

The Demographic Transition and Aggregate Savings in Less Developed Countries

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Savings functions estimated with cross section, country level data often find savings rates increasing with income. This paper argues that this result is partly due to the omission of demographic changes which occur as countries develop, rather than to an underlying convexity of the savings function. Using some concepts from demography, a theoretical connection is established between declines in child mortality and subsequential declines in birth rates on one hand and changes in aggregate saving on the other. The magnitude of this effect is then appraised by a variety of methods.

I. Introduction

There has been a great deal of empirical research on aggregate savings functions in less developed countries. A common characteristic of many of these studies is that savings rates are found to be increasing in total income, or, that the savings functions themselves are convex in income (Landau; Chenery and Syrquin), and the survey by Mikesell and Zinser. For aggregate data from say, a cross-section of countries, there may be a number of reasons for this finding. An increasing role of government saving or the shift from agriculture with nonfinancial savings instruments which has difficult to measure to industry where data is easier to obtain, may influence the recorded aggregate sav-

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ings rates. However, the aggregate results are often thought to stem from the summation of individual savings functions which are of similar, convex form. Bhalla presents some evidence from survey data this may be true, at least for a large range of incomes. This view has unfortunate implications concerning the scope of income redistribution in the development process. Since redistribution is thought to decrease total savings, by transferring income from high marginal savers to low, this conclusion is thought to be a serious impediment to egalitarian-minded development schemes.

This paper examines the implications of changing fertility patterns in less developed countries on aggregate savings. The purpose is two-fold. First, it is to provide a theoretical justification for the interaction of population structure and aggregate savings. Second, it is to show that standard empirical studies attribute a spuriously large effect to increased income *per se* on savings rates. In particular, the effect of changes in the age structure which are likely to occur (and are generally observed) in LDC's is investigated. These changes are modelled by the demographic transition, a stylized fact of development in which death rates decline at some low level of income while birth rates decline with a lag. In conjunction with the standard theory of life cycle savings, the demographic transition is used to show that personal savings functions which are linear (or slightly concave) in income can generate convexity in the aggregate. Part II presents the basic framework of analysis and essential results. In Part III a variety of methods are employed in order to appraise the probable quantitative impact of the analysis on empirical results.

II. Theoretical Framework

We assume that private savings undertaken by individuals follow an age-specific pattern of the form $s(a,y) = y(a)s(a)$, where $s(a,y)$ is total savings of an individual in age group a , $y(a)$ is income accruing to someone in that age group (assumed to be a constant fraction of aggregate income, i.e., the income distribution by age group is assumed to be constant for this analysis), and $s(a)$ is the age-specific savings rate. For now, the precise form of the functions $s(a)$ and $y(a)$ need not be specified though the common conception of these functions would give them both inverted

U-shapes; zero income (or negative savings) for young people, high for people in their productive years and low or negative for the elderly.

The applicability of this life cycle theory of savings to LDC's has come under some criticism (Landau). However, it has been used with some success in certain cases such as Italy (Modigliani and Tarantelli) where it was shown to perform very well empirically and retains a strong intuitive appeal.

To find aggregate savings it is necessary to integrate over all age groups or:

$$S = \int_0^A y(a)s(a)f(a;\bar{y})da,$$

where

$$\bar{y} = \int_0^A y(a)f(a;\bar{y})da,$$

where A is the maximum age attainable — assumed to be fixed for the remainder of the analysis, and $f(a;\bar{y})$ is the proportion of the population of people of age a for any given level of national income \bar{y} . That A is assumed to be a constant is inessential. It can be considered an arbitrarily large number.

The term $f(a;\bar{y})$ will be assumed to be generated by what is known to mathematical demographers as a stable population. It is known that a population with fixed, age-specific, birth and death rates will converge to a population with a unique growth rate and age distribution (Lotka). Since the primary source of data on LDC savings functions has been cross-sectional where observations are taken to represent long run equilibria, reliance on the stable population age distribution will be the approach most consistent with the savings side of the story. In a time-series context use of the stable distribution implies that the population convergence occurs faster than changes in the effects of income on birth rates, i.e., that the target moves slower than the approach to it. The long run, stable population distribution can be shown to be of the following form (Coale):

$$f(a;\bar{y}) = \frac{e^{-r(\bar{y})a}p(a;\bar{y})}{\int_0^A e^{-r(\bar{y})a}p(a;\bar{y})da}.$$

