Lender’s Risk Hypothesis, Rural Interest-Rate and the Shadow Wage Rate in the Urban Sector

Manash Ranjan Gupta*

This paper considers the role of 'Lender's Risk Hypothesis' and the determination of the rural interest-rate on the 'Shadow Wage Rate' in the urban sector. The 'Shadow Wage Rate' takes a higher value in this case than that it takes in the 'Standard Analysis.'

I. Introduction

In recent years, there have been some attempts to evaluate the shadow wage rate (SWR) in the urban sector in the presence of a rural-urban wage gap that induces extra migration and results into an urban unemployment equilibrium. The Harris-Todaro (1970) model first explains urban unemployment as a migration-equilibrium phenomenon with the expected income maximizing behaviour of the migrants. Assuming that there is an institutionally fixed urban wage rate, they introduce a special type of rural-urban migration mechanism in which the representative rural worker migrates to the urban sector if his probability of getting an urban job exceeds the ratio of rural wage to urban wage. The probability that a rural worker migrates to the town is defined as the ratio of urban employment to the urban labour force (including the employed and the unemployed workers in the urban sector). A migration equilibrium is established when the actual rural wage rate equals the expected urban wage rate defined as the actual urban wage rate multiplied by the migrant’s probability of getting an urban job. The existence of urban unemployment is then explained as a property of migration-equilibrium when the actual rural wage rate falls short of the urban wage rate.

The traditional view that the SWR should lie below the actual urban wage rate in the presence of a rural-urban wage-gap has been reanalyzed

* Professor, Department of Economics, Jadavpur University, Calcutta, West Bengal, India.
in such a framework. If the migration mechanism is of Harris-Todaro type, the per-capita consumption in the economy equals the rural wage rate. The standard analysis on the SWR considers a dual economy model with Harris-Todaro (1970) migration mechanism. This assumes that the urban sector's surplus is the only source of investment and the social welfare is defined as a positive function of per-capita consumption. So if the migration from the rural sector does not affect the rural wage rate, SWR equals the actual urban wage rate even if the rural wage rate falls short of the actual urban wage rate. However, with diminishing marginal productivity of labour and the marginal productivity pricing of labour in the rural sector, migration from the rural sector raises the rural wage rate and hence social welfare. So in the standard analysis the SWR lies below the actual urban wage rate in a Harris-Todaro (1970) type of model.

In two earlier papers, the present author has pointed out two factors consideration of which may lead to the higher value of SWR than what it takes in the standard analysis. First paper considers the investment of rural sector's surplus in addition to the urban sector's surplus. Additional employment in the urban sector lowers the level of employment in the rural sector. This raises the rural wage rate, but reduces the rural sector's surplus (investment). So the rise in social welfare is accompanied by a fall in the social value of rural sector's investment. The SWR takes a higher value than that it takes in the standard analysis. It exceeds the actual wage rate if the reduction in the social value of rural sector's investment is greater than the increase in the social welfare. The second paper evaluates the SWR in a Harris-Todaro (1970) model using the welfare function of Sen (1974) which is a positive function of the per-capita consumption and a negative function of the Gini-coefficient of the distribution of consumption. There are three different income groups within the working class in a Harris-Todaro model. They are: (i) the urban workers who earn a high wage rate, (ii) the rural workers who earn a relatively lower but positive wage rate and (iii) the unemployed workers in the urban sector who do not earn anything. Hence there is a positive Gini-coefficient of the distribution of consumption (worker's income). Additional urban employment worsens the problem of urban unemployment; and hence leads to a higher value of the Gini-coefficient with a marginal increase in

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1 Assuming that the entire wage income in the economy is consumed and normalizing the labour force to unity.
2 This is because in the Harris-Todaro (1970) model there is no surplus labour in the rural sector.
3 See Gupta (1986).
4 See Gupta (1988).
the per-capita consumption (rural wage rate). As a result the effect of additional urban employment on social welfare is smaller than this effect in the standard analysis. More importantly, this may lead to a possibility of a fall in social welfare with increase in the level of urban employment. Hence the SWR may be higher than the actual wage rate in the urban sector.

