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# Dynamic Interrelationship between Investment and Finance Decisions

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In this paper we discuss the general interrelationship between the investment decisions for inventory, fixed capital and working capital, and the finance decisions for retained earning, debt and external equity. These dynamic interrelated decisions are integrated in the context of the deterministic optimal control theory. My model is based on Lin (1981), and extends it by adding the debt finance and the investment for inventory and working capital. Ultimately, our discussion can contribute to the microfoundation of the interrelationship between macroeconomic fluctuations and financial structure.

## I. Introduction

Modigliani and Miller (1958) have established a powerful proposition that the optimal investment decisions of the firm can be separated from its finance and dividend decisions under the assumption of perfect capital market. Thereafter, Linter (1963) and others have analyzed the issue of optimal investment and finance in the growing firm with the different specifications of available financial resorces, factor market status, investment possibility function, and the degree of uncertainty. In these studies, however, the decisions of firm's production and investment are given and only the optimal issuance of equities in the capital market is analyzed. In addition, it is required that the firm follows a constant growth in earning or dividend. Therefore, these studies are not satisfactory to analyze the interaction of real and financial decisions at the firm level.

Apart from the literature of finance, the significance of financial impact on fixed capital investment has been a controversy in the literature of investment. Even though there are several empirical studies admitting the

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importance[1] of internal finance on the firm's fixed capital investment most previous studies based on accelerator model or neoclassical model do not accept the significance of financial impact on fixed capital investment. Even 'q' theory emphasizing the effect of firm's equity value change on the firm's investment level assumes that "investment is financed entirely by retained profit or that an exogenously given constant fraction of investment is financed by debt. This is a big contrast to the corporate finance theory whose main concern is the optimal capital structure with an exogenously given level of investment" [quoted from Hayasi(1985)].

Alternatively, Krouse (1973) and Lin (1981) have developed intertemporal dynamic model to analyze the interaction of real and financial decisions at the firm level. Krouse (1973) pioneered a discrete time model based on the optimal control theory to integrate the investment and equity finance decisions by using time varying discount rate and earning possibility function. Lin (1981) extended Krouse (1973) by accommodating the adjustment costs of investment and the transaction costs of equity finance. However, Lin (1981) has also such limitations as omitting a debt finance, and adapting a single growth rate for corporate earning and equity capital.

In this paper we will discuss the general interrelationship between the investment decisions for inventory, fixed capital and working capital, and the finance decisions for retained earning, debt and external equity. These dynamic interrelated decisions are integrated in the context of the deterministic optimal control theory. Especially, my main concern is to check mathematically the feasibility of the optimal investment for retained earning, debt and external equity and its stability condition.

My model is based on Lin (1981), and extends it by adding the debt finance and the investment for inventory and working capital. Debt, especially credit, plays a dominant role in the investment for inventory and working capital and influences the investment for fixed capital of small and medium size firms through long-term credit (see Flink (1979)). Even for large firms external equity financing is often not available because of disagrement between managers and investors or large premium required for equity issuing arisen from information asymmetry. Therefore, finance through external equity is usually treated as the last resort (see Myers and Majluf (1984) or Steigum (1983)).

The determination of optimal working capital has been discussed in the context of firm's portfolio in the finance literature (for example,

1 See Meyer and Kuh (1957), Meyer and Glauber (1963), Resek (1966), Fazzair and Mott (1986).

Bierman and Thomas (1975)). However, it is seldom analyzed in the context of firm's investment behavior related with financial availability. Richard and Smith (1973), Merville and Tavis (1973) have estimated the demand for working capital just empirically without any theoretical base which is explicitly discussed here. Ultimately, our discussion can work as a microeconomic foundation to enlighten the financial propagative process on the business cycle.

In chapter 2 we define a few variables necessary for an intertemporal dynamic model and discuss the interactive channels of investment and finance decisions at the firm level. In chapter 3 we set up the firm's dynamic intertemporal optimization model and then derive the optimal time pathes and long-run equilibrium demands for retained earning, debt, and external equity. Finally, we summarize our discussion and point out two limitations in my model for the further studies.

## II. Interrelationship between Investment and Finance Decisions

### A. *Definition of Variables*

In order to construct an intertemporal dynamic model to finance for the firm's investments, we need to define the following variables.

