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Anticipated Changes in Unemployment Compensation and Job Search Dynamics

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It is widely recognized that an increase/decrease in unemployment compensation (UC) increases/decreases unemployment duration by raising/reducing reservation wages. However, this is a comparative static finding. When we evaluate the effect of a policy which changes the level of UC, we should also take into account the effect of short run adjustments in addition to the long run effect. If a change in UC is anticipated, workers may accept temporary jobs and hence reduce reservation wages. This paper analyzes a situation in which a worker, in anticipation of a change in UC, searches for a job with the option to quit. This analysis also has some implications for introduction of UC, which will be inevitable in developing countries, and abolition of UC as well as a change in UC in developed countries.

I. Introduction

It is widely recognized that an increase/decrease in unemployment compensation (UC) increases/decreases unemployment duration by raising/reducing reservation wages (see, e.g., Katz (1977) and Mortensen (1986)). However, this is a comparative static finding. When a change in UC is anticipated, job search dynamics during the anticipation period may be different. For instance, anticipating an increase in UC may lead workers to accept temporary jobs and hence reduce reservation wages.

When we evaluate the effect of a policy which changes the level of UC, we should also take into account the effect of short run adjustments in addition to the long run effect. This paper analyzes a situation in which a worker, in anticipation of a change in UC, searches for a job with the

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option to quit.[1] This analysis also has some implications for introduction of UC, which will be inevitable in developing countries, and abolition of UC as well as a change in UC in developed countries.

II. Stationary Job Search

We consider a worker under the following circumstances. If the worker pays the search cost c, he is offered one job which pays him a constant wage each period until he quits. We say that the worker has a wage offer w when he is offered a job which pays him w each period, or when he has accepted this job and has not quit yet. At the beginning of period 0, the worker has a wage offer w and decides whether or not to accept the offer. If he rejects the offer, he searches for a new job. If he accepts the offer, then he is paid w and he will decide whether to keep the job or quit the job and then search for a new job. Whether the worker accepts the offer or rejects the offer and searches for a new job in period 0, he will have a wage offer at the beginning of period 1, and the process repeats infinitely. Neither on-the-job search nor firing is permitted. Each wage offer w is drawn from the same wage distribution F(w), with $F(0) = 0$ and $F(B) = 1$ for $B < \infty$. When the worker is looking for a job, he receives unemployment compensation b ($b \geq 0$) and pays search cost c in each period.

Let y_t be the worker's income in period t and call $\sum_{s=t}^{\infty} \beta^{t-s} y_s$ the worker's life cycle income in period t, where β ($0 < \beta < 1$) is a discount factor. We assume that $y_t = b-c$ if he searches for a job, and that $y_t = w$ if he has a job with wage w. The worker devises a strategy which maximizes the expected value of his life cycle income in period 0. Let V(w, b) be the expected value of the worker's life cycle income when the worker has offer w at hand and behaves optimally and the value of UC is b. We assume no recall. Then, Bellman's functional equation becomes

$$(1) \qquad V(w, b) = \max \{w + \beta V(w, b),\ b-c + \beta \int_0^{\infty} V(x, b) dF(x)\}.$$

It is known that the worker's optimal strategy is to accept offer w if and only if w is not less than a time invariant reservation wage in each period. Thus,

[1] Yoon (1988) considers anticipated changes in the arrival rate of job offers, but he assumes that the worker never quits a job. This unrealistic assumption removes most of the interesting parts of the dynamic analysis.

(2) $$V(w, b) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^0(b) \\ w^0(b)/(1-\beta) = b-c+\beta\int_0^\infty V(x, b)dF(x) & \text{if } w \leqslant w^0(b) \end{cases}$$

and

(3) $$w^0(b) = b-c+(\beta/(1-\beta))\int_{w^0(b)}^\infty (w-w^0(b))dF(w).$$

In (2) and (3), $w^0(b)$ is the reservation wage. From (3),

(4) $$dw^0(b)/db = (1-\beta)/(1-\beta F(w^0(b))).$$

By (4), we can conclude that an unanticipated increase/decrease in UC increases/decreases the worker's reservation wage. This is the conventional comparative static argument with which we will compare the results derived in our dynamic analysis.

