An Example of Using the BVAR Model and Not Violating the "Lucas Critique:"
An Explanation of the Recent Korean Economic Boom*

Wookyu Park**

This paper presents a historical analysis of the effects of the favorable exogenous economic environment of 1985-1986 on the 1985-1986 Korean economic boom, by using a BVAR model that allows time varying coefficients. The analysis demonstrates the empirical validity of the "Lucas Critique:" an abrupt change in the economic environment induces changes in people's behavior, and thus a reduced-form model does not help forecast the future. This paper shows how one may overcome the "Lucas Critique" in using a BVAR model to measure the effects of possible structural changes on economic variables, as long as a historical event is the focus of the analysis. This paper addresses the issue of the quantitative significance of the Lucas Critique that was not fully resolved in Miller and Robberds and thus must be viewed as a substantial improvement upon Miller and Robberds.

I. Introduction

The purpose of this paper is two-fold. One is to demonstrate

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that the "Lucas Critique" is indeed valid empirically: If there are abrupt changes in the economic environment, then a descriptive model (whether it is a purely descriptive reduced-form model as in the Vector Autoregression (VAR) approach, or a conventional reduced-form model as in the IS-LM type macroeconometric model) does not help forecast futures. The Korean economy during the past two years (1985-1986) experienced an abrupt change in its economic environment. There have been sharp reductions in the international interest rates, oil prices, and the value of the dollar. In 1986, these reductions were far past the usual fluctuation levels of the previous five years. This paper shows that a Bayesian Vector Autoregression (BVAR) model which allows time varying coefficients could not have forecast at the beginning of 1985 the actual responses (real GNP, exports, imports, and the wholesale price index) of the 1985-1986 Korean economy.

Sims argues that "the rational expectations critique of econometric policy analysis is a cautionary footnote to such analysis rather than a deep objection to its foundations," because the "Lucas Critique" is rarely applicable. The results in this paper suggest, however, that the "Lucas Critique" may be more than a footnote, because we may encounter more often than not situations in which the "Lucas critique" is relevant.

The moving average representations calculated at three points during the two-year period show that indeed there have been structural changes in the responses of the Korean economy to the abrupt changes in the economic environment. This is a situation in which the "Lucas Critique" applies. In such a case, a reduced-form model cannot be reliably used to forecast the future. The empirical results of this paper show that indeed that is the case.

The second purpose of this paper is to construct an example in which the BVAR model overcomes the "Lucas Critique," when the purpose of the analysis is to measure the structural changes of the past. By applying the structural changes which are assumed to be captured during the third quarter of 1986 retrospectively to the first quarter of 1985, the BVAR model of this paper was able to forecast quite well the real GNP, exports, imports, and wholesale price index (WPI) of 1985 and 1986. Notice that overcoming the "Lucas Critique" was possible because this paper presents a historical analysis. If the experiment is about the future on which
we do not have any available data, then we cannot use the BVAR model of this paper to forecast future fluctuations and at the same time overcome the "Lucas Critique." Thus this paper is again another example of demonstrating the empirical validity of the "Lucas Critique."

Section II describes the economic environment in which the 1985-1986 Korean economic boom occurred. The values of the three exogenous variables — the value of the dollar, the international interest rates, and the oil prices — have continued to decline for about two years. This phenomenon was quite unusual and is known as the "Three Lows" in Korea. Section II then introduces the time-varying BVAR model. Section III defines both the price effects of the "Three Lows" on the Korean economy and the effects of structural changes on the Korean economy. It is demonstrated that the effects of structural changes are measured so as not to violate the "Lucas Critique." This section also argues that the moving average (MA) representations calculated at various points in time exhibit the possibility of structural changes. Section III then calculates the price effects and the effects of the structural changes. Section IV concludes the paper.

II. Description of the Data for the Korean Economy and the Specification of the BVAR Model

The Euro-dollar rate, the price of oil (Arab light) and the value of the dollar relative to the value of the yen were the most important exogenous variables for the 1985-1986 Korean economic boom. The values of all three variables have declined since the first quarter of 1985. Moreover, in 1986, the declines were beyond the usual fluctuations of the previous five years, and this phenomenon was coined by the Koreans as the "Three Lows." Many people in Korea believe that the "Three Lows" have caused high growth rates for the real GNP, exports, and imports, and also stable WPI for 1985 and 1986.

