A Re-Examination of the Doctrine of Relative Purchasing Power Parity*

Elsworth D. Beach**, Nanch H. Cottrell***, and Noel D. Uri****

This paper investigates the doctrine of Relative Purchasing Power Parity. Mixed evidence is found supporting the concept when using a method analogous to that used by Lucas in testing the quantity theory of money. Relative Purchasing Power Parity is not consistently rejected in the long run between Canada and the United States and between Japan and the United States using quarterly data covering two separate periods 1957 QI – 1973 QII and 1973 QIII – 1989 QIV. Given the inconclusive results associated with relying on the methodology of Lucas, two alternatives are considered — first where the requisite smoothed time series are obtained via appropriate autoregressive integrated moving average filters and second where cointegration techniques are employed. In these instances, the results are unequivocal. Relative Purchasing Power Parity does not hold.

I. Introduction

The notion of Absolute Purchasing Power Parity holds that the equilibrium exchange rate (number of units of domestic currency per unit of a reference currency) is determined by the ratio of the domestic country to the price level of the country being used as the reference. Relative Pur-

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chasing Power Parity, on the other hand, asserts that the ratio of the equilibrium exchange rate in the current period, \( t \), to the equilibrium exchange rate in some base period is determined by the ratio of the domestic country's price index in period \( t \) to the reference country's price index in period \( t \), where both indexes are measured relative to the base period.

For a couple of reasons, Absolute Purchasing Power Parity is seldom realized in practice. First, from a practical standpoint, there is no perfect measurable index of prices on which to base the price level in the different countries for studying Absolute Purchasing Power Parity (Belongia (1986)). Second, considerations such as transportation costs, tariffs, and other impediments and barriers to trade also generally lead to a departure of the exchange rate from being exactly equal to the ratio of the countries' price levels (Davutyan and Pippenger (1990)). In the presence of transportation costs, the exchange rate is biased downward from what it would be if transportation costs were absent.

Relative Purchasing Power Parity relies on differences in the rate of inflation to explain deviations from Absolute Purchasing Power Parity. There are three fundamental problems with concept of Relative Purchasing Power Parity so that it might not hold in practice. (The implications of these difficulties are developed fully in Caves and Jones (1981), Crump (1925), Grennes (1984), and Keynes (1923)). First, there is the problem of non-traded goods. According to trade theory, commodity arbitrage should link the price of traded but not non-traded goods (Obstfeld and Stockman (1985)). This arises because competition in international markets exerts considerable centripetal force pulling together the prices of goods in different national markets while no such force operates on markets for non-traded goods. If this position is correct and if the price of non-traded goods in a country were to change with no change in the price of traded goods, then there would be a change in the inflation rate in that country without any corresponding change in the exchange rate. If inflation, however, is "everywhere a monetary phenomenon" as argued by most monetarists (e.g., Barro (1984) and Dornbusch and Fischer (1984)), then there is no reason that the price of non-traded goods should change without an equivalent change in the price of traded goods.

A second problem associated with Relative Purchasing Power Parity involves the impact of exogenous factors on the real values of variables as opposed to the effect on nominal values. Suppose, for example, that there is a technological breakthrough in Canadian agricultural production. Moreover, assume this development is applicable solely to Canadian production and not to comparable commodities produced in the United States. In this case there would be an increase in the supply of the affected
Canadian agricultural products (those impacted by the technological breakthrough) and a coincident increase in the demand in the United States for these Canadian products\(^1\) thereby causing a decrease in the dollar denominated exchange rate. There would be no reason, however, to expect any change in the relative rate of inflation between the two countries because the agricultural sectors in both are of modest size (Buiter and Eaton (1985) and Cooper (1968)). This is a second reason why Relative Purchasing Power Parity might not hold.

The final problem associated with Relative Purchasing Power Parity concerns capital movements and any associated impact on the rate of inflation. This is essentially an empirical question that has been addressed in a variety of studies. The research on this question has shown that capital movements can and do cause changes in the exchange rates without corresponding changes in the rates of inflation (Helliwell and Padmore (1985) and Levich (1985)). The implication of these studies is that there is no reason to expect a priori that Relative Purchasing Power Parity will hold in either the short run or in the long run.

More recent studies by, for example, Adler and Lehman (1983), Ardeni and Lubian (1991), Copeland (1991), Cumby and Huizinga (1991), Darby (1980), Frenkel (1981), and Hakkio (1982), using a variety of empirical approaches, have all demonstrated that both Absolute Purchasing Power Parity and Relative Purchasing Power Parity fail to hold in the short run. Whether they hold in the long run is another issue. A few studies find support for Relative Purchasing Power Parity. For example, studies by Galliot (1970) and Officer (1978, 1982a, 1982b) provide empirical verification of Relative Purchasing Power Parity in the long run. Unfortunately, the approach used by Galliot and Officer is problematic. First, they endeavor to identify a historical period in which Relative Purchasing Power Parity holds. They then compare empirical results from the historical period to the period of interest. The fundamental shortcoming of this approach is that there is no reason to believe, and the authors provide no convincing support for the belief, that long run Relative Purchasing Power Parity in fact held during the historical period.

One alternative to the approach used by Galliot and Officer is to use an atheoretical statistical test popularized by Lucas in his analysis of the quantity theory of money (Lucas (1980)). The Lucas approach has subsequently been used to examine such diverse issues as the relationship between money, inflation, and interest rates (Fisher (1987)) and the rela-

\(^1\) This coming about because of the changing relative prices of the agricultural commodities produced in Canada versus those produced in the United States.
tionship between inflation and exchange rates (Rush and Husted (1985)).

Of particular interest is the paper by Rush and Husted. This paper presents a test for bilateral and multilateral Relative Purchasing Power Parity between the Canada, France, Germany, Italy, Japan, Switzerland, the United Kingdom, and the United States. The multilateral tests were based on ad hoc trade weights which were used to derive "low-frequency" (Rush and Husted's term), smoothed series. The resulting series, however, are difficult to justify and interpret thereby effectively rendering the results unintelligible from both a statistical and a theoretical perspective. The bilateral tests, on the other hand, were not dependent upon trade weights and for that reason are ostensibly more credible. Using data covering 1954 Q1 to 1982 QIV, general support was found for long run Relative Purchasing Power Parity between the United States and the other countries. When bilateral comparisons were made that did not include the United States, however, Relative Purchasing Power Parity was rejected for the majority of comparisons.

