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In Pursuit of Identifying Technical Efficiency and X-Efficiency

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The objective of this paper is to show the differences between the concepts of technical efficiency and x-efficiency. Using firm-level data from the South Indian Power loom industry, the differences have been demonstrated empirically.

I. Introduction

Technical efficiency is defined as the ability and willingness of an economic unit to produce the maximum possible output from a given combination of inputs and technology regardless of market prices of outputs and inputs, and demand (Farrell, 1957). Although Leibenstein has not given a concise definition of X-efficiency, it may be argued that X-efficiency involves firms using their resources in a rational manner consistent with some empirically determined performance standards to produce the maximum possible output. (Leibenstein, 1978). Without loss of generality, profit maximization may be considered as an empirically determined performance standard. Then, X-efficiency is composed of two components — technical and allocative efficiencies. In other words, X-efficiency refers to the concept of economic efficiency. The only difference between the neoclassical concept of economic efficiency and X-efficiency is that unlike the former analysis, the latter does not assume internal (technical) efficiency within the firm. Following Leibenstein's arguments, achieving internal efficiency depends on the levels of effort

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exerted by firms. Therefore, when someone measures technical inefficiency, it need not be always X-inefficiency. In fact, if technical inefficiency is interpreted as X-inefficiency, the former may overstate the latter under certain conditions.

Studies which have synonymously used the concepts of technical efficiency and X-efficiency include Martin and Page (1983), among others. Recently, Chi (1988) discussed intra-firm differentials of X-efficiency of Taiwan's sugar industry, while estimating only technical efficiency. Although Leibenstein (1977) discusses the differences between technical efficiency and X-efficiency, his arguments appear to be ambiguous and confusing. Without detracting from the important contribution made by Leibenstein, the objective of this paper is to set up, albeit in a simpler and empirically testable form, the notions of these efficiency measures which are applicable in the context of non-price and organizational decisions of the firm, using data from the South Indian Power Loom Industry.

The next section briefly discusses the model to measure both technical and X-efficiencies which are firm-spècific using the frontier production function methodology. The following section explains the estimation procedures. The next section examines the empirical results and final section brings out the overall conclusions of this paper.

II. Technical Efficiency and X-efficiency: The Model

Consider the basic concept of the production function in the neoclassical theory of the firm.

$$(1) \qquad y^* = f_i(x_1, x_2, .. x_m)/T \qquad i = 1,2, \ldots N$$

The above production function of the i^{th} firm is in conformity with the neoclassical theory of production and shows the maximum possible output y^*, for the given levels of the inputs x_i's and technology T[1]. This is called the frontier production function of the i^{th} firm in the literature (Aigner, Lovell and Schmidt, 1977 and Meeusen and Van den Broeck, 1977). This is also named as the outer bound production function by Friedman (1967) and Leibenstein (1966), implying a limit to the inputs-output transformation for a given level of technology. Now, a modified

[1] Productivity differentials among firms are not ignored while estimating such production functions, but they are pooled with the 'usual' statistical errors and assumed to follow a normal distribution with zero mean and a constant variance.

neoclassical production function can be defined as follows. The logic behind the situation in which the i^{th} firm may be producing below the production function (1), and so lower than y^*, can be explained in two different ways. One, the conventional production analysis, generally assumes that differences between the maximum possible and actual outputs are not very important and are distributed randomly across firms.[2] Two, the frontier production methodology and Lebenstein's outer bound production approach assume that such observed differences are not just random or non-economic events, but are due to the influence of certain non-price and organizational factors.[3]

These non-price and organizational factors which are firm-specific are difficult to measure. Nevertheless, the combined influence of these factors on the difference between the maximum possible and actual outputs can be modelled as follows:

$$(2) \qquad y_i = y^*_i \exp(u_i)$$

where y_i refers to the actual output realized and u_i refers to the combined influence of non-price and organizational factors which are firm-specific. In other words, these additional factors influence the technical aspects or technical efficiency of producing y in (2). Once, this influence is accounted for, without measurement errors, the differences between the maximum possible and observed outputs should become insignificant and may actually disappear.

