \quad \Omega p \beta - w = 0$$

with respect to $\Omega$, $t$, $L$, $w$, and $p$ yields:

$$(A.2) \quad \begin{bmatrix} \mu^{-1} [x_{\Omega t} - q_{\Omega t} - M^{-1} q_{aL} L - \mu^{-1} q_{aL}^L] - \mu^{-1} [x_{l} - q_{l} - q_{aL} - \mu^{-1} [(x_{\Omega t} - p \beta - q_{\Omega t}) + x_{\Omega L}]] & 0 & \frac{d\Omega}{dt} & \frac{\partial}{\partial \Omega} \\
-\mu^{-1} q_{aL}^L & \mu^{-1} x_{l}^L & \mu^{-1} [(x_{\Omega t} - p \beta - q_{\Omega t}) + x_{\Omega L}] & 0 & \frac{d\Omega}{dL} & \frac{\partial}{\partial \Omega} \\
0 & 0 & \mu^{-1} (2x_{l} + L x_{LL}) & 0 & \frac{d\Omega}{dw} & \frac{\partial}{\partial \Omega} \\
0 & \mu^{-1} (p \beta) & 0 & -1 & \frac{d\Omega}{dp} & \frac{\partial}{\partial \Omega} \end{bmatrix} = \begin{bmatrix} \beta \\
0 \\
\Omega \delta \\
-\Omega \beta \end{bmatrix}$$

First-order conditions (5) and (6) and the independence assumptions, $x_{\Omega L} = x_{aL} = 0$, imply that,

$$(A.3) \quad \mu^{-1} [(x_{\Omega t} - p \beta - q_{\Omega t}) + x_{\Omega L}] = 0$$

and,

$$(A.4) \quad \mu^{-1} [(x_{\Omega t} - q_{aL} + x_{aL}] = 0$$
Hence, the equation system (A.2) can be simplified and rewritten as,

\begin{equation}
(A.5) \begin{bmatrix}
\mu^{-1}(x_\Omega-\Omega t) L & -M^{-1}q_\Omega L & 0 & 0 \\
-\mu^{-1}q_\Omega L & \mu^{-1}x_{\mu} L & 0 & 0 \\
0 & 0 & \mu^{-1}(2x_{\mu} + Lx_{LL}) & 0 \\
p_{\beta} & 0 & 0 & -1 \\
\end{bmatrix} \begin{bmatrix}
d\Omega \\
dt \\
dL \\
dD \\
\end{bmatrix} = \begin{bmatrix}
\beta \\
0 \\
\Omega \beta \\
-\Omega \beta \\
\end{bmatrix}
\end{equation}

Application of Cramer’s method shows that \( d\Omega/dp < 0, dt/dp < 0, dL/dp < 0, \) while \( dq/dp \) is indeterminant.

First, consider the effect of the agricultural price on the urban/rural wage ratio,

\begin{equation}
(A.6) \quad \frac{d\Omega}{dp} = \frac{-\mu^{-2}(2x_{\mu} + Lx_{L}) \begin{vmatrix} \beta & -q_\Omega L \\ 0 & x_{\mu} L \end{vmatrix}}{-\mu^{-1}(2x_{\mu} + Lx_{LL})D} = \frac{\mu^{-1}x_{\mu} L}{D} < 0
\end{equation}

The denominator of (A.6) is the determinant of the 4x4 matrix in the above equation system (A.5). It is evaluated by expanding by the fourth column of the matrix. From (14) and (15) it follows that the value of this determinant is \(-H\), and under the assumption of concavity of the profit function, \( H \) must be negative. That is,

\begin{equation}
(A.7) \quad -\mu^{-1}(2x_{\mu} + Lx_{LL})D = -H > 0.
\end{equation}

The numerator determinant of (A.6) is also evaluated by expanding by column four. It follows that \( d\Omega/dp \) is negative since \( x_{\mu} < 0 \).

A similar approach yields the following results for \( dt/dp \),

\begin{equation}
(A.8) \quad \frac{dt}{dp} = \frac{-\mu^{-2}(2x_{\mu} + Lx_{LL}) \begin{vmatrix} (x_\Omega-\Omega t) L & \beta \\ -q_\Omega L & 0 \end{vmatrix}}{-\mu^{-1}(2x_{\mu} + Lx_{LL})D} = \frac{\mu^{-1}q_\Omega L\beta}{D} < 0
\end{equation}
and for $\frac{dL}{dp}$,

\[
(A.9) \quad \frac{dL}{dp} = \frac{-\Omega \beta D}{-\mu^{-1}(2x_L + Lx_{LL})D} = \mu \frac{\Omega \beta}{(2x_L + Lx_{LL})} < 0.
\]

The numerator of the expression for $\frac{dw}{dp}$ can also be evaluated by expanding by the fourth column of the matrix in (A.5); however, in this case the fourth column has been replaced by the right-hand-side vector resulting in two terms in the numerator of (A.10).

\[
(A.10) \quad \frac{dw}{dp} = \frac{(\Omega \beta D - \rho \beta^2 \mu^{-1} x_L L) \mu^{-1}(2x_L + Lx_{LL})}{-\mu^{-1}(2x_L + Lx_{LL})D} = \Omega \beta + \frac{\rho \beta^2 \mu^{-1} x_L L}{D}
\]

Since the first term of (A.10) is positive and the second is negative the sign for $\frac{dw}{dp}$ is an empirical question.

**Definition of Symbols**

Roman:
parameters:
- $c_u$ consumption budget share for agricultural good (rural good)
- $c_m$ consumption budget share for manufactured good (urban good)
- $D$ discount factor ($e^{-\mu t}$) or see below in variables
- $p$ price of rural output
- $T_0$ initial time

variables:
- $D$ determinant of Hessian matrix of second-order partial derivatives of the profit function with respect to $\Omega$ and $t$.
- $H$ determinant of Hessian matrix of second-order partial derivatives of the profit function with respect to $\Omega$, $t$, and $L$.
- $L$ workers per urban firm
- $q$ quit rate-probability of urban worker quitting per period
- $r$ rural wage (income)
- $t$ training expenditure per urban worker
- $w$ urban wage
- $u$ urban unemployment rate
- $x$ output per urban worker
Greek:
parameters:
\(\beta\) rural labor productivity coefficient \((r = p\beta)\); may be interpreted as either average product of rural labor (e.g. Lewis) or marginal product (neoclassical).
\(\mu\) discount rate

variables:
\(\pi\) profit per worker & per period
\(\tau\) small time increment
\(\phi'\) functional relation between wage ratio and inverse of urban employment rate \((\Omega = \phi'(1/(1-u)))\); may be positive constant.
\(\Omega\) urban-rural wage ratio (w/r)

References


———, "The Efficiency Wage Hypothesis, Surplus Labour, and the Distribution of Income in L.D.C.s,