It is unclear whether the bilateral results reported are an artifact of the statistical procedures used in applying the Lucas method or whether the results actually represent an application of the Lucas method to exchange rate behavior because there is a serious problem in the mechanics of implementing the Lucas method. Exchange rates series over the sample period (which covers both the period when exchange rates were fixed under the International Monetary Fund Articles of Agreement — prior to 1973 — and the period when flexible exchange rates were the norm — after mid-1973) are treated by Rust and Husted as being homogeneous. In fact they argue that "the use of data from the two exchange rate regimes is immaterial when testing Purchasing Power Parity, because rapid and pervasive commodity arbitrage and price equalization ensure PPP holds)" (p. 139). What was observed de facto was that by the beginning of the third quarter of 1973 (by which time most industrialized countries had abandoned their reliance on fixed exchange rates), fixed exchange rates were "in some cases far from their equilibrium levels, and traders (and governments) needed to guess what levels the market would find. Since then inflation rates have been both high and variable among countries, posing the question of how fully and rapidly exchange rates would change to adapt to them" (Caves and Jones (1981, p. 370)). The conclusion that one draws from this is that there might very well have been structural changes in the underlying relationships determining exchange rates. To ignore this and to proceed without a complete investigation of the possibility of such a shift casts suspicion on the conclusions of Rust and Husted. This will be rectified in what follows.
This study used three tests to address the issue of the efficacy of the Relative Purchasing Power Parity doctrine. The reason for using several tests will become apparent later. The first relies on a direct application of the Lucas technique to Relative Purchasing Power Parity.\(^2\) The second test filters out the noise in the exchange rate time series data and uses the resulting (noise-free) series to assess whether Relative Purchasing Power Parity holds on a bilateral basis, at least in the long run. The third and final test uses cointegration techniques.

II. The Lucas Atheoretical Approach\(^3\)

A. Overview

Lucas (1980) used an atheoretical approach to test two central implications of the quantity theory of money relating the rate of inflation and the nominal interest rate to the supply of money:

\[
\begin{align*}
P_t &= a_1 M_t + e_{p_t} \\
R_t &= a_2 M_t + e_{R_t}
\end{align*}
\]

where

- \(P_t\) = the rate of inflation in period \(t\)
- \(R_t\) = the nominal interest rate in period \(t\)
- \(M_t\) = the money stock in period \(t\)
- \(a_1\) and \(a_2\) are coefficients, and
- \(e_{p_t}\) and \(e_{R_t}\) are error terms in period \(t\).

As the values of the coefficients \(a_1\) and \(a_2\) approach one, a given change in the quantity of money induces a proportional change in the rate of inflation and in the nominal rate of interest, respectively. Lucas argues that the two quantity-theoretic propositions contained in equation (1) possess a combination of "theoretical coherence and empirical verification

\(^2\) At issue is whether it is possible to reach the same conclusions obtained by Rust and Husted when they applied the Lucas technique.

\(^3\) There are a number of obvious problems with the Lucas approach. For example, it is known that price levels and exchange rates are nonstationary and that their autoregressive representations contain a unit root (see below). Consequently, if two series share a common trend, an error correction term should be incorporated in the specification (Jones and Uri (1990)). However, to remain true to the original Lucas approach and to have the results obtained compatible with those of Rust and Husted for comparison purposes, the Lucas specification will be used as it was initially presented.
shared by no other propositions in monetary economics' (p. 1005). Theoretical coherence, as Lucas defines it, results from the suggestion that the quantity theory of money is a necessary condition for well-posed models of economies in which agents optimize their behavior and markets clear. Empirical verification relies on the belief that the quantity theory of money defines a steady-state situation and for that reason can be tested by observing the long run average behavior across economies.

Lucas uses a two-sided filter to test equation (1):

\[ X_{i,t} = \gamma \sum_{k=-\infty}^{\infty} \beta^{|k|} X_{i,t+k} \]

where

\[ k = \text{the absolute value of } k \]
\[ X_{p,t} = \ln(M_{t+1}) - \ln(M_t) \]
\[ X_{t,r} = \ln(P_{t+1}) - \ln(P_t) \]
\[ \gamma = \frac{1-\beta}{1+\beta} \quad 0 \leq \beta \leq 1. \]

The length of the lag, from negative infinity to positive infinity, is a theoretical requirement. From a practical perspective, however, the effects of distant values of \( X \) on the current period value are negligible. This results from the fact that \( \beta \) is greater than zero but less than one (Kmenta (1986)).

To gain further insights into equation (2), consider the following model:

\[ (1-L)^d m_r = a(L) \mu_r \]

where

\( L \) = the lag operator,
\( m_r \) = the natural logarithm of the money stock,
\( d \) = the number of roots in the lag process, and
\( \mu_r \) = a shock to the money supply.

The effect of \( \mu_r \) on \( m_r \) is

\[ m_r = [(1-L)^d]^{-1} a(L) \mu_r \]

where

\[ m_r = (1 + L + L^2 + L^3 + \ldots) a(L) \mu_r \]
\[ m_r = a(L) \mu_r \]
\[ \alpha = (1 + L + \ldots), \text{the impulse response polynomial.} \]
Given equation (4), the infinite-horizon forecast of \( m_t \) can then be defined as a function of the information set contained in values of \( \mu_t \):

\[
E[m_{t+k} | I_t] = (\alpha_k + \alpha_{k+1} L + \alpha_{k+2} L^2 + \ldots) \mu_t
\]

Furthermore, equation (5) can be used to analyze the effect of a permanent money shock, \( \mu_{t+1} \), announced at time \( t \) on the infinite-horizon forecast of the money stock. In this case the expectation of the infinite-horizon of the natural logarithm of the money stock, \( m_t \), is conditional on current information, \( I_t \), and the size of the future shock, \( \mu_{t+1} \):

\[
E[m_{t+k} | I_t, \mu_{t+1}] = (\alpha_k + \alpha_{k+1} L + \alpha_{k+2} L^2 + \ldots) \mu_{t+1}
\]

Combining equations (5) and (6) gives the revision in the infinite-horizon forecast due to \( \mu_{t+1} \):

\[
E[m_{t+k} | I_t, \mu_{t+1}] - E[m_{t+k} | I_t] = \alpha_{k-1} \mu_{t+1}
\]

Dividing both sides of equation (7) by the money shock, \( \mu_{t+1} \), gives

\[
\frac{E[m_{t+k} | I_t, \mu_{t+1}] - E[m_{t+k} | I_t]}{\mu_{t+1}} = \alpha_{k-1}
\]

Equation (8) suggests that the revision in the infinite-horizon forecast due to a permanent increase in the money supply depends on the impulse-response polynomial \( \alpha_k \).

