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Optimal Oil Production in OPEC Members under Macroeconomic Constraints*

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This paper uses a technique of social welfare maximization under marcoeconomic constraints in order to derive the optimal level of oil production in selected members of OPEC. Most of the previous papers use simulation or optimization techniques to solve for optimal solutions, but they ignore the macroeconomic impact of oil production. This paper has attempted to overcome this shortcoming.

The optimal level of oil production is derived under different scenarios. The optimal solutions generally overestimate the actual behaviors. The paper concludes that if economic growth is the main objective of members of OPEC, they should seek ways to expand their revenue beyond the existing levels.

I. Introduction

OPEC and its oil production and pricing policies captures the attention of the public as well as academia in 1975. In those days, OPEC was strong and almost dictated its policies to the rest of the world. Despite its relative monopoly strength, many authors (Kennedy (1974), Levy (1974), Kalymon (1975), Federal Energy Administration (1974), Bohi and Russel (1975)) predicted that OPEC will have substantial surplus capacity by 1985. Following the pioneering work of Hotelling (1931) which provided an extensive treatment of the economics of exhaustible resources, in 1975 a simulation technique was utilized by many economists (Blitzer, et. al. (1975), Kalymon (1975), Abolfathi, et. al. (1977), MacAvoy (1982)) to estimate the supply and demand function for oil and the respective elasticities. Motivated by the oligopolistic power of OPEC in early 1975, many studies (Schmalensee (1976), Salant (1976), Cremer and Weitzman

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(1976), Pindyck (1977, 1978), Hynilicza and Pindyck (1976), Gilbert (1978), Gilbert and Goldmand (1978)) used the optimization technique to investigate the depletion of natural resources under oligopoly. Economists such as Kemp (1976), Loury (1978), Gilbert (1979) and Heal (1979) studied the impact of uncertainty on production decisions. The social optimality of competitive market equilibrium was confirmed by Weinstein and Zeckhauser (1975) in a discrete time model of the stochastic approach. Pindyck (1980) extended the Weinstein-Zeckhauser study to the continuous time case. His model specifies a stochastic optimization process under which producers must determine the rate of production over time in such a way that the expected value of the sum of discounted profits is maximized. The "Hotelling rule" was more or less confirmed by these studies.

Social optimality of competitive and monopolistic solutions has been examined by many of the above-mentioned studies under the strong assumption that the social discount rate is equal to the private discount rate; an assumption which forces one to use the notion of profit maximization and social welfare maximization interchangeably (Burt, et. al. 1970)). Solow (1974) highlights the shortcomings of this assumption and its implication for public intervention in resource allocation. Institutional arrangements in OPEC states are such that oil production becomes a matter of public policy which requires consideration of macroeconomic impacts of oil production policy on the economy. It is this aspect of oil production which has not fully captured the attention of past studies. The recent decline in the price of oil has already shown its devastating macroeconomic impact on the economies of OPEC states. This paper focuses on the macroeconomic impact of oil production policies by examining an intertemporal equilibrium model which maximizes social welfare of the economy, subject to macroeconomic constraints of the economy (Theil 1961)). This is a new application of policy analysis techniques used by many authors (Theil (1961), Van Eiyk and Sandee (1959), etc.).

Section II presents the optimal control model and the basic macroeconomic constraints of the model. In Section III, results of optimal solutions are discussed. Finally, Section IV concludes the paper.

II. An Optimal Control Model

The basic objective of this paper is to derive the optimal level of oil production in a way which maximizes the social welfare of the economy. To do so, a social welfare function which includes many key macroeconomic variables is specified. The process of maximization involves two steps. First, the structure of the economy and the key macroeconomic relationships are estimated. Second, the estimated macro-model is used as a constraint in the process of welfare maximization. This is an open-loop model (Theil (1961), Van Eiyk and Sandee (1959), Chow (1975, 1976)) of welfare maximization which solves for the optimal values of policy variables at the beginning of the planning period.[1] In what follows, a quadratic welfare function is specified which includes a combination of both endogenous and policy variables. The oil policy maker determines the optimal oil production in such a way that minimizes the squared deviation of actual values from the desired values of the target and control variable.[2] Constraints of the model are estimated based on a macroeconomic model of selected OPEC states. The macro-model assumes that the economy produces only three goods — namely, oil, imported goods and nontraded goods. Oil is the only export of the economy which is produced by national oil companies. The economy is small and open to international trade. The exchange rate is pegged to a major currency or a basket of major currencies. The price of imported goods and oil is given.[3] The major investor in the economy is government, which uses oil revenue to develop and expand the domestic economy. Fiscal activities of government change the money supply, creating inflation via an excess supply of money over demand for money. There are twelve endogenous and nine exogenous variables. A 2SLS technique was applied to estimate the macro-model using pooled data for Indonesia, Iran, Kuwait, Nigeria, Saudi Arabia, and Venezuela. The estimation results of the macro-model are presented in Appendix 1.[4] The period of study is 1973-1986. The period before 1973 was omitted, first because some countries included in this study did not have full control over their oil industry before early 1975. Second, major structural changes took place between the two oil price hikes of 1973 and 1979. The data were mostly taken from various issues of