- $Y(t)$ : the (pretext) net corporate profit or earning at time t,
- $N(t)$ : the stock of working capital such as cash, securities or trade credit,
- $H(t)$ : the stock of finished goods inventories at time t,
- $K(t)$ : the stock of fixed capital at time t such as plant or equipment,
- $A(t)$ : the value of total asset at time t; $A(t)=N(t)+H(t)+(t)$,
- $M(t)$ : the value of corporate debt at time t. We assume that there is a constant average interest rate charged to the total corporate debt,
- $S(t)$ : the stock of equity capital at time t, or market value of existing equities at time t; $S(t)=\int^t [R(s)+\delta E(s)]ds$
- $E(t)$ : the value of external equity acquired at time t. If $E(t)<0$, it implies retirement of existing equities,
- $R(t)$ : the value of retained earning at time t,
- $B(t)$ : the value of new debt acquired at time t,
- $D(t)$ : the value of dividends paid at time t; $D(t)=Y(t)-R(t)$,
- $\delta$ : the ratio of residual external equity after minusing the constant fractional flotation cost, $0\leqslant\delta\leqslant1$
- $\phi(t)$ : $\int \beta(s)ds$ where $\beta(s)$ is the instanttaneous discount rate at time s; $\exp(-\phi(t))$ is the discount factor at time zero for the current

values at time t,

W(t) : the ratio of the external issue price of a share to its market price at time t. $W(t) \leqslant 1$ implies issuance at a discount, and $W(t) \geqslant 1$ implies issuance of shares at a premium to market price [see Krouse (1973)].

The above variables can be summarized into a balance sheet.[2] The changes of balance sheet contents correspond to the firm's "financial capacity" which means the borrower's ability to absorb his debt without reducing either current or future spending commitments in order to avoid default or debt rescheduling.

The variables in this paper will be expressed in terms of value per total asset value at following.

y(t) : $Y(t)/A(t)$, the earning-total asset ratio at time t,
r(t) : $R(t)/A(t)$, the retained earning-total asset ratio at time t,
e(t) : $E(t)/A(t)$, the external equity-total asset ratio at time t,
c(t) : $[y(t)-r(t)]/s(t)$. dividends per share at time t. $c(t)>0$,
m(t) : $M(t)/A(t)$, the total debt-total asset ratio at time t,
s(t) : $S(t)/A(t)$, the total equity-total asset ratio at time t,
b(t) : $B(t)/A(t)$, the new debt-total asset ratio at time t,

**Balance Sheet**

| CR | DR |
|---|---|
| $N\ (=\int_{t-1}^{t} n(s)ds)$ | $M\ (=\int_{t-1}^{t} B(s)ds)$ |
| $H\ (=\int_{t-1}^{t} H(s)ds)$ | $S\ (=\int_{t-1}^{t} R(s)+\delta E(s))ds$ |
| $K\ (=\int_{t-1}^{t} I(s)ds)$ | |
| $A\ (=N+H+K)$ | $A\ (=M+S)$ |

*B. Interactive Channels of Investment and Finance Decisions*

There are two channels for investment and finance decisions to interact each other; the asset accumulation function and the earning-total

2 In the left hand side of balance sheet intangible asset (i.e. valuable opportunities for investment) should be included because equity capital(S) is not a book value but a market value of existing equities. However, this makes may model complicated unnecessarily. Therefore, I will drop this term for simplicity (for discussion with this term, see Myers (1977).

investment possibility function. The asset accumulation is the sum of investment for inventory, fixed capital and working capital, and at the same time it is the sum of the flow of reatined earning, new debt, and external equity at each period. These relations are expressed in equations (1) and (2).

$$(1)\qquad \dot{A}(t)=\dot{H}(t)+I(t)+\dot{N}(t)=R(t)+\delta E(t)+B(t)$$

$$(2)\qquad \frac{\dot{A}(t)}{A(t)}=\frac{\dot{H}(t)+I(t)+\dot{N}(t)}{A(t)}=\frac{\dot{H}(t)}{H(t)}\,\frac{H(t)}{A(t)}+\left(\frac{K(t)+\mu}{K(t)}\right)\frac{K(t)}{K(t)}+\frac{\dot{N}(t)}{N(t)}\,\frac{N(t)}{A(t)}$$

$$=g_h\cdot h+(g_k+\mu')\cdot k+g_n\cdot d$$

$$=\frac{R(t)+\delta E(t)+B(t)}{A(t)}=r(t)+\delta e(t)+b(t)$$

where $g_h$ is the growth rate of inventory investment,
$g_k$ is the growth rate of fixed capital investment,
$g_n$ is the growth rate of working capital investment,
$\mu'$ is the rate of depreciation ($\mu$) divided by capital stock (k),
h is the weight of inventory with regard to total asset,
k is the weight of fixed capital with regard to total asset,
d is the weight of working capital with regard to total asset