Given these interrelationships, the next step is to examine the relationship between money and inflation. Consider the following bivariate system:

\[
a(L)(1-L)^d m_t = b(L)(1-L) \pi_t + e_{mt} \\
g(L)(1-L)^d \pi_t = c(L)(1-L) m_t + e_{\pi t}
\]

where

- \( m_t \) = the natural logarithm of the money stock in period \( t \),
- \( \pi_t \) = the natural logarithm of the rate of inflation in period \( t \), and
- \( e_{mt} \) and \( e_{\pi t} \) are error terms in period \( t \).

Equation (9) defines the relationship between \( m_t \) and \( \pi_t \). That is, it represents an application of the atheoretic approach.
Putting equation (9) in matrix form gives

\[
\begin{bmatrix}
a(L) & -b(L) \\
c(L) & g(L)
\end{bmatrix}
\begin{bmatrix}
(1-L)^d m_r \\
(1-L)^d \pi_r
\end{bmatrix}
= 
\begin{bmatrix}
e_{mt} \\
e_{\pi t}
\end{bmatrix}
\]

Written more compactly, the relationship is given as

\[H(L) (1-L)^d Z_t = \eta_t\]

where

- \(H(L)\) = the matrix of polynomials in the lag operator \(L\),
- \(d\) = the number of roots,
- \(Z_t\) = a vector consisting of the natural logarithm of the money stock and the natural logarithm of the rate of inflation in period \(t\), and
- \(\eta_t\) = an identically and independently distributed (i.i.d.) vector with mean zero and finite variance.

Since the quantity theory of money is concerned primarily with the relationship between money shocks and prices, the error term, \(e_{\pi t}\), can be ignored. Given this simplification, the atheoretic approach can be used to analyze the long run impact of a given money shock, \(\mu_{t+1}\), on the stock of money and the rate of inflation. Specifically,

\[
(11) \quad LRI_{m\mu} = \lim_{k \to \infty} \left[ E[m_{t+k} | I_t, \mu_{t+1}] - E[m_{t+k} | I_t] \right] / \mu_{t+1}
\]

\[
LRI_{\pi\mu} = \lim_{k \to \infty} \left[ E[\pi_{t+k} | I_t, \mu_{t+1}] - E[\pi_{t+k} | I_t] \right] / \mu_{t+1}
\]

where

- \(LRI_{m\mu}\) = the long run impact of a money shock on the natural logarithm of the money stock
- \(LRI_{\pi\mu}\) = the long run impact of a money shock on the natural logarithm of the rate of inflation

The quantity theory of money as expressed in equation (1) implies that \(LRI_{m\mu}\) should equal \(LRI_{\pi\mu}\). To demonstrate this, consider equation (10). Solving this equation for \(Z_t\) gives

\[
(12) \quad Z_t = [H(L)]^{-1}[(1-L)^d]^{-1} \eta_t
\]
where

\[
[H(L)]^{-1} = \begin{bmatrix} \sigma^* & \beta^* \\ \beta^* & \alpha^* \end{bmatrix}
\]

Thus, for example, if \( d \) equals one, the long run impact of a money shock on the money stock will be nonzero:

(13) \( \frac{\text{LRI}_{m_L}}{\text{LRI}_{m_A}} = \frac{c^*(1)}{g^*(1)} = \text{the multiplier for } a_1 \) (See equation 1).

This result implies that the direction of causation is unidirectional, going from money to prices only.

The final task is to show that the atheoretic method can also be applied to inflation and exchange rates. It was suggested earlier that Relative Purchasing Power Parity in either form is an application of the quantity theory of money to exchange rate behavior. Since the quantity theory of money can be interpreted as a steady-state phenomenon, it is only necessary to prove that Relative Purchasing Power Parity is a function of the quantity theory in order to satisfy the steady-state requirement. The quantity theory of money is often expressed as (where the time subscript on each factor is implicit):

(14) \( MV = PY \)

where

- \( M \) = the money supply,
- \( V \) = the velocity of money,
- \( P \) = the price level, and
- \( Y \) = the level of economic activity.

If one assumes that velocity and economic activity can be filtered out, then equation (14) and equation (1) are equivalent.

Next, Relative Purchasing Power Parity between any two countries in any period, say the United States and Canada, can be expressed as a function of differences in money stocks between the two countries:

(15) \( \frac{\partial(ER)}{ER} = \frac{\partial(\mu_{US})}{m_{US}} - \frac{\partial(\mu_{Canada})}{m_{Canada}} \)

Finally, since as has been demonstrated, money shocks are inexorably
intertwined with the quantity theory of money, Relative Purchasing Power Parity can be expressed through the relationships that have been presented as a function of the quantity theory of money. Therefore, the atheoretical approach can be used to examine bilateral Relative Purchasing Power Parity between the United States and Canada and between the United States and Japan.

**B. Data**

In order to implement the atheoretical approach, quarterly exchange rate and inflation data for the United States, Canada, and Japan are used. For the reason indicated previously, two separate periods are examined 1957 QI to 1973 QII and 1973 QIII to 1989 QIV.\(^4\) As noted above, two separate time periods are defined in order to determine if the change from a fixed exchange rate regime to a floating exchange rate regime has any effect on concluding whether Relative Purchasing Power Parity held or not.

Relative Purchasing Power Parity is tested between the United States and Canada and between the United States and Japan using quarterly data over the period 1957 QI — 1989 QIV. Data on price indexes and exchange rates are taken from the *International Financial Statistics* and each time series is converted to a 1985 QII base period. Canadian and Japanese price indexes are converted into U.S. dollars by multiplying by the appropriate exchange rate index.