III. Anticipated Change in UC and Job Search Dynamics

At the end of period 0, it is announced and believed that b will change to b' at the beginning of period T ($T \geqslant 2$). Let $V_t(w)$ be the expected value of the worker's life cycle income at the beginning of period t when the worker has offer w at hand and behaves optimally. Then, Bellman's functional equation becomes

(5) $$V_t(w) = \max\{UA_t(w), US_t\},$$

where

$$UA_t(w) = w + \beta V_{t+1}(w),$$
$$US_t = b-c+\beta\int_0^\infty V_{t+1}(x)dF(x).$$

$UA_t(w)$ is the worker's life cycle income in period t when he accepts an offer w in period t and behaves optimally afterwards. US_t is the worker's life cycle income of the worker in period t when he rejects the offer and searches for a new job in period t and behaves optimally afterwards.

We will derive w^*_t such that the worker's optimal strategy is to accept an offer w in period t if and only if $w \geqslant w^*_t$. We can derive w^*_t from the condition that

(6) $$UA_t(w^*_t) = US_t$$

At 0, the worker's problem is stationary because the change in b is unanticipated, and hence $V_t(w) = V(w, b)$, $w^*_t = w^0(b)$ for all $t \geqslant 0$. At T, the worker's problem is also stationary, and hence $V_t(w) = V(w, b')$, $w^*_t = w^0(b')$ for all $t \geqslant T$. By the principle of optimality, we solve the worker's problem in each period if we solve his problem in period 1. In the next section, we solve the worker's problem in period 1, using the terminal condition that $V_T(w) = V(w, b')$.

IV. Results

In this section, we solve the worker's problem in two cases.

Case 1. $b' < b$.

In this case, $w^0(b') < w^0(b)$ because $dw^0(b)/db > 0$. Anticipated abolition of UC is a subcase of this case, in which $b' = 0$. The following theorem characterizes the solution of the worker's problem when $b' < b$.

Theorem 1. If $b' < b$, then $w^*_0 = w^0(b) > w^*_1 > \ldots > w^*_{t-1} > w^0(b') = w^*_T = w^*_{T+1} = \ldots$.

Proof. See Appendix A.

Theorem 1 shows that the workers' reservation wage decreases by T. Therefore, in macroeconomic context, unemployment rate decreases by T. Moreover, the maximized expected value of an unemployed worker's life cycle income decreases by T because the value in period t is $V_t(w^*_t)$ and $V_t(w^*_t) = w^*_t/(1-\beta)$. In Case 1, by T, the worker may accept an offer w which is less than b-c, which is what he can obtain by remaining unemployed, because he may want to keep such a job for the future.

Case 2. $b' > b$.

In this case, $w^0(b') > w^0(b)$. Anticipated introduction of UC is a subcase of this case, in which $b = 0$. The following theorem characterizes the solution of the worker's problem when $b' > b$.

Theorem 2. If $b' > b$, then $w^*_T = w^*_{T+1} = \ldots = w^0(b') > w^*_0 = w^0(b) > w^*_1 \quad \ldots > w^*_{T-1}$.

Proof. See Appendix B.

Theorem 2 shows that the worker's reservation wage decreases by T–1 and jumps up at T. Therefore, in macroeconomic context, unemployment rate decreases by T–1 and jumps up at T. This occurs because the worker wants to accept a temporary job which he quit at T. It is standard for the reservation wage to decrease when the time horizon is finite.

In spite of the decrease in reservation wages over time by T–1, the maximized expected value of an unemployed worker's life cycle income increases by T. For $1 \leqslant t \leqslant T-1$, the value in period t is $V_t(w^*_t)$ and

$$(13) \qquad V_t(w^*_t) = ((1-\beta^{T-t})(1-\beta))w^*_t + (\beta^{T-t}/(1-\beta))w^*_T.$$

It is not clear from (13) whether V^*_t increases or decreases in t. However, we can see that V^*_t increases in t from the optimality of the worker's strategy.