This note points out a third factor consideration of which shows that the SWR takes a higher value than that it takes in the standard analysis. The existing literature on the SWR considers a dual economy model where the rural sector is characterized by capitalist farming and competitive labour market, but does not focus on the credit market in the rural sector. The rural worker often takes loan from the professional money-lenders to finance his consumption in the lean season when he does not get any job; and pays it back to the money-lender with interest in the peak season. Hence the welfare of a rural worker is not only a positive function of the wage rate, but also a negative function of the interest rate charged by the money-lender. The effect of additional employment creation in the urban sector on the welfare of the rural worker is now determined by its effect on the rural wage-rate as well as on the rural sector's interest-rate. According to 'Lender’s Risk Hypothesis' (LRH) of Bottomley (1963, 1975) the rural money-lender charges a higher rate of interest when the probability of default of loan is higher. The probability of default of loan taken by a rural worker should be a positive function of his probability of getting an urban job because the loan can not be recovered if the rural worker migrates to the town. Addition to urban employment leads to an increase in the probability of the rural worker's getting an urban job and hence to an increase in the probability of default. Thus additional employment creation in the urban sector raises the interest rate in the rural sector. On the otherhand, because of the assumption of diminishing marginal productivity of labour and the marginal productivity-pricing of labour, addition to urban employment lowers the level of employment in the rural sector; and hence raises the rural wage rate. Since the welfare of the rural worker is now a negative function of the rate of interest, additional urban employments leads to a lower rate of increase in the welfare of the rural sector’s worker (and hence the welfare of the society) than that in the standard analysis. So the SWR in the urban sector takes a higher value than what it takes in the standard analysis.

II. The Model

As we wish to focus on the role of the 'Lender’s Risk Hypothesis' and
the determination of the rural rate of interest on the determination of the SWR, we consider a model otherwise similar to a standard model. The economy is assumed to be closed to international trade. It consists of the two sectors — urban and rural, denoted by the indices 1 and 2 respectively. Both the sectors produce the same commodity.\footnote{This is a simplifying assumption. If the terms of trade between the two sectors is exogeneous, the assumption that the two sectors produce two different commodities is analytically equivalent to the assumption that both the sectors produce the same commodity. However the terms of trade changes with change in urban sector's employment. In that case our result will be valid at the cost of some additional necessary conditions.}

The production function of the $i$th sector is given by

$$(1) \quad Y_i = F_i(K_i, L_i)$$

for $i = 1, 2$. Here $Y_i$, $K_i$ and $L_i$ represent the level of net output, capital stock (installed and utilized) and the level of employment in the $i$th sector. The production function satisfies all the standard neo-classical properties.

The planner in the urban sector employs labour at an institutionally fixed wage rate, denoted by $\bar{w}_1$. Surplus in the urban sector is the only source of investment.\footnote{We assume this because this assumption is an important feature of the standard analysis on the SWR. In one of my earlier papers (OEP, March, 1986) I have shown that if the rural sector's surplus is invested, then SWR takes a higher value than that it takes in the standard analysis. But in this paper, the objective is to show that even if the rural sector's surplus is not invested, the consideration of the determination of rate of interest rate and the lender's risk of default of loan in the rural sector leads to a higher value of SWR than that it takes in the standard analysis. That is why investment of rural sector's surplus is not considered in this paper.} Denoting, the level of investment by $I$; we have

$$(2) \quad I = Y_1 - \bar{w}_1 L_1$$

Output in the urban sector is the only source of financing the cost of labour in that sector; and hence

$$(3) \quad I \geq 0$$

We also assume that fixed-working hour system prevails in the urban sector. So the urban sector's worker can not control his labour-leisure allocation. Also it is already assumed that the urban sector's wage rate is institutionally fixed. Hence it is obvious that the utility of an urban sector's worker is exogeneous to the analysis. Let $U_1$ be the level of utility of the representative worker in the urban sector.
In the rural sector, the landlords maximize profit; and hence in equilibrium

\[ \frac{\partial Y_2}{\partial L_2} = W_2 \]

Here \( W_2 \) is the rural sector's wage rate. The landlords consume their profit.\(^7\) In the presence of diminishing marginal productivity of labour in the rural sector, and given \( K_2 \), from equation (4), we have

\[ W_2 = W_2(L_2) \text{ with } W_2'(L_2) < 0 \]

So the wage-rate and the level of employment in the rural sector are inversely related. Since there does not exist any unemployment in the rural sector,\(^8\) migration from the rural sector lowers its level of employment and hence raises the wage rate.