**C. Statistical Techniques and Results**

Natural logarithms (denoted by \(\ln\)) are taken of each variable to yield the final equations to be used in testing Relative Purchasing Power Parity:

\[
\begin{align*}
\ln (\text{P}_{ct} \cdot \text{E}_{(US,C)t}) &= \delta_c \ln (\text{P}_{(US)t}) + \varepsilon_{ct} \\
\ln (\text{P}_{jt} \cdot \text{E}_{(US,J)t}) &= \varepsilon_{jt} \ln (\text{P}_{(US)t}) + \varepsilon_{jt}
\end{align*}
\]

where

- \(\text{P}_{ct}\) = Canadian price index in period \(t\),
- \(\text{P}_{(US)t}\) = United States price index in period \(t\),
- \(\text{P}_{jt}\) = Japanese price index in period \(t\),
- \(\text{E}_{(US,C)t}\) = Exchange rate index, U.S. dollars per Canadian dollar in period \(t\),
- \(\text{E}_{(US,J)t}\) = Exchange rate index, U.S. dollars per Japanese yen in period \(t\).

\(^4\) Qi, i = I, II, III, and IV denotes quarter one, quarter two, etc.
$\delta_c$ and $\delta_j$ are coefficients, and $\varepsilon_{Ct}$ and $\varepsilon_{jt}$ are error terms in period $t$.

Evidence supporting the doctrine of Relative Purchasing Power Parity between the United States and Canada will be found if it is not possible to reject the null hypothesis that $\delta_c$ is (statistically) different from one. Likewise, evidence in support of Relative Purchasing Power Parity between the United States and Japan will be found if there is failure to reject $\delta_j$ being statistically significantly different from one. Note that in testing the concept of Relative Purchasing Power Parity, an implicit assumption is that the United States price index serves as an independent variable since the United States is cast in the role of the reference country.

The filter defined in equation (2) is used to aid in testing the long run relationships expressed in equation (16). The basic idea behind using this filter to test the nature and extent of the long run relationships is that it is not uncommon for high frequency data to possess a considerable amount of noise (see, e.g., Mills (1990)). Consequently, filtering the data enables one to extract the long run signal component for each variable which, in turn, permits an identification of any underlying structural relationships that might otherwise be obfuscated by the noise component. It is these signaling time series which are used in testing for the presence of the long run relationships.

The filter defined in equation (2) is a two-sided exponentially weighted filter. It is used to smooth the original time series data on the United States price index and the exchange rate adjusted Canadian and Japanese price indexes. Since the original time series data for these variables were nonstationary, as shown in Figure 1 for the relative exchange rates between the United States and Canada and the United States and Japan$^5$ and Figure 2 for the price indexes of the three countries ($1985 = 100$), first differences of the data were required.$^6$ This, in effect

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$^5$ Note that the exchange rates are denominated in each country's respective currency. Thus, the Canadian exchange rate is denominated in Canadian dollars while the Japanese exchange rate is denominated in Yen and the United States exchange rate is denominated in United States dollars.

$^6$ An Augmented Dickey-Fuller test was also conducted to examine for the presence of autoregressive unit roots in the data (Phillips (1987)). The values of the test statistic without first differencing the series are $-1.23$, $-1.99$, and $-1.55$ for Canada, Japan, and the United States, respectively for the 1957 Q1 to 1973 QII period and $-1.43$, $-1.50$, and $-1.87$ for the 1973 QIII to 1989 QIV period. With first differencing, the values of the test statistic are $-4.22$, $-4.51$, and $-4.59$ for the 1957 Q1 to 1973 QII period and $-4.99$, $-5.02$, and $-4.74$ for the 1973 QIII to 1989 QIV period. Thus, all three series for both time periods must be first differenced to induce stationarity.
Figure 1
Relative Exchange Rates between the United States and Canada and the United States and Japan
means that the concept of Relative Purchasing Power Parity is being tested in terms of changes in the rates of inflation instead of in terms of changes in the price level.

In filtering the data, the lag length used, \( k \), and the smoothing coefficient, \( \beta \), must be predetermined. In this effort, different lag lengths were considered. The results are consistently robust across different lag lengths. For expository purposes, the results are reported only for a lag length of 24. (Results for other lag lengths are available from the authors upon request.) The smoothing coefficient is allowed to range from 0.50 to 0.95. The closer the smoothing coefficient is to one, the greater the degree of smoothing. For example, when \( \beta = 0.95 \), the smoothed series approaches the sample mean of the original data. A smoothed series is obtained for each smoothing coefficient, \( \beta \), and for each variable in the original time series for each of the two time periods under investigation (i.e., 1957 QI to 1973 QII and 1973 QIII to 1989 QIV).\(^7\)

The filtered series are used to test long run Relative Purchasing Power Parity using equation (16). The results are presented in Table 1a for the 1957 QI to 1973 QII period and in Table 1b for the 1973 QIII to 1989 QIV period. The estimates were obtained via ordinary least squares. The Durbin-Watson statistics were consistently in the indeterminant range and the coefficients of determination (R\(^2\)) range between 0.40 and 0.70. For the first time period, there is little support for accepting the null hypothesis that Relative Purchasing Power Parity holds in the long run. That is, since the estimates are statistically significantly different than one (with one exception), they do not lend credence to the Relative Purchasing Power Parity doctrine. The results for the second period are mixed. In some instances (i.e., for selected values of \( \beta \)), it is not possible to reject the null hypothesis that the values of \( \delta_C \) and \( \delta_Y \) are different than one thereby implying that Relative Purchasing Power Parity does hold in the long run. In other instances, the null hypothesis of Relative Purchasing Power Parity is rejected outright.

The variability in the estimates of \( \delta_C \) and \( \delta_Y \) as the value of the smoothing coefficient, \( \beta \), changes from 0.50 to 0.95 is quite sizeable. Moreover, the wide departure of the estimates from the neighborhood of one (where they would be expected to lie) raises a degree of consternation. Does the Lucas approach to filtering the data to obtain just the signal component from the time series really accomplish this objective? This is properly the subject of future research.