[1] Instead of an open-loop model, one may use a closed loop optimal control model which specifies the policy variables as a function of future observations (Chow 1975)).

[2] A major shortcoming of a quadratic function is that it penalizes both positive and negative deviations equally, but it has many nice mathematical properties. For example, one can obtain optimal solutions by applying the certainty equivalence theorem (Simon (1956)) to the linear econometric model with an additive random disturbance used in the present study (Chow 1975)).

[3] OPEC as a whole may exert some impact on the international price of oil, but individual countries are price takers.

[4] A covariance model of pooled-data estimation is employed which requires to include (N−1) + (T−1) dummy variables to account for the possibility of variations in intercepts over time and over cross-sections. N and T stand for the number of cross-sections and time series respectively. The significan dummies are reported in the appendix.

International Financial Statistics, a United Nations publication, and the *Petroleum Economist.*

Table 1

THE COMPLETE MODEL

$$\text{Max } W = -1/2\,(Y_{t+1} - Y^0_{t+1})\; E\; (Y_{t+1} - Y^0_{t+1})^1 - 1/2\; (Z_t - Z^0_t)\; F\; (Z_t - Z_t)^1$$

subject to:

(1) $Y = C + I + G + (X - M)$

(2) $C_t = C(Y_t, P_t, PN_t)$

(3) $I_t = I\,(t)$

(4) $X^d_t = X\,(PO_t, YE_t)$

(5) $M^d_t = M\,(PM_t, P_t, Y_t)$

(6) $G_t = g\,(X^s_t, PO_t, G_{t-1})$

(7) $R_t = r\,(X^d_t, PO_t, R_{t-1})$

(8) $L^s_t = L^1\,(G_t, R_t, B_{t-1}, E_t)$

(9) $L^d_t = L^2\,(P_t, Y_t, \pi_t)$

(10) $P_t = P^1\,(L^s_t, L^d_t, PM_t, \pi_t)$

(11) $\pi_t = \alpha P_t + (1-\alpha)\,\pi_{t-1}$

(12) $PN_t = P_2\,(PM_t, Y_t, PN_{t-1})$

(13) $Y_t = Y\,(X^s_t, PO_t, Y_{t-1})$

where

Y, Y^0 = a (lxn) vector of actual and target values of n endogenous variables, respectively

E = a (nxn) given constant positive-definite symmetric matrix

Z, Z^0 = a (lxl) vector of actual and target values of control variable

F = a (lxl) vector of weight given to deviation of actual value of policy variable from its target value

Endogenous Variables

Y = National Income

C = Consumption

I = Investment

X^d = Quantity of Oil Exported

M^d = Demand for Imports

G = Total Government Expenditures
R = Government Revenue
L^s = Money Supply
L^d = Money Demand
PN = Price of Nontraded Goods
P = Price Level
π = Expected Price

Exogenous Variables
PO = Price of Oil
B = Monetary Base
PM = Price of Imports
M = Money Multpilier
X^s = Quantity of Oil Produced
e = Exchange Rate
YF = Income of Industrial Countries
t = Time
E = Monetary Variable
DIR = Dummy Variable for Iran
DIN = Dummy Variable for Indonesia
DS = Dummy Variable for Saudi Arabia
DV = Dummy Variable for Venzuela
DK = Dummy Variable for Kuwait