Here $r(\bar{y})$ is the equilibrium growth rate of the population and $p(a;\bar{y})$ is the probability of survival to age a , i.e., the product of all survival probabilities up to age a . Note that since $f(a;\bar{y})$ is always nonnegative and integrates to unity, it possesses all the characteristics of a probability density function. Therefore, aggregate personal savings can be expressed as

$$S = \frac{\int_0^A y(a)s(a)p(a;\bar{y}) e^{-r(\bar{y})a} da}{\int_0^A e^{-r(\bar{y})a} p(a;\bar{y}) da},$$

and the savings rate as

$$(1) Q = \frac{\int_0^A q(a;\bar{y}) p(a;\bar{y}) e^{-r(\bar{y})a} da}{\int_0^A p(a;\bar{y}) e^{-r(\bar{y})a} da},$$

where $q(a;\bar{y}) = \frac{s(a)}{\bar{y}}$ and it remains to be seen how this term changes during the demographic transition, or, as a result of increased income.

The effect of the growth of income on birth and death rates which is commonly observed in the development process can be decomposed into two successive regimes (Chenery and Syrquin). In the first place, the growth of income reduces mortality rates in the population. This may come from the elimination of communicable diseases which effects the entire mortality schedule but the main effects have been primarily in the reduction of infant and child mortality rates. This is due to factors such as improved nutrition with higher incomes and better hygiene as water supplies are increased and improved with public investment. For the purpose of this paper, we will assume that the effect is entirely on the mortality rates of infants and children.

The second aspect of the transaction is the subsequent reduction in birth rate. Again, there are a variety of possible explanations for this phenomenon and for the fact that the reduction occurs with a lag. On the one hand, the birth rate may decline for reasons independent of the changes in the mortality rate. Thus, increasing urbanization and participation of women in the workforce will tend to reduce birth rates and it may just be an empiri-

cal regularity that these effects occur at income levels higher than those levels which reduce mortality rates. Alternatively, the reduction in death rates (of children particularly) may directly lead to fertility declines. If families calculate a target family size and wish to end the woman's child-bearing years with a certain number of children, it will be necessary to have more children than is ultimately desired since a relatively high proportion are likely to die. As mortality rates are reduced parents may operate on the assumption that they are still up high as in their own generation until new information is available. Thus, the learning of the true probabilities may take as long as a generation or two and results in the lagged decline.

In either case, the problem is to determine the effect on aggregate savings rates of an increase in income which also accompanies the demographic transition. The first part is the decline in child mortality, the second, a decline in fertility.

For the early stage, expression (1) is differentiable with respect to income, noting the effect on the population growth rate and the function $p(a)$. Simplification leads to:

$$\frac{\partial Q}{\partial y} = \int_0^A q(a)f(a;\bar{y}) \left\{ -a \frac{dr}{dy} + \frac{1}{p(a)} \frac{dp(a)}{dy} \right\} da$$

$$-Q \int_0^A f(a;\bar{y}) \left\{ -a \frac{dr}{dy} + \frac{1}{p(a)} \frac{dp(a)}{dy} \right\} da.$$

To simplify this expression further, a commonly used (Coale) approximation to the equilibrium growth rate can be applied.

$$r \approx \frac{\log(G) + \log p(\bar{m})}{\bar{m} - \sigma^2} - \frac{\log(G)}{2\bar{m}}$$

where

- G : gross rate of reproduction
- \bar{m} : mean child-bearing age
- $\sigma^2 = \int_{m_i}^{m_2} (m - \bar{m})^2 c(a) da$: variance of child bearing age

m_1, m_2 : earliest and latest ages of child bearing

$$\bar{m} - \sigma^2 \frac{\log G}{2\bar{m}} = T: \text{ length of generation.}$$

In the first regime of the demographic transition, only $P(\cdot)$ is affected. Differentiation and substitution yields

$$\begin{aligned} \frac{\partial Q}{\partial y} &= \int_0^A q(a)f(a;\bar{y}) \left\{ \frac{1}{p(a)} \frac{dp(a)}{dy} - \frac{a}{T} \cdot \frac{1}{p(\bar{m})} \cdot \frac{dp(\bar{m})}{dy} \right\} da \\ -Q &= \int_0^A f(a;\bar{y}) \left\{ \frac{1}{p(a)} \frac{dp(a)}{dy} - \frac{a}{T} \cdot \frac{1}{p(\bar{m})} \cdot \frac{dp(\bar{m})}{dy} \right\} da. \end{aligned}$$

Note that the expression in brackets depends on the proportionate decrease of every age group relative to the decrease of the mean of the child-bearing age group. To simplify this expression we make use of the following result:

Lemma. If changes in the survival function are limited to an interval $[0, b]$, then the proportionate change in survival is the same for all ages greater than b . Let $\mu(i)$ be the death rate at age i .