If we take the linear approximation of differential equations with specific form of earning-total investment possibility function, $g_h$, $g_k$ and $g_n$ can be expressed as the function of discrepancy between the equilibrium and the current stock of each factor as below. (For the derivation of equation (3) and the sign of coefficient, see Shin (1990))

$$(3a)\qquad g_h=a_{11}(\ln H^*-\ln H)+a_{12}(\ln K^*-\ln K)+a_{13}(\ln N^*-\ln N)$$

$$(3b)\qquad g_k=a_{21}(\ln H^*-\ln H)+a_{22}(\ln K^*-\ln K)+a_{23}(\ln N^*-\ln N)$$

$$(3c)\qquad g_h=a_{31}(\ln H^*-\ln H)+a_{32}(\ln K^*-\ln K)+a_{33}(\ln N^*-\ln N)$$

where $H^*$, $K^*$, $N^*$ are the long-run equilibrium stocks of inventory, fixed capital, and working capital and H, K, N are the current stocks of each factor. In turn, the weights of inventory and production factor stocks with regard to total asset, h, k, d, are determined through $g_h$, $g_k$ and $g_n$ in time. Hence, investment decisions in this model are directly related to finance decisions.

The earning-total investment possibility function is the relationship showing how earning is influenced by specific investment and finance decisions.

(4) $Y = F(A, \dot{A}, R, E, B)$, given $Y(0) = Y_0$, $A(0) = A_0 > 0$

Then, we can get the derivative of equation (4) like equation (5)

(5) $\dot{Y} = \dot{F}(A, \dot{A}, R, E, B,)$ $\dot{Y} = dY/dt$, $\dot{F} = dF/dt$

which is assumed to be continuous, twice differentiable, strictly concave and whose the first and second order partial derivatives satisfy the following properties.

$$\dot{F}_A,\ \dot{F}_R,\ \dot{F}_E,\ \dot{F}_B > 0,\ \dot{F}_{AA},\ \dot{F}_{RR},\ \dot{F}_{EE},\ \dot{F}_{BB},\ \dot{F}_{\dot{A}},\ F_{\dot{A}\dot{A}} < 0$$

The variable A in equation (5) corresponds to the conventional literature of investment based on the adjustment cost[3] and denotes the effect of each stock variable, H, K, N on earning through production function and inventrory accumulation equation. The proposition, $F_A > 0$ and $F_{AA} < 0$ means that the marginal earning of stock variables, H, K, N is positive, but diminishing. The property $F_{\dot{A}}$, $F_{\dot{A}\dot{A}} < 0$ is arisen from Eisner and Strotz (1963) assumption that the cost of investment depends on the rate of expansion and on the length of time interval in which expansion actually occurs. The feasibility of the firm's total investment (= the sum of investments for inventory, fixed capital and working capital) is determined by the firm's financial availability as well as the marginal contribution of the firm's total investment on earning. The separate incorporation of R, E and B reflects the different transaction cost associated with competing financial resources.

I will take two more assumptions for the earning-total investment function which correspond to conventional assumptions in the literature of investment. First, function F is assumed to be homogeneous of degree one for its five variables. Then, the equation (4) can be rewritten as below.

(6) $$\frac{\dot{Y}}{A} = \dot{F}\left(\frac{R}{A}, \frac{E}{A}, \frac{B}{A}, \frac{\dot{A}}{A}\right)$$

3 The adjustment cost can be either external or internal. For example, these costs are either the value of certain resources used exclusively in the planning and installation prosess (internal cost) or the premium to obtain the factor at more rapid (external) rate.

Second, it is assumed that the adjustment costs function can be separated from the earning-total investment possibility function like the equation (7).

$$(7) \qquad \frac{\dot{Y}}{A} = f\left(\frac{R}{A}, \frac{E}{A}, \frac{B}{A}\right) - q\left(\frac{\dot{A}}{A}\right)$$

$$= f(r,\ e,\ b) - q(r+\delta e+b)$$

where q( ) reflects the function of adjustment costs.
Also, we assume that the following conventional assumptions are met.

$$(8) \qquad q(0)=0,\ q,'\ q'',\ f_r,\ f_e,\ f_b > 0,\ frr,\ fee,\ f_{bb} < 0$$

## III. The Dynamic Optimization Model of Firm's Decisions

### A. *Firm's Dynamic Intertemporal Optimization*

In the literature of corporate investment connected with finance decisions the usual objective function is defined as below.