III. Optimal Oil Policy

Table 2 reports optimal oil production in the six selected OPEC members. These results were obtained under three different scenarios (A, B, C). The vector Y contains all or some of the endogenous variables of the macro-model. The target values (desired values) in vector Y^0 are set to grow at 5%, starting from their 1973 values. This is based on the assumption that a higher growth rate is not realistic and a lower rate is not desirable. The matrix E includes the weights given to each policy objective. In scenario A, all endogenous variables were included in the social welfare function and were assumed to be equally important to policy makers, which results in the matrix F becoming an identity matrix. Scenario B assumes that economic growth is twice as important as other variables, and it carries a weight equal to two. Finally, in scenario C all endogenous variables were dropped from the welfare function except the national income. In the studies cited before (Van den Bogard and Theil (1959), etc.), the vector Z includes traditional policy variables such as government expenditures and money supply. Here, based on institutional arrangements in oil exporting countries (e.g. the oil ministry functions independent of other government bodies), we assume that the oil ministry authorities make their output decisions independent of government fiscal or monetary authorities. In doing so, they try to take into consideration macroeconomic impacts of oil output policy on the economy. Therefore, from their point of view, the only policy variable under their control is the level of oil production. This explains why vector Z includes only one control variable, namely oil production.[5] The target values of control variables were set to grow by 2% annually, starting from 1973. A larger growth rate may undermine the health of the international oil market as well as the conservationist efforts of the country. The deviations of actual oil production from their target values are penalized by a factor of 14. This eliminates the discrepancy between different units of measurement and assigns to the oil variable the same weight which was assigned to the target variables.[6]

Comparisons of optimal and actual production in Indonesia show that they are compatible under scenario A, but generally actual production is

Table 2

ACTUAL AND OPTIMAL SOLUTIONS

(Million Barrels)

Year	Indonesia				Iran			
	Actual	Optimal A	Optimal B	Optimal C	Actual	Optimal A	Optimal B	Optimal C
1974	501.7	721.3	947.4	776.4	2,197.9	3,062.5	3,940.2	3,059.7
1975	476.9	596.3	681.8	616.9	1,952.8	2,398.6	2,554.5	2,397.7
1976	550.3	606.6	728.8	636.4	2,153.1	2,599.7	3,206.9	2,599.1
1977	615.4	683.5	807.6	714.0	2,066.9	2,437.2	2,676.9	2,435.9
1978	596.8	366.6	101.6	301.8	1,913.2	1,806.4	1,505.3	1,807.2
1979	580.6	781.0	955.6	823.5	1,156.3	2,191.5	2,432.2	2,192.2
1980	576.7	725.2	860.9	759.0	540.2	1,306.1	1,433.7	1,307.1
1981	578.5	753.0	920.2	793.9	495.0	629.6	708.1	629.5
1982	482.8	549.9	509.0	539.8	682.6	823.2	1,141.6	823.2
1983	465.1	290.1	84.6	239.9	938.1	1,178.1	1,661.1	1,178.6
1984	479.5	507.8	541.7	516.1	779.9	870.4	784.2	870.7
1985	439.0	477.3	465.3	474.3	820.4	-2,614.7	-6,025.4	-2,615.2

5 This does not require that fiscal and monetary authorities give up their policy variables.

6 Results of sensitivity analysis indicate that the general conclusions and optimal solutions do not change significantly when different values within the feasible range are tried.

Table 2 (continued)

(Million Barrels)

Year	Kuwait Actual	Kuwait Optimal A	Kuwait Optimal B	Kuwait Optimal C	Nigeria Actual	Nigeria Optimal A	Nigeria Optimal B	Nigeria Optimal C
1974	929.3	1,283.5	1,442.2	1,283.1	823.1	760.9	690.4	760.4
1975	760.7	925.4	902.4	924.9	650.9	839.5	821.0	838.4
1976	785.2	805.8	835.8	805.9	756.5	661.9	623.1	661.5
1977	718.7	812.7	824.8	813.0	761.1	772.6	781.2	772.2
1978	778.0	740.2	747.2	740.0	692.4	775.1	782.7	776.8
1979	912.6	951.8	1,110.1	951.9	840.3	704.6	678.1	704.6
1980	608.9	949.0	967.0	948.9	753.2	852.5	785.0	852.9
1981	412.3	568.4	515.7	568.3	525.5	770.2	799.3	770.1
1982	243.0	280.7	225.0	280.6	476.6	503.2	509.7	503.1
1983	328.3	237.3	226.8	237.3	443.5	488.0	519.7	488.1
1984	333.0	319.7	304.8	319.9	509.0	453.4	471.6	453.5
1985	294.6	308.8	277.9	308.8	520.4	518.4	501.5	518.1