The rural worker takes consumption loan from the money-lender at the beginning of each period and pays it back with interest at the end of it when he gets his wage-income. The rate of interest charged by the money-lender is denoted by \( i \). The worker has no access to the organized credit market where the rate of interest is \( r \). We assume that the rural worker has an utility function\(^9\) given by the following:

\[ U_2 = U_2(i, W_2) \]

with the following properties

\[ (\partial U_2 / \partial i) < 0, \text{ and } (\partial U_2 / \partial W_2) > 0. \]

Rural workers migrate to the urban sector and the migration-decision is determined by the expected utility maximizing behaviour of the migrant. Let \( I_u \) be the number of workers remaining unemployed in the urban sector. We ignore population growth over time\(^10\) and assume that the size of the labour force of the entire economy is equal to unity. So

\[ L_1 + L_2 + I_u = 1 \]

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\(^7\) The logic behind this assumption is given in footnote no. 6.

\(^8\) Harris-Todaro (1970) model is based upon the assumption that there is no unemployment in the rural sector.

\(^9\) This is an indirect utility function.

\(^10\) The assumption of zero growth rate of population should not seriously bias the results because the model is not a dynamic one.
Following Harris and Todaro (1970) we define \( \left( \frac{L_1}{L_1 + L_u} \right) \) as the probability of the representative rural migrant of getting an urban job. Hence \( \left( \frac{L_1}{L_1 + L_u} \right) \) is his expected utility if he migrates to the urban sector. Rural-urban migration continues so long as \( \left( \frac{U_1}{L_1 + L_u} \right) > U_2 \) and the reverse migration takes place when \( \left( \frac{U_1}{L_1 + L_u} \right) < U_2 \). In migration-equilibrium each migrant is indifferent between the indirect utility he derives staying in the rural sector and the expected utility he obtains migrating to the urban sector. Mathematically,

\[
(7) \quad U_1 L_1 = (L_1 + L_u) U_2
\]

Now we turn to the determination of the rate of interest, \( i \), in the rural sector; and while explaining this we consider the 'Lender's Risk Hypothesis' of Bottomley (1963, 1975). The money-lender in the rural sector faces a positive risk of default. Suppose that the money-lender finds that \( q \) is the probability that the loan will not be repaid. Then if the money-lender gives loan to the worker at the rate of interest, \( i \), his effective rate of interest is \( (i(1-q) - q) \). In equilibrium, his effective rate of interest in the unorganized credit market is equal to the rate of interest in the organized credit market to which he has access.

Hence,

\[
(8) \quad r = i(1-q) - q
\]

Since \( q < 1, i > r \). This explains how the existence of the lender's risk of default keeps the rural rate of interest above the rate of interest in the organized sector.

The money-lender faces the positive risk of default because the rural worker has a positive probability of leaving the rural sector. Once the rural worker migrates to the urban area, the loan can not be recovered. We assume that the probability of default of loan is a positive function of the probability of the rural worker's getting an urban job. Mathematically,

\[
(9) \quad q = q\left( \frac{L_1}{L_1 + L_u} \right) \text{ with } q' (\cdot) > 0
\]

Note that here equations (4a), (5), (6), (7), (8) and (9) close the model. The planner determines the value of \( L_1 \) subject to the constraint (3) and the institutionally fixed urban wage rate, \( \bar{W}_1 \). Given the value of \( L_1 \), the equations (4a), (5), (6), (7), (8) and (9) determine the equilibrium values of \( W_2, L_2, L_u, i, q \) and \( U_2 \). So if the planner changes the level of
employment in the urban sector, automatically the equilibrium values of \( W_2, L_2, L_x, i, q \) and \( U_2 \) are changed.

The standard analysis on SWR does not focus on the determination of the rural interest rate. So if we assume \( i \) to be fixed and drop the equations (8) and (9), the model considered in this paper is reduced to a standard model.