$$V_0 = \int_0^\infty D^0(t)\ \exp(-\phi t)dt$$

where $D^0(t)$ is the dividend paid to the firm's initial shareholders. By following Krouse (1973), we can express as the difference between the present value of total dividends and the market value of investment by new shareholders.

$$V_0 = \int_0^\infty [Y(t) - R(t)]\cdot\exp(-\phi(t))dt - \int_0^\infty [W(t)\cdot E(t)]\cdot\exp(-\phi(t))dt$$

$$= \int_0^\infty [Y(t) - R(t) - W(t)\cdot E(t)]\cdot\exp(-\phi(t))dt$$

Unlike Lin (1981), we want to maximize the discounted stream of dividends per share, not per unit of fixed capital stock because equity capital and fixed capital need not be the same. Then, we can express in terms of firm value per share like the equation (9).

$$\frac{V_0}{S} = \int_0^\infty \left[\frac{Y(t) - R(t) - W(t)\cdot E(t)}{S(t)}\right]\cdot\exp(-\phi(t))dt$$

$$= \int_0^\infty [\frac{Y(t)-R(t)-W(t)E(t)}{S(t)} \cdot \frac{A(t)}{S(t)}] \exp(-\phi(t))dt$$

(9) $$V_0 = \int_0^\infty [t(t)-r(t)-W(t)e(t)] \cdot \frac{1}{s(t)} \exp(-\phi(t))dt$$

In addition to dividend constraint we take another constraint for the above objective function (9); a debt constraint. Debt capacity is specified by the maximum debt-equity ratio ($\Delta^*$) which is determined by such many factors as corporate taxes, management policy, institutional constraints, the cost of financial distress, and so on.[4]

$$\Delta^* \geqslant M(t)/S(t)$$

Dividing M(t) and S(t) by A(t) and substituting m(t) and s(t) for those gives

(10) $$\Delta^* s(t) - m(t) \geqslant 0$$

In addition, there are constraints coming from the transition equations. First, the definition of z gives us the differential equation for y as below.

$$y(t) = Y(t)/A(t)$$

$$y(t) \cdot A(t) = Y(t)$$

Differentiation both sides with regard to time provides as $y \cdot \dot{A} + y \cdot \dot{A} = \dot{Y}$, and then dividing both sides by A and rearranging it provides the equation (11).

(11) $$\dot{y} = \frac{\dot{Y}}{A} - y \frac{\dot{A}}{A}$$

$$= f(r, e, b) - q(r + \delta e + b) - y[r + \delta e + b]$$

4 Debt constraint in my modelis a mixture of endogenous and exogenous constraints. See Donaldson (1961) or Meyers (1977) for more discussion. The debt constraint in my model is a manipulable rationing because the firm can lessen the severity of debt constraint through delaying dividend payments. However, the firm should pay the delayed dividend sooner or latter and this point is not discussed in my model because of its mathematical complexity.

Similar manipulations give us the differential equations for and s.

(12) $\dot{m} = b - m[r + \delta e + b]$

(13) $\dot{s} = r + \delta e - s[r + \delta e + b]$

where $y_0$, $m_0$ and $s_0$ are given as $y_0 = Y_0 / A_0$, $m_0 = M_0 / A_0$ and $s_0 = S_0 / A_0$

Finally, I can set up the firm's dynamic intertemporal optimization model such as the objective function (14) with several constraints like (15)—(19).

(14) $\mathrm{Max}_{r,e,b}\ v_0 = \int_0^\infty [y(t) - r(t) - W(t)e(t)] \frac{1}{s(t)} \exp(-\phi(t))dt$

subject to

(15) $\dot{y} = f(r,\ e,\ b) - q(r + \delta e + b) - y[r + \delta e + b]$

(16) $\dot{m} = b - m\,[r + \delta e + b]$

(17) $\dot{s} = r + \delta e - s[r + \delta e + b]$

(18) $\Delta^* s - m \geqslant 0$

(19) $[y(t) - r(t)] / s(t) = c \geqslant 0$

where initial values $y_0$, $m_0$, $s_0$ are given.

The above optimization model requires selecting the optimal time schedule for dividends, debt, equity, and corporate earning which results from the investment decisions for inventory, fixed capital, and working capital. From the above conditions we can get the relationship of $\varrho_1$ like the below equation (20).