Year	Saudi Arabia Actual	Saudi Arabia Optimal A	Saudi Arabia Optimal B	Saudi Arabia Optimal C	Venezuela Actual	Venezuela Optimal A	Venezuela Optimal B	Venezuela Optimal C
1974	3,095.1	2,672.2	2,563.2	2,695.5	1,086.4	1,722.5	1,949.5	1,550.1
1975	2,582.5	2,960.1	2,993.6	3,074.5	956.4	1,095.4	965.2	1,036.6
1976	3,139.3	2,618.4	2,558.8	2,596.6	839.7	938.5	883.4	878.4
1977	3,358.0	3,091.0	3,074.9	3,138.2	816.8	1,077.3	1,050.6	953.4
1978	3,029.9	3,366.2	3,342.9	3,383.9	790.4	929.4	957.1	895.3
1979	3,479.3	3,011.9	3,062.9	3,076.3	860.1	910.1	1,081.0	943.7
1980	3,623.6	3,441.1	3,340.8	3,444.5	792.4	1,034.6	1,077.1	977.1
1981	3,579.9	3,607.3	3,520.2	3,605.2	769.5	1,021.2	892.2	849.9
1982	2,334.2	3,491.2	3,608.9	3,571.7	657.4	732.8	737.6	761.3
1983	1,753.9	2,521.2	2,628.1	2,504.6	644.8	533.1	568.8	619.8
1984	1,636.2	1,887.2	1,932.3	1,860.8	620.6	115.4	483.5	370.0
1985	1,186.3	1,761.3	1,799.9	1,734.6	600.8	559.3	507.8	570.0

less than optimal under scenarios B and C. Solution B requires more production than the other two solutions. In Iran, actual and optimal results are very close up to 1979. Scenarios A and B involve less error before 1979, but after this year the actual pattern of production (ups and downs in production) follows the optimal pattern except in 1985. This discrepancy may be attributed to strikes by oil workers and other political difficulties experienced in Iran since 1978 particularly the Iran-Iraq War. Kuwait's actual oil production is systematically less than optimal solutions, but it follows the optimal pattern after the recession of 1981-82. Optimal output Bs are greater than optimals A and C during the period before the recession of 1981-82. Nigeria's actual behavior is the opposite of the optimal path, and generally less than it for the period before 1981-82. Later it follows the optimal paths. For Saudi Arabia, the optimals are consistently greater than the actual output for the period after the recession of 1982. In Venezuela actual and optimals have similar patterns after 1978, and scenario B outperforms the other scenarios.

In conclusion, optimal solutions generally overestimate actual behavior, but they have similar patterns and turning points. Except for Saudi Arabia, optimal behaviors reflect the impact of the 1975 and 1981-82 recession on the level of oil production. Actual output shows reduction in all these nations in that year. Scenario B requires more production than scenarios A and C. This indicates that if the major objective of the OPEC nation is economic growth, a very liberalized oil production policy is due. Finally, it must be realized that the discrepancy between the actual and optimal solutions does not necessarily mean that policy makers have behaved suboptimally. It may be simply that they do not share the same objectives and social welfare function as this research specifies. For example, at times one nation or another has tried to win the leadership of OPEC by adjusting its oil output to meet the market requirements.

IV. Conclusion

This paper uses an optimal-control approach in order to determine the optimal level of oil production in six OPEC states. It is assumed that the oil company (ministry) authorities try to maximize the social welfare of the economy by going through a constraint process of maximization. The constraints of the model come from a macroeconomic model of the nation, which was built and estimated in a previous paper.

Optimal levels of oil production generally overestimate actual behaviors but they reflect the impact of the 1975 and 1981 recessions on oil policy. If economic growth is the main objective of OPEC, some of them should expand their oil production (revenue) beyond the existing level. Implementation of this seems extremely difficult under the current market situation.

This study was somewhat hindered by a lack of data, making pooling inevitable. If data were available for individual countries, it would have been desirable to consider the differences among these nations and estimate the model for each country separately. Also, a study of the impact of oil revenue on the components of government expenditures rather than the aggregate level can be very helpful. A comparison of macroeconomic optimal solutions (Hotelling rule) with the results of this study could shed light on the merits of this macroeconomic approach.

Appendix 1

Estimation Results

$$C_t = 3{,}984.2 + 0.503\, Y_t - 83.9\, P_t + 87.5\, PN_t$$
$$(0.09) \quad (4.93) \quad (-0.25) \quad (0.19)$$
$$R^2 = 0.84 \quad \bar{R}^2 = 0.82 \quad D.W. = 1.99$$
$$RHO = 0.09 \quad F = 37.39 \quad SER = 9{,}299.6$$

$$I_t = -10{,}819.8 + 2{,}876.9\, t + 16{,}276\, DIR$$
$$(-1.87) \quad (3.49) \quad (2.19)$$
$$R^2 = 0.28 \quad \bar{R}^2 = 0.2 \quad D.W. = 2.201$$
$$RHO = 0.33 \quad F = 3.43 \quad SER = 8{,}709.4$$