We refer strategy profile at t to the worker's strategy, decided in period t, for the current and future period. If the unemployed worker uses optimal strategy profile at 0 in period 1, his expected life cycle income is $V_1 = V_0(w^*_0) + (b'-b) \sum_{i=T}^{\infty} \beta^{i-1} PR_i$, where PR_i is the probability that the worker will be unemployed in period i when he uses the optimal strategy profile at 0 in period 1. By the suboptimality of the strategy considered, $V_1(w^*_1) \geqslant V_1 > V_0(w^*_0)$. If the unemployed worker uses optimal strategy profile at t–1 in period t ($2 \leqslant t \leqslant T$), his expected life cycle income is $V_t = V_{t-1}(w^*_{t-1}) + (b'-b)\beta^{T-t} PR_T(t)$, where $PR_T(t)$ is the probability that the worker will be unemployed in period T when he uses optimal strategy profile at t–1 in period t. By the suboptimality of the strategy considered, $V_t(w^*_t) \geqslant V_t > V_{t-1}(w^*_{t-1})$.

V. Conclusions

It is widely recognized that an increase/decrease in unemployment compensation increases/decreases unemployment duration by raising/reducing reservation wages. However, this is a comparative static finding. When a change in UC is anticipated, job search dynamics during the anticipation period may be different. When we evaluate the effect of a policy which changes the level of UC, we should also take into account the effect of short run adjustments in addition to the long run effect. This paper considers a situation in which a worker, in anticipation of a change in UC, searches for a job with the option to quit.

We first consider the case when a worker anticipates at the end of period 0 a decrease in UC from b to b′ at the beginning of period T, and show that the worker's reservation wage, unemployment rate in macroeconomic context, and the maximized expected value of an unemployed worker's life cycle income decrease by T. We also find that by T, the worker may accept an offer w which is less than b-c, which is what he can obtain by remaining unemployed, because he may want to keep such a job for the future. These results also have some implications for abolition of UC because abolition of UC is a special case of a decrease in UC with $b' = 0$.

Then, we consider the case when a worker anticipates at the end of period 0 an increase in UC from b to b′ at the beginning of period T, and show that the worker's reservation wage and unemployment rate in macroeconomic context decrease by T–1 and jumps up at T. This occurs because the worker wants to accept a temporary job which he will quit at T. It is standard for the reservation wage to decrease when the time horizon is finite. In spite of the decrease in reservation wages over time by T–1, the maximized expected value of an unemployed worker's life cycle income increase by T. These results have particularly relevant implications for introduction of UC, which will be inevitable in developing countries, because introduction of UC is a special case of an increase in UC with $b = 0$.

Appendix A

Theorem 1. If $b' < b$, then $w^*_0 = w^0(b') > w^*_1 > \dots > w^*_{T-1} > w^0(b') = w^*_T = w^*_{T+1} = \dots$.

Proof. For $t = 0$ and $t \geqslant T$, the worker's problem is stationary and the results in Section 2 apply. We will prove for $1 \leqslant t \leqslant T-1$ by backward induction on t. At T,

$$\text{(A1)} \qquad V_T(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_T \\ w^*_T/(1-\beta) & \text{if } w \leqslant w^*_T. \end{cases}$$

Thus,

$$\text{(A2)} \qquad UA_{T-1}(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_T \\ w + (\beta/(1-\beta))w^*_T & \text{if } w \leqslant w^*_T, \end{cases}$$

$$\text{(A3)} \qquad US_{T-1} = b - c + \beta \int_0^\infty V_T(w)dF(w).$$

We have $w^*_0 > w^*_{T-1}$ because $UA_{T-1}(w)$ increases in w and

$$\text{(A4)}\quad \begin{aligned} UA_{T-1}(w^*_0) &= b-c+\beta\int_0^\infty V(w,\ b)dF(w) \\ &> US_{T-1} = b-c+\beta\int_0^\infty V(w,\ b')dF(w) \\ &> UA_{T-1}(w^*_T) = b'-c+\beta\int_0^\infty V(w,\ b')dF(w). \end{aligned}$$