(20) $$\rho_1 = \frac{\rho_2\, m - \rho_3(1-s) + 1/s}{(f_r - q' - y)} = \frac{\rho_2\, m\delta - \rho_3(1-s)\delta + w/s}{(f_e - \delta q' - \delta y)}$$

$$= \frac{\rho_3\, s - \rho_2(1-m)}{(f_b - q' - y)}$$

where $\rho_1$, $\rho_2$, $\rho_3$, are the marginal (shadow) value of y, m, s respectively. From equation (19), we can make it sure that two common marginal propositions of neoclassical economics hold in this model. First, the implicit marginal value of the change in total investment at time t is equal to the implicit marginal cost of each financial source. Second, the implicit marginal costs of competing finance sources are the same each other. These criteria are independent of firm's size or growth rate. Meanwhile, these propositions are affected by the joint behavior of $\rho_i$, y, m, and s and also by adjustment cost and the prevailing marginal external issue market rate.[5]

## B. *Optimal Time Pathes and Long-Run Equilibrium Decisions*

If we assume that the debt and the dividend constraint are not binding; $\Delta^* \cdot s - m$, $(y-r)/s \geq 0$ and hence $\rho_4 = \rho_5 = 0$, we can get the optimal time path of r, e, and b from the optimality conditions to solve the simultaneous equations. (see mathematical appendix)

(21) $r = r(\rho_1, \rho_2, \rho_3, y, m, s)$

(22) $e = e(\rho_1, \rho_2, \rho_3, y, m, s)$

(23) $b = (\rho_1, \rho_2, \rho_3, y, m, s)$

Applying the matrix Ricatti equation provides the solutions of $\rho_1$, $\rho_2$, $\rho_3$.

(24) $\hat{\rho}_1 = \hat{\rho}_1\ (\hat{y}, \hat{m}, \hat{s})$

(25) $\hat{\rho}_2 = \hat{\rho}_2\ (\hat{y}, \hat{m}, \hat{s})$

(26) $\hat{\rho}_3 = \hat{\rho}_3\ (\hat{y}, \hat{m}, \hat{s})$

where $\hat{y}$, $\hat{m}$, $\hat{s}$, $\hat{\rho}_1$, $\hat{\rho}_2$, $\hat{\rho}_3$ are optimal solutions, however, they are not the long-run equilibrium yet. By transferring $\hat{y}$, $\hat{m}$, $\hat{s}$, $\rho_1$, $\hat{\rho}_2$, $\hat{\rho}_3$ from (24)-(26) into (21)-(23), we can get the demand for retained earning, external equity, and debt per value of total asset like equations (27)-(29).

(27) $\hat{r} = \hat{r}(\hat{y}, \hat{m}, \hat{s})$

[5] By following Lin (1981), we assume $0 \leq \delta \leq 1$. When $\delta = 1$, flotation cost is zero and it implies that there is no difference of finance cost between retained earning and external equity. However, this is not often feasible in real world.

(28) $\hat{e} = \hat{e}(\hat{y},\ \hat{m},\ \hat{s})$

(29) $\hat{b} = \hat{b}(\hat{y},\ \hat{m},\ \hat{s})$

In order to get the long-run equilibrium we set $\dot{y}$, $\dot{m}$, $\dot{s}$, $\dot{\rho}_1$, $\dot{\rho}_2$, $\dot{\rho}_3$ equal to zero and then we get the following equations from optimality condition.

(30) $f(r,e,b) = q(r+\delta e+b) + y[r+\delta e+b]$

(31) $b = m[r+\delta e+b]$

(32) $r+\delta e = s[r+\delta e+b]$

(33) $\{f+r+\delta e+b\} = 1/(s\rho_1)$

(34) $\{f+r+\delta e+b\}\rho_2 = 0$

(35) $\{f+r+\delta e+b\} = 1/(s^2\rho_1)\{y-r-We\}$

At equation (34) we can see $\rho_2{}^* = 0$ since $\{f+r+\delta e+b\} > 0$. This proposition seems to be reasonable because B(t) influences the initial shareholders' wealth implicitly via its ability to create future earning while R(t) and E(t) do it explicitly. Then, substituting for r, e, and b from (21)-(23) into (30)-(35) provides us the five simultaneous equations with five unknowns y, m, s, $\rho_1$, $\rho_3$. Then, we can get the unique solutions of $y^*$, $m^*$, $s^*$, $\rho_1{}^*$. Inserting $y^*$, $m^*$, $s^*$, $\rho_1{}^*$, and $\rho_3{}^*$ back into equations (21)-(23) gives us final solutions

(36) $r^* = r^*(y^*,\ m^*,\ s^*)$

(37) $e^* = e^*(y^*,\ m^*,\ s^*)$

(38) $b^* = b^*(y^*,\ m^*,\ s^*)$

Also, we can get the long-run corporate earning function, (39) by inserting $r^*$, $e^*$, $b^*$ from equations (36)-(38) into equation (30).