When u takes the value zero, the firm realizes its technical efficiency fully and produces the maximum possible output. u takes a value less than zero, if the firm does not realize its technical efficiency fully (that is, technically inefficient), and produces the output smaller than its maximum possible output. Further, the maximum possible output can vary either randomly across firms, or over time for the same firm. On this interpretation, the maximum possible output is stochastic and accordingly, the firm-specific actual output is given by adding a statistical random disturbance term v to (2) as,

2 This kind of analysis does not deny the importance of these productivity differences, but offers little to explain these differences. For some probable explanations for such differences, in the conventional approach, see Marschak and Andrews (1944), and Zellner, Kmenta and Dreze (1966).

3 Carlson (1972) has briefly discussed about the possible non-price and organizational factors which are sources of inefficiency.

$$(3) \qquad y_i = f_i\ (x_1,\ x_2,..x_m)\ \exp\ (u_i + v_i)$$
$$= y_i^*\ \exp\ (u_i + v_i)$$

A measure of technical efficiency[4] is then defined as follows:

$$(4) \qquad \exp\ (u_i) = \frac{y_i / u_i}{y^*_i\ \exp\ (v_i) / u_i = 0}$$

This measure necessarily varies between 0 and 1 and does not involve the components of price efficiency. Therefore, technical inefficiency can be measured as $[1-\exp\ (u_i)]$. Two observations should be made about this measure of technical efficiency. One, as this measure compares the firm's actual output with its own maximum possible output, given its resources and environment, it provides a realistic measure of the firm's technical efficiency. Two, even though technical efficiency measures may be the same for a number of firms, this does not mean that all these firms have obtained the same actual output levels. Because, firms have different resource endowments, specifically, with respect to non-measurable inputs, which enable them to reach different production levels.

Although Leibenstein (1978) claims that the concept of technical efficiency is within the domain of neoclassical economics and that X-efficiency is a much broader concept, both these approaches can be reconciled. The argument that the maximizing behaviour can be interpreted as reaching empirically determined performance standards still leaves intact the fundamental equilibrium concept of economics, while this need not lead to the automatic presumption that firms always intend to achieve their best. (Rozen, 1985). Thus, it can be argued that the X-efficiency theory does not reject the neoclassical framework but facilitates explanations for differences in performance observed. Therefore, the above modified neoclassical paradigm, which is the frontier production function methodology, can be used to explain and measure X-efficiency at the firm level. Basically, X-inefficiency means the extent to which firms fail to use their resources consistent with the determined performance standards to realize their production potential given by the maximum possible output. The maximum possible output which is rationally consistent with the firm's resources is identified with

[4] Here, technical efficiency is measured in terms of firm-specific outputs, where it is defined as being the maximum increase in outputs that could be obtained from the same set of inputs.

the point of tangency between the frontier production function defined above in (3) and the firm-specific price or the budget constraint indicating the set performance standards. This involves two steps.[5] First, solving the first-order marginal productivity conditions for profit maximization by using the firm-specific price data and the estimated parameters of the firm-specific frontier production function showing the firm's maximum possible output. Second, obtaining the profit maximizing input levels and substituting them into the firm's maximum possible frontier function to arrive at the level of x-efficient output. Let this output be called as $\tilde{y}_i$. Now, a measure of X-efficiency of the i^{th} firm in terms of output level can be defined as the ratio of actual output to the x-efficient output. Let this ratio be named as K.

$$(5) \qquad \text{X-eff: } K = \frac{y_i \text{ given } u_i}{\tilde{y}_i \text{ given } u_i = 0 \text{ and } MVP_{ij} = MC_{ij}}$$

The above measure of K can be $\gtreqless 1$, depending on both the actual level, and the point of tangency of the price plane with the frontier function. When $K > 1$, this means that firms' productions are not consistent with the empirically determined performance standards (profit maximization) because they are overutilizing their inputs, even if they are on their frontiers. Next, $K < 1$ implies that firms are underutilizing their inputs, even if they are producing on their frontiers. When $K = 1$, firms are said to be producing on their frontiers using optimal combination of their inputs which is consistent with the determined performance standards of profit maximization. Therefore, only when $K = 1$, firms are said to be x-efficient. When $K \neq 1$ (either greater or less than 1), this means that firms are x-inefficient. Therefore a measure of x-inefficiency can be obtained as $|1\text{-}K|$.