Proof. Let $\mu(i)$ be the death rate at age i :

$$p(a) = \prod_{i=0}^a (1 - \mu(i)) = \prod_{i=0}^b (1 - \mu(i)) \prod_{i=b+1}^a (1 - \mu(i))$$

$$dp(a) = d \prod_{i=0}^b (1 - \mu(i)) \prod_{i=b+1}^a (1 - \mu(i))$$

$$dp(a) = dp(b) \prod_{i=b+1}^a (1 - \mu(i))$$

$$\frac{dp(a)}{p(a)} = \frac{dp(b) \prod_{i=b+1}^a (1 - \mu(i))}{\prod_{i=0}^b (1 - \mu(i)) \prod_{i=b+1}^a (1 - \mu(i))}$$

$$\frac{dp(a)}{p(a)} = \frac{dp(b)}{p(b)} \quad \text{for all } a > b.$$

Hence

$$\frac{1}{p(a)} \cdot \frac{dp(a)}{d\bar{y}} = \frac{1}{p(m)} \cdot \frac{dp(m)}{dy} = \varepsilon.$$

This indicates that if mortality rates decline up to a maximum age b , that is, only infants and children are affected by the decline, the proportionate increase in the probability, evaluated at birth, of surviving to all ages older than b is the same. If we make the further assumption that children up to age b do not themselves save (as is reasonable since b should be about five years of age), then

$$\begin{aligned} \frac{\partial Q}{\partial y} &= \int_0^A a(q)f(a) 1 - \frac{a}{T} \varepsilon da - Q \int_0^A f(a) 1 - a \frac{A}{T} \varepsilon da \\ &= -\frac{\varepsilon}{T} \left\{ \int_0^A a q(a) f(a) da - \int_0^A q(a) f(a) da \int_0^A a f(a) da \right\} \end{aligned}$$

Since the function $f(a; \bar{y})$ has all the properties of a probability density function, this last term may be interpreted as

$$E(a \cdot s(a)) - E(a)E(s(a)) = \text{cov}(a, s(a))$$

or

$$\frac{\partial Q}{\partial y} = -\frac{\varepsilon}{T} \text{cov}(a, s(a)).$$

Thus, the effect on the savings rate of the decrease in infant and child mortality due to the demographic transition depends on the covariance of age and the age-specific savings function. If the covariance is positive, the decrease in mortality will tend to decrease savings by increasing the proportion of the population in the younger, low saving age groups. If the covariance is negative the effect of the first part of the transition is to increase savings as

the proportion of people in the middle ranges of the age distribution (high savers) increases.

While a priori, any savings function is admissible, the empirically relevant ones almost certainly would yield a positive covariance. If, as a stylized fact, the life cycle savings functions is the inverted U mentioned before, and, since the stable age distribution is a monotonically decreasing function of age, the early positive correlation of age and savings will outweigh the later, negative relation. Larger probability mass is associated with early ages and this region will hold sway. Thus, the aggregate savings rate should decline in this stage of economic development due to demographic shifts.

For the second phase of the transition, the analysis is considerably easier. Since there is no further change in mortality rates, the only effect of increasing income is to decrease the fertility or gross reproduction rate.

So:

$$\frac{\partial Q}{\partial y} = \frac{\int_0^A -aq(a)(e^{-ra})p(a) \frac{dr}{dy} da - Q \int_0^A -p(a)(e^{-ra}) \frac{dr}{dy} da}{\int_0^A p(a)(e^{-ra}) da}$$

$$= -\frac{dr}{dy} \text{cov}(a, s(a)).$$

Using the approximation for r mentioned above it is clear that $\frac{dr}{dy} < 0$, or, that the savings rate varies directly with the covariance with age. This is understandable since the only effect of income in this stage is to decrease the birth rate. This should decrease the proportion of young people and bias the age distribution toward high savers again.