This paper attempts to verify whether the economic boom was purely due to the price effects (the decline in the prices of the dollar, oil and foreign financial assets). For that purpose, we use a BVAR model which allows time varying coefficients. The BVAR model is specified as follows1:
\[ Y_t = A_t(L)Y_{t-1} + B_t(L)X_t + C_t + \varepsilon_t \]
\[ A_t(L) = A_{0t} + A_{1t}L + A_{2t}L^2 + \ldots + A_{kt}L^{k-1} \]
\[ B_t(L) = B_{0t} + B_{1t}L + B_{2t}L^2 + \ldots + B_{kt}L^k \]

\[ \theta_t = \theta_{t-1} + \mu_t \]

where

\[ Y_t^1 : (\log y_{1t}, \log y_{2t}, \log y_{3t}, \log y_{4t}) \]
\[ X_t^1 : (x_{1t}, \log x_{2t}, \log x_{3t}) \]
\[ y_{1t} : \text{real GNP} \]
\[ y_{2t} : \text{exports} \]
\[ y_{3t} : \text{imports (excluding oil imports)} \]
\[ y_{4t} : \text{WPI} \]
\[ x_{1t} : \text{Euro-dollar rate} \]
\[ x_{2t} : \text{the price of oil (Arab light)} \]
\[ x_{3t} : \text{the price of the dollar (yen/$)} \]

\( L \) is the lag operator, and \( \varepsilon_t \) and \( \mu_t \) are uncorrelated error term vectors. \( A_t, B_t \) and \( C_t \) are the matrices and vectors that consist of time varying coefficients.

Notice that the logarithms of the variables are used except for the Euro-dollar rate. This is to impose the Bayesian prior that log \( y_{it} - \log y_{it-1} \) is stationary once the trend in the variable is removed.\(^2\) This use of the Bayesian prior is necessary in order to resolve the overfitting issue in a way to improve the forecasting performance.\(^3\) \( \theta_t \) is the parameter vector obtained by stacking up all the time varying coefficients of equation (1). For estimation, we use the Kalman filter period-by-period updates, and the Bayesian prior parameters described in Doan, Litterman and Sims.\(^4\)

In equation (1), the current values of the exogenous variables are included to reflect the presumption that those exogenous

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1 This specification is based on Doan, Litterman, and Sims. The author benefited from the "RATS" program, "Bayesian Vector Autoregression Model of the U.S. Economy" obtained from the Research Department of the Federal Reserve Bank of Minneapolis. The author obtained the data from the Korea Development Institute data tape.

2 The trend is removed by adding a time varying coefficient vector \( C_t \) in equation (1).

3 For a discussion on the overfitting issue and the use of the Bayesian prior for forecasting, see Litterman; Sims; Doan, Litterman and Sims; Todd among others.

4 The actual values of the prior parameters and the associated theil's U-statistics for the model's forecasting performance are available from the author upon request. A detailed analysis is described in Park.
variables affect the endogenous variables both contemporaneously and after a few periods.

Because $\theta_t$ varies over time, the BVAR model — equations (1) and (2) — takes into account the possibility that people's behavior will change in response to changes in environment ($X_t$). In the BVAR model of this paper, the time varying coefficient vector $\theta_t$ represents people's behavior.\(^5\)

III. Price Effects and the Effects of Structural Changes

This section assumes that the continuous decline in the value of the dollar, the international interest rates, and oil prices during 1985-1986 has caused two effects on the Korean economy: First, the decline in the values of the three exogenous variables does have a price effect on the Korean economy even if the changes in the prices do not change the pattern of people's behavior. For instance, in equation (1) with a fixed value of $\theta_t$, $Y_t$ will be different in response to different values of $X_t$. Second, since the pattern of changes in $X_t$ for the past two years was quite unusual, $\theta_t$ may change. In other words, people's behavior may be of a different pattern, which can be viewed as changes in the model structure. Thus the changes in $X_t$ induce the effects of structural changes (changes in $\theta_t$) on the Korean economy.

In this paper, those effects are defined by using the following three forecasting experiments.

Experiment A: This experiment assumes that the values of $X_t$ in 1985: I are maintained through 1986: IV. That is, there are no changes in exogenous prices. This experiment then forecasts the values of $Y_t$ from 1985: II to 1986: IV, based on the data up to 1985: I.

Experiment B: Assuming that $X_t$ assumes the actual values of

\(^5\) The random walks specification for $\theta_t$ is used in Sims; Doan, Litterman and Sims; Miller and Roberts among others. This specification is general enough to capture the systematic changes in people's behavior under a changing environment. For example, a permanent change in tax policy will induce a permanent change in parameter values in an investment function as in Lucas; Sargent (pp. 344-345).
$X_t$ from 1985: II to 1986: IV, this experiment forecasts $Y_t$ from 1985: II to 1986: IV based on the data up to 1985: I.

The differences in the forecasts in Experiments A and B are due to the pure price effects, because $\theta_t$ was estimated based on the data up to 1985: I and the possibility of changes in $\theta_t$ after 1985: I was not incorporated.

Experiment A is a forecasting exercise conditioned on the naive assumption that there will be no changes in the exogenous variables. Experiment B is almost identical to the conditional forecasting exercise in Miller and Roberds, except that Experiment B is conditioned on the future values of the exogenous variables whereas the other is conditioned on the future values of the policy variables. Miller and Roberds concluded that the conditional forecasts were unable to explain actual data because "the conditional forecasts ignore potential future variation in the model coefficients." Therefore, as they conclude, "(their exercise does) not provide any evidence that the Lucas Critique is invalid in an empirical sense."