Thus, it is evident that the X-efficient output is both technically and allocatively efficient. However, there is a distinction between this technical efficiency component of the X-efficiency and the technical efficiency measured by equation (4). The former involves efficient employment (technical efficiency) of optimal combination of factor proportions (allocative efficiency), while the latter concerns efficient use of the actual combination of inputs employed which may or may not be the appropriate factor combinations at market prices. It is apparent now that the

[5] Alternatively, the frontier production function and the first order conditions can be estimated simultaneously. For further details see Kalirajan (1990).

measure of technical inefficiency, when used as the measure of X-inefficiency, may lead to overestimation of the latter.

III. Estimation

Given the data on actual levels of inputs used, output produced, and prices paid and received which are all firm-specific, the above two measures of technical efficiency (4) and X-efficiency (5) can be estimated, using the maximum likelihood methods. It becomes, now, necessary to specify functions for both the random variables u_i and v_i

It is assumed that u follows a half normal distribution[6] and v follows a full normal distribution with mean zerc and variances σ_v^2. u and v are assumed to be independently distributed.

The density function of u and v can respectively be written as:

$$(6) \qquad f(u) = \frac{1}{\sqrt{\frac{1}{2}}\,\sigma_u} \exp\left(-\frac{u^2}{2\sigma_u^2}\right) \quad u \leqslant 0$$

$$(7) \qquad f(v) = \frac{1}{\sqrt{2\pi}\,\sigma_v} \exp\left(-\frac{v^2}{2\sigma_v^2}\right) \quad -\infty \leqslant v \leqslant \infty$$

The likelihood function of y is the product of density function of (u + v).

By convolution formula, the joint density function of (u + v) can be written as,

$$(8) \qquad f(u+v) = \frac{1}{\sqrt{\frac{1}{2}\pi(\sigma_u^2 + \sigma_v^2)}} \exp\left(-\frac{1}{2}\frac{(u+v)^2}{(\sigma_u^2 + \sigma_v^2)}\right) \left[1 - F\left[(u+v)\left(\frac{\sigma_u}{\sigma\sigma_v}\right)\right]\right]$$

where $F(\cdot)$ is the cumulative distribution function of the standard normal random variable:

(9) Specifying i) $\sigma^2 = \sigma_u^2 + \sigma_v^2$

ii) $\gamma = \dfrac{\sigma_u}{\sigma_u^2 + \sigma_v^2}$ where γ lies in the interval (0,1)

6 This facilitates the assumption that there are no observations above the frontiers and all the observations are either on or below it.

iii) $u + v = e$

The density function of y is:

$$(10) \qquad f(y)\frac{1}{\sigma\sqrt{\pi/2}}\exp\left(-\frac{1}{2}\frac{e^2}{\sigma^2}\right)\left[\left(1-F\left(\frac{e}{\sigma}\sqrt{\frac{\gamma}{1-\gamma}}\right)\right]$$

The likelihood function of the sample can now be written as:

$$(11) \qquad L^*(y,\theta) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{\pi/2}} \exp\left(-\frac{1}{2}\frac{e^2}{\sigma^2}\right)\left[1-F\left(\frac{e}{\sigma}\left(\frac{\gamma}{1-\gamma}\right)^{1/2}\right)\right]$$

where θ is the parameter to be estimated and is equal to the production parameters, σ^2 and γ.