Combining these two regimes we can generate the intended result. If age and savings rates are positively correlated in the population, the demographic transition will tend to first depress and then boost the aggregate savings rate generated from linear personal savings functions. As most of the world's nations have already experienced some drop in death rates, the major impact of these changes is observed from unusually low to higher savings

rates. In cross-section, country level data, this will show up as convexity in aggregate savings.

III. Quantitative Impact of Age Distribution

Having established the theoretical possibility of the effect of the demographic transition on aggregate savings, the probable size of the effect can be appraised. For this it is necessary to have information on the stable age structure of countries passing through to demographic transition. Age specific savings functions are also required. Since this information is difficult to obtain, three methods will be used to approximate these effects. These three approximations are: (1) reconstruction of age structures by empirical birth and death rates, (2) use of published tables of stable populations and (3) estimation of aggregate savings functions with population structure taken into account.

A. Method 1

The first technique employs the construct of the Leslie matrix from the demography literature (Leslie). If the population is divided into a finite number of discrete age groups and the number of members of each group arrayed in a vector, the Leslie matrix summarizes the birth and death rates which generate the population vector in the next period. Formally:

$$\begin{bmatrix} k_1 \\ \vdots \\ k_i \\ \vdots \\ k_{n_t} \end{bmatrix} = \begin{bmatrix} b_1 & \dots & b_i & \dots & b_n \\ & f_1 & & & \\ & & \dots & & 0 \\ & & & f_i & \\ & & & & \dots \\ 0 & & & & f_{n-1} & f_n \end{bmatrix} \begin{bmatrix} k_1 \\ \vdots \\ k_i \\ \vdots \\ k_{n_{t-1}} \end{bmatrix} \quad \text{or } k_t = Mk_{t-1},$$

where b_i is the birth rate for age group i and f_i is the proportion of people in age group i who survive for the next period, i.e., it is one minus the death rate. The k_i 's are the proportion of the population in age group i . The population growth rate and the stable age distribution can be derived by solving the determinant equation:

$$|M - \lambda I| = 0$$

in which the largest real root is the population growth rate and its corresponding right eigenvector is the stable age distribution, k .

Data on births and deaths at this level of detail for a cross-section of countries is not available. Therefore, the method used here was to derive families of matrices consistent with aggregate birth and death statistics. From the Chenery and Syrquin study, birth rates, death rates and infant mortality rates for countries at three levels of per capita income were computed. Countries designated as low income are those with a per capita income level of below \$200 in 1970. High income countries (high relative to those generally included in cross-section savings studies) have incomes above \$800.

From this aggregate data, an attempt was made to reconstruct the population transition matrix in such a way as to conform to population growth characteristics of each of the two hypothetical countries. In order to keep the number of unknown parameters to a minimum, a population model with three generations was chosen as a compromise with desired detail of population specification. Each generation, therefore, is on the order of twenty-five years. In addition, a "holding" generation of one year olds was included in order to concentrate on the effect of declines in infant mortality and to keep large changes in infant mortality from unduly affecting the calculation of generation size.

For the three generation model, the system becomes

$$k_1 = b_2/G$$

$$k_2 = \sqrt{b_2 f_1}/G$$

$$k_3 = f_2 \sqrt{b_2 f_1} / ((\sqrt{f_1 b_2} - f_3)G)$$

$$G = b_2 + \sqrt{b_2 f_1} + f_2 \sqrt{b_2 f_1} / (\sqrt{b_2 f_1} - f_3)$$

$$\text{Birth rate} = b_2 k_2$$

$$\text{Death rate} = k_1 (1 - f_1) + k_2 (1 - f_2) + (m/1 - b_2) + k_3 (1 - f_3)$$

$$\text{Population growth rate} = \sqrt{b_2 f_1}$$

where m is the infant mortality rate. Since there are still too many free parameters, it is necessary to specify the value of f_3 beforehand. These values were varied within a wide empirically relevant range and the results were found to be quite insensitive to changes in these parameters. The age distribution and remaining variables result from the solution of the above equations using empirical birth, death and population growth rates. The typical age distribution to result from this procedure is:

	<u>Low income</u>	<u>High income</u>
k_1	0.643	0.547
k_2	0.268	0.311
k_3	0.089	0.142

From this point, the effect of the alternative age distributions on predicted savings rates can be calculated if age specific savings functions were known. Here, again, the data leave something to be desired. To overcome this deficiency, a variety of alternative savings functions can be used. With three assumptions the range of possible effects can be explored. These are: (1) that only people in the oldest two generations save, (2) the middle generation saves at least as much as the oldest taking both income and propensity to save into account (relaxing this assumption strengthens the predicted effect) and (3) the savings rate of the poorer country is normalized to its empirical level of about 13% (again using World Bank analyses). If the total savings rate of the economy is $\sum_i s_i k_i$, the results below show the savings rates when: (A) $s_2 = s_3$, (B) $s_2 = 3s_3$, and (C) $s_3 = 0$, i.e., when the middle generation performs all the savings.