(39) $f(r^*,\ e^*,\ b^*) = q(r^*+\delta e^*+b^*) + y[r^*+\delta e^*+b^*]$

Unfortunately, we can not get a closed form of demand functions for internal finance, debt, external equity, and corporate earning without specifying f(r, e, b) and q ( ). Nevertheless, we can discuss qualitatively some characteristics of the solution via the phase-space analysis by following Lin (1981). [see Mathematical appendix]

Up to now, we have shown how to derive the demand functions for retained earning, debt and external equity via the earning-total investments possibility function. Then, if we know the available amount of retained earning, debt and external equity, and do not know the investment demand for inventory. fixed capital or working capital, can we determine the investment demand schedule? This issue is analogous to the so-called "inverse optimal control" problem [see Kurz (1968)] and it is mathematically possible to derive the investment demand schedules for inventory, fixed capital or working capital from the available financial resources.[6] The next question is what happen to the investment demand schedules when there are constraints on financial availability? This question is exhaustively discussed in Shin (1990) where a specific production function enables us to get a closed form of the investment demand functions for H, K, and N.

## IV. Concluding Remarks

Recently, the interest on the interrelationship between macroeconomic fluctuations and financial structure is getting bigger (for survey on this topic, see Gertler (1988)). First, investment has been studied as a main

[6] A simple example of inverse optimal control problem was discussed by Kurz (1968) in the literature of economic growth as following

$$\text{Max } \int_0^\infty U(C)\cdot\exp(-rt)\, dt$$

$$\text{subject to } \dot{K} = f(K) - nK - C$$

where k(0) is given and U(C) is a strictly concave, twice differentible function. i.e. $U_c > 0$, $U_{cc} < 0$ for all C. K is the capital-labor ratio, n is the constant growth rate of population, and f(k) is the output per capital. If we know consumption level C, then we can get an utility function U. Similarly, if we know r, e, and b, we can get y, m, and s from the above relationship and in turn we can get total investment, A(t).

$$\text{Max}_{r,e,b}\ v_0 = \int^\infty [y(t) - r(t) - W(t)e(t)]\ \frac{1}{s(t)}\ \exp(-\phi(t))dt$$

subject to

$$\dot{y} = f(r, e, b) - q(r + \delta e + b) - y[r + \delta e + b]$$
$$\dot{m} = b - m[r + \delta e + b]$$
$$\dot{s} = r + \delta e - s[r + \delta e + b]$$
$$\Delta^* s - m^* \geqslant 0$$
$$[y(t) - r(t)]/s(t) = c \geqslant 0$$

where initial values $y_0$, $m_0$, $s_0$ are given.

factor of business cycle. Second, advanced capitalist economy is characterized by highly developed financial markets. The more sophisticated financial markets become, the more tightly the decisions of investment and finance are integrated.

The final goal of this paper is to contribute to the microfoundation of the interrelationship between macroeconomic fluctuations and financial structure. For this purpose we have investigated the general interrelationship between the investment decisions for inventory, fixed capital and working capital, and the finance decisions for retained earning, debt and external equity. Especially, my main concern is to check mathematically the feasibility of optimal investment for retained earning, debt and external equity and its stability condition.

There are two major limitations in my model. One is the absence of uncertainty and hence there is no role of the firm's expectation. However, it is too complicated to incorporate an expectation factor in intertemporal dynamic model, discussing the interrelationship between investment and finance decisions. This issue is discussed in Shin (1990) qualitatively by using phase diagram. The other limitation is that the corporate tax is not incorporated. However, I believe that this limitation is not critical because the main goal in this paper is to investigate the (mathematical) feasibility of effect of financial constraints on firm's investments, not the optimal combination of different financial sources.

## Mathematical Appendix

From equations (14)-(18) we can get the Lagrangian function as (A-1).

$$\text{(A-1)}\ \pi = \exp(-\phi t)\{(y-r-we)/s + \rho_1[f(r,e,b) - q(r+\delta e+b) - y\{r+\delta e+b\}] + \rho_2[b - m\{r+\delta e+b\}] + \rho_3[r+\delta e - s\{r+\delta e+b\}]\} + \rho_4[\Delta^* s - m] + \rho_5[(y-r)/s]$$

where $\rho_1$, $\rho_2$, $\rho_3$, $\rho_4$, $\rho_5$ are the marginal (shadow) value of y, m, s, (endogenous) debt constraint and dividend constraint respectively.