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\(^7\) The authors would like to thank Robert Lucas for providing the computer software used to filter the data.
Table 1a

ESTIMATES OF $\delta_C$ AND $\delta_I$ FOR DIFFERENT VALUES OF THE SMOOTHING COEFFICIENT $\beta$ FOR PERIOD I (1957 Q1-1973 QII)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\delta_I^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Canada</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.2818 (0.0102)</td>
</tr>
<tr>
<td>0.55</td>
<td>0.3617 (0.0098)</td>
</tr>
<tr>
<td>0.60</td>
<td>0.6769 (0.0154)</td>
</tr>
<tr>
<td>0.65</td>
<td>0.8631 (0.0153)</td>
</tr>
<tr>
<td>0.70</td>
<td>0.7778 (0.0065)</td>
</tr>
<tr>
<td>0.75</td>
<td>0.6917 (0.0077)</td>
</tr>
<tr>
<td>0.80</td>
<td>0.7968 (0.0187)</td>
</tr>
<tr>
<td>0.85</td>
<td>0.9717 (0.0122)</td>
</tr>
<tr>
<td>0.90</td>
<td>0.8535 (0.0319)</td>
</tr>
<tr>
<td>0.95</td>
<td>0.5339 (0.0226)</td>
</tr>
</tbody>
</table>

II. Japan

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\delta_I^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.9112 (0.0183)</td>
</tr>
<tr>
<td>0.55</td>
<td>0.9767 (0.0162)</td>
</tr>
<tr>
<td>0.60</td>
<td>0.7851 (0.0199)</td>
</tr>
<tr>
<td>0.65</td>
<td>0.6709 (0.0081)</td>
</tr>
<tr>
<td>0.70</td>
<td>0.6348 (0.0119)</td>
</tr>
<tr>
<td>0.75</td>
<td>0.5906 (0.0110)</td>
</tr>
<tr>
<td>0.80</td>
<td>0.8838 (0.0349)</td>
</tr>
<tr>
<td>0.85</td>
<td>1.1801 (0.0310)</td>
</tr>
<tr>
<td>0.90</td>
<td>0.6622 (0.0564)</td>
</tr>
<tr>
<td>0.95</td>
<td>-0.0416 (0.0383)</td>
</tr>
</tbody>
</table>

$^1$ Standard errors of the estimates are in parentheses. Also, $i = C$ (Canada) and $J$ (Japan).

$^2$ Not statistically significantly different than one at the 95 percent level.

Based on these results applying the Lucas' approach, Relative Purchasing Power Parity can be rejected for the 1957 Q1 to 1973 QII period between Canada and the United States and between Japan and the United States. For the 1973 QIII to 1989 QIV period, on the other hand, Relative Purchasing Power Parity can neither be conclusively accepted nor rejected.

These mixed results are especially unsatisfying. They do nothing to provide support (or lack thereof) for the Relative Purchasing Power Parity
Table 1b

ESTIMATES OF $\delta_C$ AND $\delta_J$ FOR DIFFERENT VALUES OF THE SMOOTHING COEFFICIENT $\beta$ FOR PERIOD II (1973 QIII-1989 QIV)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\delta_i^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Canada</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>2.8053 (0.2731)</td>
</tr>
<tr>
<td>0.55</td>
<td>1.0367 (0.0843)$^2$</td>
</tr>
<tr>
<td>0.60</td>
<td>0.6730 (0.4069)$^2$</td>
</tr>
<tr>
<td>0.65</td>
<td>0.4585 (0.0297)</td>
</tr>
<tr>
<td>0.70</td>
<td>0.7687 (0.0196)</td>
</tr>
<tr>
<td>0.75</td>
<td>-0.1306 (0.0698)</td>
</tr>
<tr>
<td>0.80</td>
<td>0.2604 (0.1094)</td>
</tr>
<tr>
<td>0.85</td>
<td>1.1490 (0.1214)$^2$</td>
</tr>
<tr>
<td>0.90</td>
<td>3.2965 (1.4679)$^2$</td>
</tr>
<tr>
<td>0.95</td>
<td>5.4789 (0.2571)</td>
</tr>
</tbody>
</table>

II. Japan

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\delta_i^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>2.7467 (0.4094)</td>
</tr>
<tr>
<td>0.55</td>
<td>-1.5693 (0.1988)</td>
</tr>
<tr>
<td>0.60</td>
<td>1.4863 (0.7049)$^2$</td>
</tr>
<tr>
<td>0.65</td>
<td>1.0922 (0.1208)$^2$</td>
</tr>
<tr>
<td>0.70</td>
<td>1.5936 (0.0536)</td>
</tr>
<tr>
<td>0.75</td>
<td>-0.4208 (0.1589)</td>
</tr>
<tr>
<td>0.80</td>
<td>-2.2550 (0.2591)</td>
</tr>
<tr>
<td>0.85</td>
<td>-7.6463 (0.2694)</td>
</tr>
<tr>
<td>0.90</td>
<td>1.7774 (0.8119)$^2$</td>
</tr>
<tr>
<td>0.95</td>
<td>3.6497 (0.4577)</td>
</tr>
</tbody>
</table>

$^1$ Standard errors of the estimates are in parentheses. Also, $i = C$ (Canada) and $J$ (Japan).

$^2$ Not statistically significantly different than one at the 95 percent level.

...
proach uses the actual series themselves to delineate the nature of the smoothing filters to be used in developing the signalling time series as opposed to imposing an artificial structure to generate the requisite smoothed series as does the Lucas approach. The second uses cointegration techniques.

Note that the rationale behind the Lucas approach and the use of data determined ARIMA filters to smooth the series is basically the same. The idea is to test Relative Purchasing Power Parity with regressions of the form specified in equation (16) after first using ARIMA filters to extract the maximum amount of noise from the original time series.

III. An ARIMA Approach

The decision to use ARIMA methods to let the data indicate which filter is most desirable (in the sense discussed above) is developed elsewhere (see, e.g., Box, Hilmer, and Tiao (1978)). To gain some appreciation of its mechanics, however, a brief outline of the approach is given here. First, any stationary time series, say of prices \( P_t \), can be decomposed into a signal component, \( S_t \), and a noise component, \( N_t \), as follows:

\[
(17) \quad P_t = S_t + N_t.
\]

It is not possible to observe \( S_t \) or \( N_t \), but one can identify and estimate a standard ARIMA model for \( P_t \). Given this and assuming that \( N_t \) is a white noise series with mean zero and a finite variance, a two-sided filter can be estimated for \( P_t \) that maximizes the variance of \( N_t \) taking as much noise as possible out of the original series leaving only the signal component which, in turn, can be used in testing for any long run relationships between variables.