<u>Case</u>	<u>Low income</u>	<u>High income</u>
A	0.13	0.165
B	0.13	0.156
C	0.13	0.151

The effect of age structure changes consistent with empirical population growth data can range from 2.1 to 3.5 percent between poor and rich countries. The empirical difference between these countries is about 8 percent (Chenery and Syrquin). So,

while not accounting for the entire increase in personal savings rates, the effect of shifting age distribution may be an important component even if the underlying personal savings functions are linear in income.

B. Method 2

The second approach to the problem is to use complete theoretical stable age distributions calculated from empirical birth and death rates. The work of Coale and Demeny on stable age structures was based on data derived from European sources. While not covering countries at the level of income encountered in the current developing world, those tables employing birth and death schedules which resemble current values may be instructive. For the low income countries, the main criteria for the choice of age distributions were high infant mortality in the underlying mortality schedule and an overall fertility and mortality rate close to empirical values. For the high income countries, those schedules in which the main reduction in mortality was due to higher infant and child survival rates were chosen. Of the tables reported in that work, the two schedules most reflective of current poor and relatively well-off LDC's are those reproduced in Table 1 (Coale and Demeny, pp. 482 and 282). Again, a variety of schedules were tried and the results are quite insensitive to the particular choice. Recent work (Kotlikoff and Summers) shows that life-cycle savings functions are relatively flat for adults. The results here, then, should depend primarily on the proportion of people under age 15. A variety of functions were used for these calculations as illustrated in Figure 1. With these savings functions used (the level of the function being normalized so that the poorer country again saves 13 percent of income) the difference in savings rates between the two countries varies from 1.9 to 2.8 percent. This is similar to the constructed age distribution results.

C. Method 3

In an influential study, Leff (1971) showed the effect of dependency rates (the fraction of the population younger than 15 and older than 65) on the aggregate savings rates of a cross-section of 47 LDC's. Here, that study is reproduced with two modifications on a sample of 86 LDC's in 1960-1970. The first modifica-

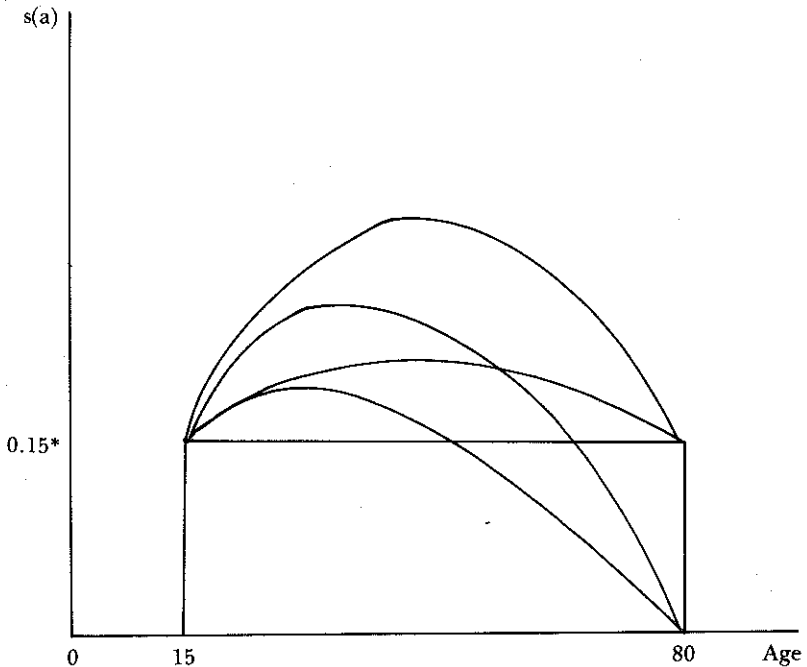
Table I

	Age	0-1	1-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
Income										
Low		0.039	0.129	0.137	0.118	0.103	0.088	0.075	0.064	0.054
High		0.024	0.090	0.103	0.095	0.088	0.081	0.074	0.068	0.062
	Age	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80+
Low		0.046	0.039	0.032	0.026	0.020	0.014	0.009	0.005	0.002
High		0.057	0.051	0.046	0.041	0.036	0.030	0.024	0.017	0.015

Entries are the proportion of the population in each age group.