The optimality conditions for equation (20) are as follows:

(A-1a) $y = f(r,e,b,) - q(r+\delta e+b) - y\{r+\delta e+b\}$

(A-1b) $m = b - y\{r+\delta e+b\}$

(A-1c) $s = r+\delta e - y\{r+\delta e+b\}$

(A-1d) $\rho_1 = \rho_1[\phi + r+\delta e+b] - 1/s$

(A-1e) $\rho_2 = \rho_2[\phi + r+\delta e+b]$

(A-1f) $\rho_3 = \rho_3[\phi + r+\delta e+b] - 1/s^2[y - r - We]$

(A-1g) $\rho_1(f_r - q' - y) - \rho_2 m + \rho_3(1-s) - 1/s = 0$
(A-1h) $\rho_1(f_e - \delta q' - \delta y) - \rho_2 m\delta + \rho_3(1-s)\delta - w/s = 0$
(A-1i) $\rho_1(f_b - q' - y) + \rho_2(1-m) + \rho_3 s = 0$
(A-1j) $\lim \rho_1 \cdot y \cdot \exp(-\phi t) = \lim \rho_2 \cdot m \cdot \exp(-\phi t) = \lim \rho_3 \cdot s \cdot \exp(-\phi t) = 0$
(A-1k) $\rho_1(f_{rr} - q'') < 0$
(A-1m) $\rho_2(f_{ee} - \delta^2 q'') < 0$
(A-1n) $\rho_1(f_{bb} - q'') < 0$
(A-1o) $\partial\rho_1/\partial A > 0$
(A-1p) $\rho_4, \rho_5 \geqslant 0$, and $\rho_4[\delta^* s - m] = \rho_5[(y-r)/s] = 0$
where w in (A-1f) is the marginal external issue market value.

In order to get the optimal time pathes we rewrite (A-1g)-(A1i) as
(A-2a) $f_r = 1/(s\ \rho_1) + (m\ \rho_2)/\rho_1 - (1-s)\rho_3/\rho_1 + q' + y$
(A-2b) $f_e = w/(s\ \rho_1) + (\delta m\ \rho_2)/\rho_1 - \delta(1-s)\rho_3/\rho_1 + \delta q' + \delta y$
A-2c) $f_b = (s\ \rho_3)/\rho_1 - (1-m)\ \rho_2/\rho_1 + q' + y$
Solutions of (A-2a) – (A-2c) yields the functions of r, e, b with w, r, $\delta$,
(A-3a) $r = r(\rho_1, \rho_2, \rho_3, y, m, s)$
(A-3b) $e = e(\rho_1, \rho_2, \rho_3, y, m, s)$
(A-3c) $b = b(\rho_1, \rho_2, \rho_3, y, m, s)$

*The stability condition of optimal solution*

In my model the interrelationship of y and $\rho_1$ is of interest because that expresses the interaction of total investment-finance decisions. Hence, I will concentrate to sketch the interrelationship of y and $\rho_1$. By applying the implicit function rule to the equation (A-1g) we can get the slope of the curve $\dot{y} = 0$ as

$$\text{(A-4)} \quad \left.\frac{d\rho_1}{dy}\right| = -\frac{[(f_r - q' - y)\dfrac{\partial r}{\partial y} + (f_e - \delta q' - \delta y)\dfrac{\partial e}{\partial y} + (f_b - q' - y)\dfrac{\partial b}{\partial y} - (r + \delta e + b)]}{[(f_r - q' - y)\dfrac{\partial r}{\partial \rho_1} + (f_e - \delta q' - \delta y)\dfrac{\partial e}{\partial \rho_1} + (f_b - q' - y)\dfrac{\partial r}{\partial \rho_1}]}$$