The estimated ARIMA models used in computing the signalling component of the various time series used in the tests for the presence of Realtime Purchasing Power Parity are given in Table 2. Note that each of the variables is transformed logarithmically. Preliminary analyses of the series based on an examination of the estimated ARIMA series residuals indicated that this was appropriate. The reported values of the modified

---

8 If the series is not stationary, differencing can be used to induce stationarity. Generally, some low order (i.e., \( d = 1 \) where \( d \) indicates the order of differencing) is sufficient for most economic time series.
Table 2

ARIMA Filters for the Exchange Rate and Prices Series Data Used in Testing for Long Run Purchasing Power Parity

<table>
<thead>
<tr>
<th>Series</th>
<th>ARIMA Estimates</th>
<th>Q²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period I (1957 Q1-1973 QII)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. U.S. Price Index</td>
<td>(1-L)Xₜ = (1-0.4635 L)uₜ</td>
<td>13.67</td>
</tr>
<tr>
<td></td>
<td>(0.1254)</td>
<td></td>
</tr>
<tr>
<td>b. Canadian Price Index and Canada/U.S.</td>
<td>(1-L)(1-0.8953 L)Xₜ = (1-0.8262 L) uₜ</td>
<td>18.91</td>
</tr>
<tr>
<td>U.S. Exchange Rate</td>
<td>(0.0231)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0799)</td>
<td></td>
</tr>
<tr>
<td>c. Japanese Price Index and Japan/U.S.</td>
<td>(1-L)(1-0.6010 L²)Xₜ = (1 + 0.9387 L) uₜ</td>
<td>16.98</td>
</tr>
<tr>
<td>U.S. Exchange Rate</td>
<td>(0.0963)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1372)</td>
<td></td>
</tr>
</tbody>
</table>

Period II (1973 QIII-1989 QIV)

| a. U.S. Price Index                        | (1-L) Xₜ = (1-0.3620 L²) uₜ                            | 23.51|
|                                             | (0.1176)                                              |      |
| b. Canadian Price Index and Canada/U.S.    | (1-L)(1-0.5877 L-0.6266 L²) Xₜ = (1-0.6266 L²) uₜ     | 16.48|
| U.S. Exchange Rate                         | (0.1208)                                              |      |
|                                             | (0.1216)                                              |      |
| c. Japanese Price Index and Japan/U.S.     | (1-L)(1-0.4201 L) Xₜ = (1-0.3102 L²)uₜ                | 22.48|
| U.S. Exchange Rate                         | (0.1134)                                              |      |
|                                             | (0.0563)                                              |      |

1 Standard errors of the estimates are in parentheses. Note that Xₜ and uₜ are used to represent generically the univariate series of interest and the residual white noise series, respectively. The uₜ corresponds to the Nₜ in the text.

2 Ljung-Box Q-statistic based on 24 degrees of freedom. See Ljung and Box (1978) for a discussion of this statistic.

Ljung-Box Q-statistic indicate that each of the estimated filters results in the residual series being reduced to white noise. The filters are used to generate smooth series as suggested by Box, Hilmer, and Tiao.

These smoothed series (the signal component and not the noise component of the respective time series) are used in estimating the coefficients
specified in equation (16). These coefficient estimates are obtained via ordinary least squares with correction for first order serial correlation by the approach of Beach and MacKinnon (1978) and are reported in Table 3.

The results clearly indicate that Relative Purchasing Power Parity does not hold between the Canada and the United States and between Japan and the United States. That is, the estimates (together with their standard errors) do not support the null hypothesis that the values of $\delta_c$ and $\delta_j$ are (statistically) equal to one at the 5 percent level of significance.

IV. The Cointegration Approach

The final approach that will be used in examining the doctrine of Relative Purchasing Power Parity involves the use of cointegration techniques. A number of studies focusing on various aspects of Absolute and Relative Purchasing Power Parity using cointegration techniques exist (Ardeni and Lubian (1991), Beng (1991), Canarella, et al. (1990),

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
</thead>
</table>

VALUES OF $\delta_c$ AND $\delta_j$ FOR TESTING WHETHER RELATIVE PURCHASING POWER HOLDS PAIRWISE BETWEEN CANADA AND THE UNITED STATES AND JAPAN AND THE UNITED STATES

<table>
<thead>
<tr>
<th>Country Pair</th>
<th>Coefficient Estimate$^1$</th>
<th>$R^2(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period I (1957 Q1-1973 QII)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Canada/United States</td>
<td>1.0540</td>
<td>0.9771</td>
</tr>
<tr>
<td>b. Japan/United States</td>
<td>0.9388</td>
<td>0.4309</td>
</tr>
<tr>
<td>Period II (1973 QIII-1989 QIV)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Canada/United States</td>
<td>1.0310</td>
<td>0.9451</td>
</tr>
<tr>
<td>b. Japan/United States</td>
<td>1.0532</td>
<td>0.7029</td>
</tr>
</tbody>
</table>

$^1$ Standard errors of estimates are in parentheses.

$^2$ Coefficient of Determination.
Enders (1988), Nachane and Chrissanthaki (1991), Taylor (1988), and
Taylor and McMahon (1988)). It is not the purpose here to review these.
Rather, the approach will be considered with the sole objective of compar-
ing the results obtained with those coming from the ARIMA approach.

Cointegration exists between two nonstationary time series (a non-
stationary time series is one whose mean and/or variance change over time
and whose covariance between values at two time points of the same
distance vary when alternative time points are considered) that are both
integrated of the same order, I(d), if there is a linear combination of the
two series which is itself stationary. By definition, a variable (e.g., a price
index) is integrated of order d if a \(d^{th}\) difference of the series is stationary.
A stationary series is denoted as I(0). Two price indexes, \(s_x\) and \(s_y\), are
cointegrated if the relationship between the two can be written as

\[
(18) \quad z_t = s_{xt} - k s_{yt}
\]

where \(z_t\) is I(0) and \(k\) is the cointegrating constant. The equilibrium error
process, \(z_t\), represents the deviation of the price indexes for \(x\) and \(y\) away
from the long run equilibrium.\(^9\)

One test for cointegration involves ordinary least squares (OLS) estima-
tion of the following static cointegrating regression:

\[
(19) \quad s_{yt} = u + \phi s_{xt} + u_t
\]

where \(a\) and \(b\) are coefficients to be estimated and \(u_t\) is the stochastic
term. The null hypothesis of no cointegration is rejected if both \(\phi\) is
statistically significant and if \(u_t\) is I(0).\(^10\) This test is not very powerful,
however, in the event that \(u_t\) is stationary but highly serially correlated
(Jenkinson (1986)).