The maximum likelihood (ML) estimators of θ, maximizing the above likelihood function are obtained by setting its first order partial derivatives with respect to the elements of θ equal to zero, and solving them simultaneously.

The first issue here is to estimate the firm-specific technical efficiency measures for each observation in the sample. Measurement of exp(u) for individual firms is obtained from the conditional distribution of u_i, given $(u_i + v_i)$ (Jondrow et al. 1982, Kalirajan and Flinn, 1983).

$$(12) \qquad E(u/u+v) = -\frac{\sigma_u\alpha_v}{\sigma}\left[\frac{f(\cdot)}{1-F(\cdot)} - \frac{e}{\sigma}\left[\frac{\gamma}{1-\gamma}\right]^{1/2}\right]$$

where $f(\cdot)$ and $F(\cdot)$ are respectively the standard normal density function and distribution function which are evaluated at $\left[\frac{e}{\sigma}\sqrt{\frac{\gamma}{1-\gamma}}\right]$

The next issue is to estimate firm-specific X-inefficiency measures for individual observations. Using the firm-specific prices and the production frontier, these measures are calculated as explained above.

IV. Data and Empirical Results

A random sample of 100 power loom units were selected from Coimbatore District which is an important centre of power loom concentration in Tamil Nadu, India. The annual figures of various varieties woven and production costs were collected for the financial year 1981-82.[7] A Cobb-Douglas type of production function was estimated, using the above data.[8]

$$\ln(\text{output}) = \alpha_0 + \alpha_1 \ln(\text{Labour}) + \alpha_2 \ln(\text{Material inputs}) + \alpha_3 \ln(\text{Capital}) + u + v. \tag{13}$$

The maximum likelihood estimates of the parameters of the above function are given in Table 1. The production parameters are significant at the 5 percent level and they do have theoretically acceptable signs. The parameter γ, which is the ratio of variance of firm-specific characteristics to total variance, is significant at the 1 percent level. This means that the variation of firms' actual outputs from their maximum possible outputs (technical inefficiency) are not just due to unexplained random shocks but due to their unidentifiable specific characteristics, which are denoted by the random variable u. Now, the significance of the inclusion of the random variable u, in explaining the variations of the dependent variable in equation (13), is further tested using a generalized likelihood ratio. If the random variable u is absent from the model, then the ordinary least squares estimates of the remaining parameters of the equation (13) are maximum likelihood estimates. Therefore, the negative of twice the logarithm of the generalized likelihood ratio has approximately χ^2-distribution with parameter equal to 1. The calculated ratio worked out to be 15.22 which is significant at the 1 percent level. This implies that the inclusion of the observation-specific random variable u in equation (13) is significant for explaining the variations of outputs from the maximum possible levels.

The firm-specific individual technical efficiency measures were calculated using equation (12). The measures are presented in Table 2, in the form of a frequency distribution. The results show that there is a wide variation in technical efficiency among the sample participants. Technical efficiency measures vary from 0.6815 to 0.9812. Firm-specific technical inefficiency measures can easily be worked out as $(1-e^{u})$ because when $e^{u} = 1$, this means that firms are technically efficient. Technical inefficiency measures vary from 2% to 32% for the sample firms.

The firm-specific individual measures of K are given in the form of a frequency distribution in Table 3. Again, a wide variation in the ratio of

7 The nature of the data set used in this study is explained in the appendix.

8 Capital is assumed to be the fixed input, and profit is defined as the difference between total revenue and total variable cost (Yotopoulos and Lau, 1973).

Table 1

MAXIMUM LIKELIHOOD ESTIMATES OF THE FIRM-SPECIFIC FRONTIER PRODUCTION FUNCTION FOR SAMPLE PARTICIPANTS

Variables	Unit of Measurement	Parameter	Estimates
Constant	—	α_0	4.1263 (1.0254)
Labour	Rupees	α_1	0.2716 (0.0902)
Material inputs	Rupees	α_2	0.3541 (0.1168)
Capital	Rupees	α_3	0.3683 (0.1247)
Ratio of variances σ_u^2/σ^2	—	γ	0.8525 (0.1632)
Total variance	—	σ^2	0.8319 (0.1587)
Log Likelihood Function			–18.1673

Notes: 1. Figures in parentheses are asymptotic standard errors of estimates.
2. All the coefficients are significant at the 1% level.