From (A-1g)-(A-1i), I can rewrite $f_r - q' - y$, $f_e - \delta q' - \delta y$, $f_b - q' - y$ as

$$\text{(A-5)} \quad f_r - q' - y = \frac{1}{s\rho_1} + \frac{m\rho_2}{\rho_1} - \frac{(1-s)\rho_3}{\rho_1} = \frac{1 + s^2\rho_3 - s(\rho_3 - m\,\rho_2)}{s\rho_1}$$

$$\text{(A-6)}\quad \phi_e - \delta q' - \delta y = \frac{w}{s\rho_1} + \frac{\delta m \rho_2}{\rho_1} - \frac{(1-s)\delta\rho_3}{\rho_1} = \frac{w + s^2\delta\rho_3 - \delta_s(\rho_3 - m\rho_2)}{s\rho_1}$$

$$\text{(A-7)}\quad f_b - q' - y = \frac{s\rho_3}{\rho_1} + \frac{(1-m)\rho_2}{\rho_1} = \frac{s\rho_3 - (1-m)\rho_2}{\rho_1}$$

Under the assumption that dividend and debt constraint are not binding, $\rho_2 = \rho_3 = 0$. Then, we know that the signs of $f_r - q' - y$, $f_e - \delta q' - \delta y$, $f_b - q' - y$ and $r + \delta e + b$ are all positive. If we assume that the increase of profit brings the increase of financing demand through the increase of total investment then $\partial r/\partial y$, $\partial e/\partial y$, $\partial b/\partial y$, $\partial r/\partial \rho_1$, $\partial e/\partial \rho_1$ are all positive. Thus, the slope of $\dot{y}=0$, $[d\rho_1/dy]|_y$, depends on the sign of numerator, $[(f_r - q' - y)\partial r/\partial y + (f_e - \delta q' - \delta y)\ \partial e/\partial y + (f_b - q' - y)\partial b/\partial y + (r + \delta e + b)$. It is reasonable to assume that the slope of $y=0$ is monotonic regardless that it is positive or negative. The curve of $\rho_1 = 0$ is convex, negative and would not fall below the minimum marginal cost of total investment if the Legendre condition is met. Then, the figure 1 and 2 shows the optimal trajectory in the $(y, \rho_1)$ space. If the eigenvalues of the linear approximated differential equations of (A-1a) and (A-1d), then the equilibrium point $(y^*, \rho^*_1)$ is a saddle point (this is discussed algebraically in Shin (1990)).

**Figure 1**

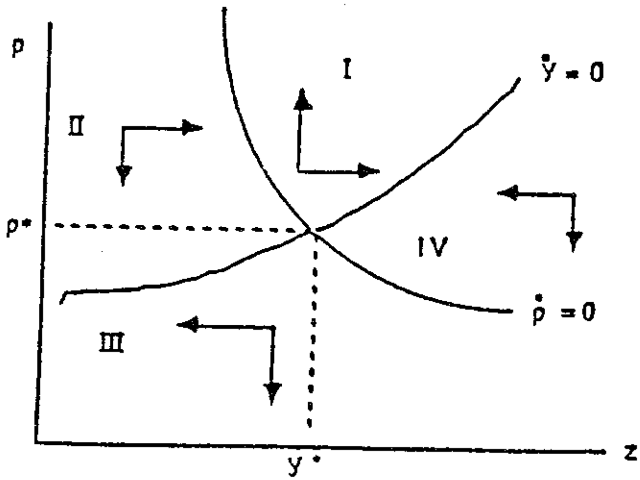
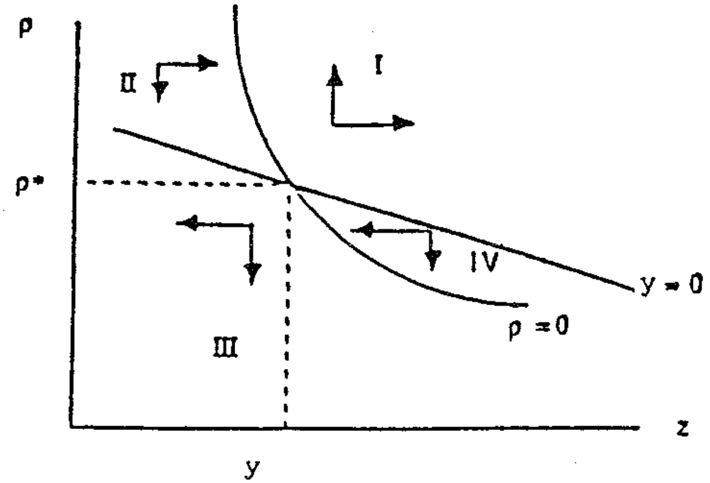


**Figure 2**




If the initial point ($y_0$, $\rho^0{}_1$) is in the region II or IV, intertemporal movement will converge to the equilibrium. Otherwise, it will move away from the equilibrium point.

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