Error correction models provide a second test for cointegration. This
approach models both the short run dynamics and the long run equi-
librium between variables suggested by economic theory. It is also possible
to draw causal inferences on the basis of error correction models. For two
price index series that are cointegrated, causality must run in at least one

\(9\) For a further discussion of cointegration and of alternative testing procedures, the in-
terested reader is referred to the excellent collection of papers in the Oxford Bulletin of

\(10\) Stock (1984) shows that the estimates in relationship (2) will be superconsistent if
cointegration holds.
direction since one price index can be used to help forecast the other.\textsuperscript{11} (Granger (1986, 1988) discusses the causal implications of cointegration.)

According to the Granger Representative Theorem (Granger and Weiss (1986), Engle and Granger (1987), and Engle and Yoo (1987)), if two price index series are cointegrated, then there is an error correction model (ECM) of the following form:

\begin{align}
(1-L) s_{yt} &= -p_1 z_{x,t-1} + A(L) (1-L) s_{yt} + B(L) (1-L) s_{xt} + u_{1t} \\
(1-L) s_{xt} &= -p_2 z_{x,t-1} + C(L) (1-L) s_{xt} + D(L) (1-L) s_{yt} + u_{2t}
\end{align}

where $z_{x,t-1} = s_{y(t-1)} - k s_{y(t-1)}$, $p_1$ and $p_2$ are nonzero parameters, and $u_{1t}$ and $u_{2t}$ are both $I(0)$. The one sided lag polynomials, $A(L)$, $B(L)$, $C(L)$, and $D(L)$ are stable such that the roots of the associated polynomial are outside the unit circle. If two price index series are cointegrated, the coefficient on the error correction term, $z_{x,t-1}$, must be statistically significant in at least one of the error correction equations (i.e., equations (20) and (21)).

Causal inferences are based on the statistical significance of $p_1$ and $p_2$ and the elements in $B(L)$ and $D(L)$. For example, $p_1$ and the elements in $B(L)$ equal to zero supports the conclusion that the price index of country $x$ does not Granger-cause the exchange rate adjusted price index of country $y$.\textsuperscript{12}

Summary results for the static cointegrating regressions (i.e., relationship (19)) are presented in Table 4. Provided in the table are Cointegrating Regression Durbin-Watson statistics (as proposed by Sargan and Bhargava (1983)) as well as the $t$-statistics for both Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests for stationarity of the regression residuals for the 1957 Q1 to 1973 QII period and for the 1973 QIII to 1989 QIV period.\textsuperscript{13,14} Although not reported (they are available upon re-

\textsuperscript{11} Since the error correction term is a function of the levels of the price indexes, lagged values of the other price indexes are significant in explaining movements in the price indexes that serves as the dependent variable when the lagged error correction term is significant.

\textsuperscript{12} Granger (1988) discusses the possibility of viewing the causal impact of the error correction term as occurring at low frequencies (i.e., in the long run). For example, $p_1$ different from zero would indicate long run causality from price index of country $x$ to the price index of country $y$, while $B(L)$ different from zero would indicate short run causality. While such an interpretation is attractive, Granger warns that it is unclear whether such a view is justified until analysis similar to that of Geweke (1982) is completed for the error correction test being considered here. Such an effort would explore the frequency decomposition of the error correction term.

\textsuperscript{13} Based on a Monte Carlo study, Engle and Granger (1987) recommend the ADF test. Both tests are used here, however, since the DF test is frequently used in applied econo-
Table 4

STATIC COINTEGRATING REGRESSIONS —
DICCAY-FULLER AND AUGMENTED DICCAY-FULLER TESTS

<table>
<thead>
<tr>
<th>Country Pair</th>
<th>CRDW(^1)</th>
<th>DF(^2)</th>
<th>ADF(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period I (1957 QI-1973 QII)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Canada/United States</td>
<td>0.0776</td>
<td>-1.03</td>
<td>-1.71</td>
</tr>
<tr>
<td>b. Japan/United States</td>
<td>0.0616</td>
<td>-0.35</td>
<td>-1.17</td>
</tr>
<tr>
<td>Period II (1973 QIII-1989 QIV)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Canada/United States</td>
<td>0.0723</td>
<td>-1.15</td>
<td>-1.31</td>
</tr>
<tr>
<td>b. Japan/United States</td>
<td>0.0984</td>
<td>-1.06</td>
<td>-1.27</td>
</tr>
</tbody>
</table>

1 The Cointegrating Regression Durbin-Watson statistic. The critical values for the CRDW are 0.386 and 0.511 at the 5 percent and the one percent levels, respectively. The critical values are reported in Engle and Granger (1987, p. 269).
2 Dickey-Fuller test statistic.
3 Augmented Dickey-Fuller test statistic.

quest), the OLS estimates of cointegrating constants were statistically significant and positive. In addition the Ljung-Box modified Q-statistic (Ljung and Box (1978)) for each regression was very large with marginal significance levels less than 0.001 percent in each case. (The same result was obtained when longer lag lengths (of 24 and 36) of the dependent variable were considered.)

The relatively small values for the CRDW statistic as well as the low DF and ADF t-statistics indicate a lack of cointegrating relationships between the price index for the United States and the exchange rate adjusted price index for Canada and between the price index for the United States and the exchange rate adjusted price index for Japan.

A closer examination of the results, however, from the static cointegration tests leads one to be concerned with the relatively large computed Q-statistics.\(^{13}\) These values suggest the possibility that the inability to re-

14 Lags of length 24 were used in the estimation for the Augmented Dickey-Fuller test.
15 The computed Q-statistics based on 48 degrees of freedom for the 1957 QI-1973 QII
ject the null hypothesis of no cointegration may be due to the low power of the test because of autocorrelated residuals.