Table 2

FIRM-SPECIFIC TECHNICAL EFFICIENCY MEASURES FOR THE SAMPLE PARTICIPANTS

Efficiency Measure	Frequency	Minimum Value	Maximum Value
0.65-0.70	3	0.68	0.70
0.71-0.75	7	0.72	0.75
0.76-0.80	10	0.76	0.79
0.81-0.85	32	0.82	0.85
0.86-0.90	21	0.87	0.90
0.91-0.95	18	0.91	0.95
0.96-1.00	9	0.96	0.98
Total	100		

Table 3

FIRM-SPECIFIC MEASURES OF K FOR THE SAMPLE PARTICIPANTS

Measures of K	Frequency	Minimum Value	Maximum Value
0.75-0.80	10	0.76	0.80
0.81-0.85	11	0.82	0.84
0.86-0.90	13	0.87	0.90
0.91-0.95	18	0.92	0.94
0.96-1.00	20	0.96	0.98
1.01-1.05	9	1.02	1.05
1.06-1.10	7	1.06	1.09
1.11-1.15	6	1.12	1.15
1.16-1.20	4	1.16	1.20
1.21-1.25	2	1.23	1.25
Total	100		

actual to the X-efficient output at the firm level may be noted here. As expected, some sample firms seem to be overutilizing their inputs, while some appear to be underutilizing, as K is greater than and less than 1 respectively. The ratios range from 0.7616 to 1.25. Firm-specific X-inefficiency measures can be obtained as $|1-K|$, because when $K=1$, this means that firms are X-efficient. The X-inefficiency measures range from 2% to 24%. When these results are compared with technical inefficiency measures, it is obvious that the possibility of the latter measure overestimating the former measure cannot be ruled out.

V. Conclusion

The measurement of firm-specific technical inefficiency is determined by examining whether firms are able to produce on their production frontiers. The measurement of firm-specific X-inefficiency is determined by not only examining whether firms are producing on their frontier, but also examining whether they are using the combination of inputs consistent with some set performance standards. This means that the concept of technical efficiency is different from the concept of X-efficiency. The hypothesis tested in this paper is that the possibility of the measure of X-inefficiency being overestimated by the measure of technical inefficiency cannot be ruled out. Empirical results from the South Indian Power Loom Industry significantly support this hypothesis. This study also demonstrates that it is possible to measure X-efficiency within a modified neoclassical framework, using the frontier production methodology.

Appendix

Data for the present study come from a larger project examining the economic performance of the power loom industries in Coimbatare district in the state of Tamilnau, India financed by the Indian Council of Social Sciences Research. The second author was one of the co-ordinators of the research project. Although the project surveyed 310 power loom industries, a random sample of 100 industries was chosen for the present study in order to maintain the hypothesis of more of less similar technology being used by the sample industries. Further, the product mix and size are similar in all these sample industries, so that the impact of economies of scope and economies of scale on technical efficiency can be considered as negligible. The number of non-managerial workers in the sample industries varies between 100 and 150 which has been used to define the size of the industries.

Output refers to total revenue which is measured in rupees. Labour variable is measured by the total wage bill, expressed in rupees which includes the salaries of both managerial and non-managerial workers and other benefits such as the provident fund. In order to take care of the problem of differential labour quality, the labour variable has been represented by the total wage bill with the assumption that the latter may represent quality variations in the labour force. Capital refers to the net tangible fixed assets which include land and buildings, installations, machines and equipment, furniture and selling stock, expressed in rupees. The straight line method of depreciation has been used. Material inputs refer to the value of raw materials used, fuel, electricity charges and machinery maintenance costs, expressed in rupees.

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