The contribution of this paper is that it has improved upon their results cited above by defining the following Experiment C. Experiment C allows one to measure the extent of the structural changes in response to policy changes and/or external shocks without violating the Lucas Critique.

Experiment C: This experiment uses actual data for $X_t$ and $Y_t$ up to 1986: III, when the continuous decline in the values of $X_t$ moderated. The value of $\theta_t$ obtained from the above estimation — $\theta_t$ for 1986: III — is assumed to be the value of $\theta_t$ for 1985: I. This $\theta_t$ is then used to forecast $Y_t$ from 1985: II to 1986: IV based on the data up to 1985: I, under the assumption that $X_t$ takes actual values, as in Experiment B.

The presumption in Experiment C is that the possibility of changes in people's behavior is not revealed in the data up to 1985: I. Yet the manner in which people actually change their behavior is revealed in the 1985-1986 data. Experiment C is
meant to capture that change. The differences in the forecasts in Experiments B and C are due to the effects of structural changes.

Strictly speaking, unlike Experiments A and B, Experiment C is not a forecasting exercise. Rather, it is an ex post “fitting” exercise. Notice that this approach of analyzing the effects of abrupt changes in $X_t$ on the Korean economy does not violate the “Lucas Critique,” only because the analysis were about past data. If the analysis were about the effects on future $Y$ for which we do not have any data, then the best alternative would be Experiment B, which has the possibility of violating the “Lucas Critique.”

In order to see if carrying out Experiment C is worthwhile, we calculated the moving average representations (MAR) at 1985: I, 1985: IV, and 1986: III. The MARs in Appendix A-C show that there may have been a structural change in the Korean economy between 1985: IV and 1986: III. If that is the case, then Experiment B alone would not fully capture the effects of changes in $X_t$ on the Korean economy.

The results of the experiments are compared with the actual data in Figures 1-4 and Tables in Appendix D. These figures show that Experiment B cannot explain the 1985-1986 Korean economic boom. One may argue that this is exactly what happened in Miller and Roberds: Conditional forecasts cannot explain actual data during a substantial policy (and/or environment) change because conditional forecasts ignore potential future variation in the model coefficients. Experiment C, however, explains the actual data quite well, which testifies the empirical validity of the “Lucas Critique.”

IV. Conclusion

This paper shows an empirical example using a BVAR model while not violating the “Lucas Critique.” The results in this paper

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6 MAR evaluates $\frac{\partial \log y_{t+1}}{\partial \log x_t}$, $i = 1, ..., 24$. If $x_t$ is Eurodollar rate, $\frac{\partial \log y_{t+1}}{\partial x_t}$.

7 MARs are shown in respective figures in Appendix A-C. These figures show that the effects of the exogenous shocks on the economy do not easily dissipate over time. This is consistent with Nelson and Plosser; Campbell and Mankiw to list only a few. The charts in Doan, Litterman and Sims also exhibit similar responses.
also suggest that one should be careful about using a reduced-form model for forecasting and/or policy evaluation for one may encounter more often than not cases where the "Lucas Critique" is truly relevant.

The empirical analysis of this paper shows that favorable exogenous economic environment alone cannot explain the 1985-1986 Korean economic boom. It turns out that a consideration of structural changes induced by the improvements in the economic environment is necessary for an explanation of the recent Korean economic boom. The effect of structural changes is defined so as not to violate the "Lucas Critique" in a BVAR model.

Such a use of BVAR model, however, is possible only because the experiment herein is about a past event: By definition, the details of structural changes cannot be captured if the object of the experiment is about the future. Hence, this paper demonstrates an example of the empirical validity of the "Lucas Critique" — an abrupt change in the economic environment induces changes in people's behavior, and thus a reduced-form model does not help forecast the future.

This paper addresses the issue of the quantitative significance of the "Lucas Critique" that was not fully resolved in Miller and Roberds and thus must be viewed as a substantial improvement upon Miller and Roberds.
Appendix A

MA REPRESENTATIONS
RESPONSES OF VARIABLES TO A 1 PERCENT POINT RISE IN THE EURO-DOLLAR RATE

Figure A.1
REAL GNP

□: MAR calculated at 85: I
+ : MAR calculated at 85: IV
○: MAR calculated at 86: III
Appendix B

MA REPRESENTATIONS

RESPONSES OF VARIABLES TO A 1 PERCENT POINT RISE IN THE PRICE OF OIL (Arab Light)
Appendix C

MA REPRESENTATIONS
RESPONSES AT VARIABLES TO A 1 PERCENT POINT
RISE IN YEN/$

Figure C.1
REAL GNP
### Appendix D

#### Table D.1.

**REAL GNP**

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#### Table D.2.

**EXPORTS**

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Table D.3.

IMPORTS

(Million U.S. Dollars)

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Table D.4.

WHOLESALE PRICE INDEX

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