Jenkinson (1986) notes the low power of the CRDW test as well as of both the DF and ADF tests when the residuals in static cointegrating regressions display stationarity by exhibiting an autoregressive pattern. For example, the values of the CRDW statistic in Table 4 imply a first order autoregressive coefficient that is very close but not equal to one in each of the cointegrating regressions. These values range from 0.953 to 0.969 and show the autoregressive nature of the residuals.\textsuperscript{16} Sargent and Bhargava (1983) find that the power of the CRDW test for random walk behavior (the null hypothesis) against the alternative hypothesis that \( u_t = \Psi u_{t-1} + \xi_t \) becomes very low as \( \Psi \) approaches one.\textsuperscript{17} As an alternative to the cointegrating regression, Jenkinson suggests using an error correction model to test for cointegration in the presence of autoregressive residuals. This is done in what follows.

Table 5 presents the final, restricted error correction models for the Canada/United States and Japan/United States pairwise combinations. The error correction equations for each pairwise relationship are estimated in order to draw correct inferences regarding cointegration. Included in the tables are OLS parameter estimates with standard errors of the estimates in parentheses, Ljung-Box modified Q-statistics, and the standard errors of the regressions, SEE.

The basic model-building strategy delineated by Granger and Weiss (1983) and Engle and Granger (1978) was followed in deriving the final parsimonious error correction models (ECMs) from the initial over-parameterized specifications.\textsuperscript{18} This procedure involves dropping all insignificant lagged values of both variables and imposing the restrictions implied by the lagged error correction term, \( EC_{t-1} \). The initial error correction model were specified such that they included a constant, 12 lags of both differenced dependent and independent variables, and lagged values of the level (i.e., untransformed values of the prices) of each variable. For

---

\textsuperscript{16} These values are computed from the expression for the CRDW statistic given as \( CRDW = 2(1 - \Psi) \). This is solved for \( \Psi \).

\textsuperscript{17} Note that the \( \xi_t \) are assumed to be identically and independently distributed with a mean of zero and a finite variance.

\textsuperscript{18} Note that it would be tempting to use data based criteria like the Akaike Information Criterion (AIC) (Akaike (1974)) for selecting the model specification. Such a procedure, however, is likely to result in reduced power of the test. See Engle and Granger (1987) for a discussion of this.
Table 5
FINAL RESTRICTED JOINT ERROR CORRECTION MODEL REPRESENTATIONS

<table>
<thead>
<tr>
<th>Country Pair</th>
<th>Period I</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1957 QI-1973 QII)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a.</td>
<td>Canada/United States</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1-L) CAN_t = -0.299 -0.067 (1-L) CAN_t-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.132) (0.030)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 0.197 (1-L) US_t + 0.036 EC_t-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.052) (0.038)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SEE = 0.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Q(48) = 54.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>Japan/United States</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1-L) JAP_t = 3.263 + 0.251 (1-L) JAP_t-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.596) (0.126)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.491 (1-L) JAP_t-2 + 0.832 (1-L) US_t-4 - 0.262 EC_t-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.176) (0.268)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SEE = 0.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Q(48) = 52.21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country Pair</th>
<th>Period II</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1973 QIII-1989 QIV)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a.</td>
<td>Canada/United States</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1-L) CAN_t = -0.300 + 0.441 (1-L) CAN_t-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.056) (0.132)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 0.287 (1-L) CAN_t-3 + 0.651 (1-L) US_t-5 + 0.021 EC_t-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.142) (0.433)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SEE = 1.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Q(48) = 53.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>Japan/United States</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1-L) JAP_t = 4.372 + 0.142 (1-L) JAP_t-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.651) (0.661)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 0.070 (1-L) JAP_t-2 + 1.203 (1-L) US_t-3 + 0.016 EC_t-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.036) (0.549)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SEE = 6.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Q(48) = 48.27</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^1\) Standard errors of the estimates are in parentheses. SEE denotes the standard error of the regression. Q is the computed Ljung-Box modified Q-statistic with the number of degrees of freedom in parentheses. L is the lag operator. CAN\(_t\), JAP\(_t\), and US\(_t\) denote the price indexes in period \(t\) for Canada, Japan, and the United States, respectively. Note that the price indexes for Canada and Japan are exchange rate adjusted. EC is the error correction term.
both sets of error correction models estimated for the two time periods, the error correction term is not statistically significantly different than zero. They must be negative and statistically significant, however, for cointegration to hold (see, e.g., Engle and Yoo (1987)). Thus, the price index series for both of the paired combinations for each period are not cointegrated.

The results from using the error correction model specification are consistent with those obtained when the static cointegrating regression specification was used. Based on the absence of any significance of the estimated coefficient on the error correction term in the previous period, the price index of the United States and the exchange rate adjusted price index for Canada as well as the price index of the United States and the exchange rate adjusted price index for Japan are not cointegrated.

V. Conclusion

In this paper mixed evidence has been found supporting the concept of Relative Purchasing Power Parity using a method analogous to that used by Lucas in testing the quantity theory of money. Relative Purchasing Power Parity was not consistently rejected in the long run between the Canada and the United States and between Japan and the United States using quarterly data covering two separate periods — 1957 QI-1973 QII and 1973 QIII and 1989 QIV. Given the mixed results associated with relying on the methodology of Lucas, two alternative approaches were considered the first whereby the requisite smoothed time series were obtained via appropriate ARIMA filters and the second where the results were not dependent on smoothed series. In these instances, the results were unequivocal. Relative Purchasing Power Parity did not hold in the long run.

Finally, while there is no objective justification for preferring one approach over the others, the very ad hoc nature of the Lucas approach and the variability in the resulting coefficient estimates as the value of the smoothing coefficient changed together with the reliance of the ARIMA approach on letting the evolution of the time series itself define the nature of the signal component of the series suggests that the latter is probably preferable to the former. In terms of preferring the ARIMA approach to the cointegration approach, the consistency of the conclusions reached by both approaches precludes the selection of one over the other based on their relative performance in testing for the presence of Relative Purchasing Power Parity using the Canada, Japan, and the United States as the basis of the analysis.
References


RELATIVE PURCHASING POWER PARITY


———, and A. Weiss, "Time Series