III. The SWR

Let \( V \) be the level of social welfare and \( P \) be the marginal social value of investment. So the SWR in the urban sector is to be evaluated by maximizing \((V + PI)\) by the choice of \( L_1 \) subject to the constraint (3). Assuming that an interior solution exists, we have

\[
(10) \quad SWR = \frac{dP}{dL_1} = \frac{\Delta W_1}{P} - (1/P) \frac{dV}{dL_1}
\]

Since \( P \) and \( \Delta W_1 \) are given, it is obvious that the value of SWR depends on the effect of additional urban employment on the level of social welfare. We define

\[
(11) \quad V = \Delta U_1 L_1 + L_2 U_2 (i, W_2)
\]

This is a standard welfare function generally used in evaluating the SWR.\textsuperscript{11} It implies that the welfare of the society is an weighted average of the welfare of the rural and urban workers, weight being the level of employment in the corresponding sector.

Using equations (6), (7) and (11), we have,

\[
(11a) \quad V = U_2 (i, W_2)
\]

\textsuperscript{11} Note that there are three different income-groups in a Harris-Todaro (1970) type of model. They are: (i) the urban workers who earn a high wage rate, (ii) the rural workers who earn a relatively lower but positive wage rate, and (iii) the unemployed workers in the urban sector who do not earn anything. Hence there is a positive degree of inequality of income (consumption) of the labourers. In one of my earlier papers (OEP, September, 1988), I have shown that if social welfare is defined not only as a positive function of per-capita consumption, but also as a negative function of the degree of inequality, then the SWR takes a higher value than that it takes in the standard analysis. But the objective of this paper is to show that the consideration of the rate of interest determination and the lender's risk of default of loan raises the value of SWR even if one considers a standard welfare function. That is why we consider a standard welfare function in this paper.
(12) \[ \frac{dV}{dL_1} = \frac{\partial U_2}{\partial i}(\frac{dL_1}{dL_1}) + \left( \frac{\partial U_2}{\partial W_2}W_2'(L_2) \right) \frac{dL_2}{dL_1} \]

We know that \( \frac{\partial U_2}{\partial i} < 0; \quad \frac{\partial U_2}{\partial W_2} > 0 \); and \( W_2'(L_2) < 0 \). So the sign and the magnitude of \( \frac{dV}{dL_1} \) depends on the sign and magnitude of \( \frac{dL_1}{dL_1} \) and \( \frac{dL_2}{dL_1} \).

From equation (8), we have,

(13) \[ \frac{di}{dq} = \frac{(1+r)}{(1-q)^2} > 0 \]

So the higher the probability of default of loan, the higher is the rural interest rate.

Also from equations (6) and (9), we have,

(14) \[ \frac{dq}{dL_1} = \frac{q'(-)}{(1-L_2)} \left( 1 + \frac{L_1}{(1-L_2)} \frac{dL_2}{dL_1} \right) \]

Here \( q'(-) > 0 \); and \( (1-L_2) > 0 \). So the sign of \( \frac{dq}{dL_1} \) depends on the sign of \( \frac{dL_2}{dL_1} \).

Using equations (13) and (14) we have,

(15) \[ \frac{di}{dL_1} = \frac{(1+r)}{(1-q)^2 \,(1-L_2)} \left( 1 + \frac{L_1}{(1-L_2)} \frac{dL_2}{dL_1} \right) \]

Now we evaluate \( \frac{dL_2}{dL_1} \). Using equations (5), (6) and (7), we have,

(7a) \[ \bar{u}_1 \quad L_1 = (1-L_2) \quad U_2 \quad (i, \quad W_2) \]

Hence,

(16) \[ \frac{dL_2}{dL_1} = \frac{\bar{u}_1 - \left( \frac{\partial U_2}{\partial i} \left( 1 + \frac{L_1}{(1-L_2)} \frac{dL_2}{dL_1} \right) \right)}{U_2 - \left( \frac{\partial U_2}{\partial i} \left( 1 + \frac{L_1}{(1-L_2)} \frac{dL_2}{dL_1} \right) \right)} \]

Here \( \frac{\partial U_2}{\partial i} < 0, \quad q'(\cdot) > 0, \quad \frac{\partial U_2}{\partial W_2} > 0 \) and \( W_2'(L_2) < 0 \).
So, \((dL_2/dL_1)<0\). This implies that an increase in the level of urban employment lowers the level of employment in the rural sector. Now equations (4a) and (16) show that the addition to urban employment raises the rural wage-rate.