$$X_t^d = 316.16 - 14.99\, PO_t + 0.00045\, YF_t$$
$$(0.04) \quad (-0.14) \quad (0.27)$$
$$R^2 = 0.73 \quad \bar{R}^2 = 0.72 \quad D.W. = 1.2$$
$$RHO = 0.85 \quad F = 59.56 \quad SER = 495.12$$

$$M_t^d = 7{,}536.8 + 0.54\, Y_t - 41.4\, P_t - 148.9\, PM_t$$
$$(0.40) \quad (2.81) \quad (-0.98) \quad (-0.98)$$
$$R^2 = 0.27 \quad \bar{R}^2 = 0.16 \quad D.W. = 1.99$$
$$RHO = -0.03 \quad F = 2.48 \quad SER = 13{,}473.4$$

$$G_t = -6{,}686.56 + 196.7\, PO_t + 6.29\, X_t^s + 0.779\, G_{t-1}$$
$$(-2.59) \quad (2.26) \quad (3.57) \quad (9.74)$$
$$R^2 = 0.72 \quad \bar{R}^2 = 0.71 \quad D.W. = 1.97$$
$$RHO = -0.16 \quad F = 34.72 \quad SER = 4{,}934.1$$

$$R_t = -25{,}093.9 + 614.67\, PO_t + 19.39\, X_t^d + 0.73\, R_{t-1}$$
$$(-2.00) \quad (1.82) \quad (2.15) \quad (5.07)$$
$$R^2 = 0.81 \quad \bar{R}^2 = 0.78 \quad D.W. = 1.98$$
$$RHO = 0.04 \quad F = 29.38 \quad SER = 8{,}625.3$$

$$L_t^s = -2{,}986.6 + 1.08\, G_t - 0.39\, R_t + 0.74\, E_t \;\; 0.136\, B_{t-1}$$
$$(-0.62) \quad (0.76) \quad (-0.84) \quad (2.19) \quad (2.57)$$
$$R^2 = 0.8 \quad \bar{R}^2 = 0.79 \quad D.W. = 1.65$$
$$RHO = 0.59 \quad F = 53.42 \quad SER = 7{,}398.6$$

$$L_t^d = -8{,}885.2 + 117.64\, P_t + 0.025\, Y_t - 10.89\pi_t + 25{,}511.1\, DIR$$
$$(-0.17) \quad (1.68) \quad (0.05) \quad (-0.49) \quad (1.72)$$

$R^2 = 0.78 \quad \bar{R}^2 = 0.75 \quad D.W. = 1.82$
$RHO = 0.50 \quad F = 24.24 \quad SER = 8{,}015.3$

$$P_t = \underset{(0.48)}{71.74} + \underset{(0.081)}{0.081}\, PM_t + \underset{(2.68)}{0.01}\, L_t^s - \underset{(-1.09)}{0.005}\, L_t^d + \underset{(1.19)}{0.468}\pi_t$$

$R^2 = 0.75 \quad \bar{R}^2 = 0.71 \quad D.W. = 1.93$
$RHO = 0.22 \quad F = 17.67 \quad SER = 55.42$

$$\pi_t = \underset{(6.49)}{0.60}\, P_t + \underset{(4.58)}{0.4}\pi_{t-1}$$

$R^2 = 0.68 \quad \bar{R}^2 = 0.67 \quad D.W. = 2.42$
$RHO = -0.29 \quad F = 71.58 \quad SER = 58.59$

$$PN_t = \underset{(2.69)}{425.64} + \underset{(0.41)}{0.417}\, PM_t + \underset{(1.70)}{0.018}\, Y_t - \underset{(-0.17)}{0.029}\, PN_{t-1}$$
$$- \underset{(-3.88)}{324.58}\, DIR - \underset{(-6.38)}{405.88}\, DIN - \underset{(-4.56)}{550.95}\, DS$$
$$- \underset{(-2.96)}{325.51}\, DV - \underset{(-3.21)}{327.29}\, DK$$

$R^2 = 0.82 \quad \bar{R}^2 = 0.79 \quad D.W. = 1.7$
$RHO = 0.86 \quad F = 30.12 \quad SER = 52.43$

$$Y_t = \underset{(-0.58)}{-10{,}683.82} + \underset{(0.88)}{961.87}\, PO_t + \underset{(2.14)}{13.05}\, X_t^s + \underset{(2.23)}{0.56}\, Y_t$$

$R^2 = 0.72 \quad \bar{R}^2 = 0.71 \quad D.W. = 2.01$
$RHO = 0.23 \quad F = 43.89 \quad SER = 20{,}418.4$

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