Now, suppose that $w^*_0 > w^*_{T-i} > w^*_{T-i+1}$ for $i=1,\ \ldots,\ s$, where $s<T-1$. Then,

$$\text{(A5)}\quad V_{T-s}(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_{T-s} \\ w^*_{T-s}/(1-\beta) & \text{if } w \leqslant w^*_{T-s}, \end{cases}$$

$$\text{(A6)}\quad UA_{T-s-1}(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_{T-s} \\ w + (\beta/(1-\beta))w^*_{T-s} & \text{if } w \leqslant w^*_{T-s}, \end{cases}$$

$$\text{(A7)}\quad US_{T-s-1} = b-c+\beta\int_0^\infty V_{T-s}(w)dF(w).$$

We have $w^*_0 > w^*_{T-s-1} > w^*_{T-s}$ because $UA_{T-s-1}(w)$ increases in w and

$$\text{(A8)}\quad UA_{T-1}(w^*_0) > US_{T-s-1} > UA_{T-1}(w^*_{T-s}).$$

Q.E.D.

Appendix B

Theorem 2. If $b'>b$, then $w^*_T = w^*_{T+1} = \ldots = w^0(b') > w^*_0 = w^0(b) > w^*_1 > \ldots > w^*_{T-1}$.

Proof. For $t=0$ and $t \geqslant T$, the worker's problem is stationary and the results in Section 2 apply. At T,

$$\text{(B1)}\quad V_T(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_T \\ w^*_T/(1-\beta) & \text{if } w \leqslant w^*_T. \end{cases}$$

Thus,

$$\text{(B2)}\quad UA_{T-1}(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_T \\ w + (\beta/(1-\beta))w^*_T & \text{if } w \leqslant w^*_T, \end{cases}$$

$$\text{(B3)}\quad \begin{aligned} US_{T-1} &= b-c+\beta\int_0^\infty V_T(w)dF(w) \\ &= b-b'+w^*_T/(1-\beta). \end{aligned}$$

We have $w^*_0 > w^*_{T-1}$ because $UA_{T-1}(w)$ increases in w and

(B4) $$UA_{T-1}(w^*_0) - US_{T-1} = (w^*_0 - b) - (w^*_T - b') > 0.$$

If T = 2, the proof is completed.

Suppose that $T \geqslant 3$ and we prove this theorem to be true when $1 \leqslant t \leqslant T-2$ by backward induction on t. At T−1,

(B5) $$V_{T-1}(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_T \\ w + (\beta/(1-\beta))w^*_T & \text{if } w^*_{T-1} \leqslant w \leqslant w^*_T \\ w^*_{T-1} + (\beta(1-\beta))w^*_T = b-c+\int_0^\infty V_T(w)dF(w) & \text{if } w \leqslant w^*_{T-1}. \end{cases}$$

Thus,

(B6) $$UA_{T-2}(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_T \\ (1+\beta)w + (\beta^2/(1-\beta))w^*_T & \text{if } w^*_{T-1} \leqslant w \leqslant w^*_T \\ w + \beta w^*_{T-1} + (\beta^2/(1-\beta))w^*_T & \text{if } w \leqslant w^*_{T-1}, \end{cases}$$

(B7) $$US_{T-2} = b-c+\beta\int_0^\infty V_{T-1}(w)dF(w).$$

We have $w^*_T > w^*_{T-2} > w^*_{T-1}$ because $UA_{T-1}(w)$ increases in w and

(B8) $$\begin{aligned} UA_{T-2}(w^*_T) &= b'-c+\beta\int_0^\infty V_T(w)dF(w) \\ &> US_{T-2} = b-c+\beta\int_0^\infty V_{T-1}(w)dF(w) \\ &> UA_{T-2}(w^*_{T-1}) = b-c+\beta\int_0^\infty V_T(w)dF(w) \\ &\quad -\beta(w^*_T - w^*_{T-1}). \end{aligned}$$

The second inequality of (B8) follows from the fact that $\int_0^\infty (V^*_T(w) - V^*_{T-1}(w))dF(w) < w^*_T - w^*_{T-1}$.