Since \((dL_2/dL_1)<0\), \(W'_2(L_2)<0\) and \((\partial U_2/\partial W_2)>0\), the second term of the R.H.S. in equation (12) is positive. But the sign of the first term of the R.H.S. of equation (12) depends on the sign of \((dL/\partial L_1)\); and this in turn depends on the sign of \((1+7L/(1-L_2)(dL_2/dL_1))\).

From equation (16), we have,

\[
1 + \frac{L_1}{1-L_2} \left( \frac{dL_2}{dL_1} \right) = 1 - \frac{\tilde{U}_1 L_1}{(1-L_2)} \left( \frac{\partial U_2/\partial i}{(1-q)^2 (1-L_2)} \right)
\]

\[
-\left[ U_2 - (\partial U_2/\partial i) \frac{(1-t) q'(\cdot) L_1}{(1-q)^2 (1-L_2)} \right]
\]

\[-(\partial U_2/\partial W_2) W'_2(L_2) (1-L_2)\]

Since \(W'_2(L_2)<0\), \((\partial U_2/\partial W_2)>0\), \((\partial U_2/\partial i)<0\) and \(q'(\cdot)>0\), then it is obvious that

\[0 < 1 + \frac{L_1}{1-L_2} + \left( \frac{dL_2}{dL_1} \right) < 1\]

Hence from (15) we have \((dL/\partial L_1)>0\). So the additional urban employment raises the interest rate in the rural sector.

Now using equations (12), (15), (16) and (17), we have,

\[
\frac{dV}{dL_1} = \frac{-\left(\partial U_2/\partial W_2\right) W'_2(L_2) \cdot \tilde{U}_1}{\left[ U_2 - (\partial U_2/\partial i) \frac{(1-t) q'(\cdot) L_1}{(1-q)^2 (1-L_2)} \right]}
\]

\[-(\partial U_2/\partial W_2) W'_2(L_2) (1-L_2)\]

Now from the restrictions imposed on the different derivatives, it is obvious that \((dV/\partial L_1)>0\). So from equation (10), we have \(\text{SWR}<\bar{W}_1\).

But in the standard analysis, \(i\) is treated as exogeneous. Hence the second term in the denominator does not appear. So
\[
\frac{dV}{dL_1} = \frac{-(\partial U_2/\partial W_2) W_2' (L_2)}{U_2-(\partial U_2/\partial W_2) W_2' (L_2)(1-L_2)} \tilde{U}_1
\]

Note that the second term in the denominator of R.H.S. of equation (18) is negative and there is one minus sign in front of it. So the R.H.S. of (18) takes a lower value than the value taken by the R.H.S. of (19). So the marginal contribution of urban employment on the welfare of the society in the present model is lower than that in a standard model. Hence from equation (10), it is clear that the SWR in the present model is higher than that in the standard analysis.

We now turn to the intuitive explanation of why SWR in the present model takes a higher value than that in the standard model. The Harris-Todaro (1970) migration mechanism makes the welfare of the society to be equal to the welfare of the representative rural worker. In a standard model additional urban employment lowers the level of rural employment and hence raises the rural wage rate. So the welfare of the society (welfare of the rural worker), being a positive function of the rural wage rate, rises when the addition to the urban employment takes place. But in the present model, on the one hand, additional urban employment raises the rural wage rate; and on the other hand leads to an increase in the probability of the rural worker's migrating to the urban sector. The latter increases the probability of default of loan in the rural sector. So the rural interest rate increases. Since the welfare of the rural worker is a positive function of the rural wage rate and a negative function of the rural interest rate, the positive effect of a rise in the wage rate on the social welfare is partly off set by the negative effect of an increase in the interest rate. So the SWR in the present model takes a higher value than that it takes in the standard analysis.

References


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