Now, suppose that $w^*_T > w^*_{T-i} > w^*_{T-i+1}$ for $i = 1, \ldots, s$, where $s < T-1$. Then,

(B9) $$V_{T-s}(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_T \\ ((1-\beta^s)/(1-\beta))w + (\beta^s/(1-\beta))w^*_T & \text{if } w^*_{T-s} \leqslant w \leqslant w^*_T \\ ((1-\beta^s)/(1-\beta))w^*_{T-s} + (\beta^s/(1-\beta))w^*_T = b-c+\beta\int_0^\infty V_{t-s+1}(w)dF(w) & \text{if } w \leqslant w^*_{T-s}, \end{cases}$$

(B10) $UA_{T-s-1}(w) = \begin{cases} w/(1-\beta) & \text{if } w \geqslant w^*_T \\ ((1-\beta^{s+1})/(1-\beta))w + (\beta^{s+1}/(1-\beta))w^*_T & \text{if } w^*_{T-s} \leqslant w \leqslant w^*_T \\ w + ((\beta-\beta^{s+1})/(1-\beta))w^*_{T-s} + (\beta^{s+1}/(1-\beta))w^*_T & \text{if } w \leqslant w^*_{T-s}, \end{cases}$

(B11) $US_{T-s-1} = b-c+\beta\int_0^\infty V_{T-s}(w)dF(w).$

We have $w^*_T > w^*_{T-s-1} > w^*_{T-s}$ because $UA_{T-1}(w)$ increases in w and

(B12) $$\begin{aligned} UA_{T-s-1}(w^*_T) &= b'-c+\beta\int_0^\infty V_T(w)dF(w) \\ &> US_{T-s-1} = b-c+\beta\int_0^\infty V_{T-s}(w)dF(w) \\ &> UA_{T-s-1}(w^*_{T-s}) = b-c+\beta\int_0^\infty V_{T-s+1}(w)dF(w) \\ &\quad -\beta^s(w^*_T-w^*_{T-s}). \end{aligned}$$

The second inequality of (B12) follows from the fact that

$$\int_0^\infty (V^*_{T-s+1}(w)-V^*_{T-s}(w))dF(w) < \beta^{s-1}(w^*_T-w^*_{T-s}).$$

We will complete the proof if we show that $w^*_0 > w^*_1$. By the condition that $UA_{T-s-1}(w^*_{T-s-1}) = US_{T-s-1}$,

(B13) $$\begin{aligned} w^*_{T-s-1} = b-c+(\beta/(1-\beta))\Big[&\int_0^{w^*_{T-s}}(1-\beta^s)(w^*_{T-s}-w^*_{T-s-1})dF(w) \\ &+\int_{w_{T-s}}^{w^*_T}(1-\beta^s)(w-w^*_{T-s-1})dF(w) \\ &+\int_{w^*_T}^\infty\{(1-\beta^s)(w-w^*_{T-s-1})+\beta^s(w-w^*_t)\}dF(w)\Big]. \end{aligned}$$

Let $w^0_T = w^*_T$ and define inductively for $i \geqslant 1$,

(B14) $$\begin{aligned} w^0_{T-i} = b-c+(\beta/(1-\beta))\Big[&\int_0^{w^0_{T-i+1}}(1-\beta^{i-1})(w^0_{T-i+1}-w^0_{T-i})dF(w) \\ &+\int_{w^0_{T-i+1}}^{w^0_T}(1-\beta^{i-1})(w-w^0_{T-i})dF(w) \\ &+\int_{w^0_T}^\infty\{(1-\beta^{i-1})(w-w^0_{T-i})+\beta^{i-1}(w-w^0_T)\}dF(w)\Big]. \end{aligned}$$

Then, $w^0_{T-i} = w^*_{T-i}$ for $1 \leqslant i \leqslant T$. Let $w^0 = \lim_{i\to\infty} w^0_{T-1}$. By taking limit i on both sides of (B14), we have

(B15) $w^0 = b-c+(\beta/(1-\beta))\int_{w^0}^\infty (w-w^0)dF(w).$

Thus, $w^0 = w^0(b) = w^*_0$. Because w^0_{T-i} increases in i, $w^*_0 > w^*_1$. Q.E.D.

References

Katz, A., ed., "The Economics of Unemployment Insurance: A Symposium," *Industrial and Labor Relation Review,* 30, 1977, 431-526.

Mortensen, D.T., "Job Search," in *Handbook of Labor Economics,* 2, Ashenfelter O. and R. Layard, eds., North-Holland, Amsterdam, 1986, 849-919.

Yoon, B-J., "Continuous Time Optimization, Reservation Wage Path and External Shocks," *Seoul Journal of Economics,* 1, 1988, 417-430.