Figure 1

ALTERNATIVE SHAPES OF SAVINGS FUNCTIONS



*Actual level of functions vary to yield observed national average (see text).

tion is that personal savings rates (rather than the gross national savings rates in the original) are used since the theory proposed here is not related to the behavior of governments. Second, since one motivation of this study is to show how the omission of the population parameter can indicate a convex savings relation, the equations are run with and without the dependency rates. The hypothesis is that the effect of per capita income on the savings rate will be smaller when the dependency rate is included. Following Leff, the estimated equation is

$$\ln \frac{S}{Y} = 0.4512 + 0.1559 \ln \frac{Y}{N} + 0.0328 \ln(G) - 0.9454 \ln(DP)$$

(2.1937) (0.0828) (0.1097) (0.4156)

$$S.E. = 0.4741$$

$$R^2 = 0.1323$$

where S/Y: savings rate

Y/N: per capita income (in 1970 dollars)

G : income growth rate for the previous decade

DP : dependency rate (proportion of the population younger than 15 and older than 65).

Standard errors are in parentheses. Data is from The World Bank.

When the dependency rate is omitted, the regression equation becomes

$$\ln \frac{S}{Y} = -3.4943 + 0.2172 \ln \frac{Y}{N} + 0.0195 \ln(G)$$

(0.4340) (0.0768) (0.1110)

$$\text{S.E.} = 0.4808$$

$$R^2 = 0.0967.$$

These results conform quite well to the original study.

To illustrate the direct effect of income on demographic structure, the following regressions were run:

$$\ln(DP) = 59.1027 - 2.0778 \ln \frac{Y}{N}$$

(3.6172) (0.5954)

$$\text{S.E.} = 3.741$$

$$R^2 = 0.1898$$

$$\ln(DP) = -4.7018 + 19.8548 \ln \frac{Y}{N} - 1.8455 \left(\ln \frac{Y}{N} \right)^2$$

(19.471) (6.6182) (0.5550)

$$\text{S.E.} = 3.4247$$

$$R^2 = 0.3341$$

The second equation illustrates the entire demographic transition in the sample. Dependency (mostly due to large numbers of children) increases at low levels of income and decreases thereafter as outlined in the theoretical discussion. The turning point occurs at income levels of about \$215 per capita in the lower end of the sample. The first equation, without the quadratic term, shows the direct effect of income on dependency as it is specified in the savings equations. The negative coefficient indicates that over most of the sample, the second phase of the demographic transition dominates.

While not significantly different between the two equations, the coefficient on the per capita income term is lower in the version where the dependency rate is included, as expected. To gauge the importance of this difference, the savings rates for poor and rich countries can be computed with the other variables held at their means. The gap between the savings rates of the poor and rich countries narrows by just over one percent when the population parameter is included. While a bit low, this is in reasonable conformity with the results of the two previous methods.

IV. Conclusion

This paper has had two goals, first, to provide a theoretical justification of the hypothesis that population changes during the development process should have an effect on aggregate savings rates conformable to observed rates and, second, to show that empirical studies of aggregate savings that ignore the effect of population changes will tend to conclude that the marginal propensity to save increases more than in the underlying individual savings functions. It is important to note that this paper has focused on the purely passive response of savings to age structure. There are two ways in which further adjustments in the savings rate may be affected by population changes. Relaxing the assumption of a constant income distribution as a function of age is possible by making the share of income of a generation a decreasing function of its relative size. Such an effect has been discussed by Easterlin; Welch. The effects in the above model are, in general, ambiguous but insofar as the results are sensitive to the relative size of the young and middle-aged cohorts, inclusion of this consideration should dampen the results somewhat as increases in the size of a generation reduce its relative income.

More importantly, the paper has not explored the possible (and perhaps more interesting) behavioral approach which would treat both fertility and savings as joint decisions of individual agents. In that case, it would not be possible to specify separate functions for specific savings and birth rates. One might conjecture that expanding this study to a choice theoretic formulation would strengthen the present results. If children and financial savings directly compete as alternative uses for income of parents, one would expect that a decline in fertility will be accompanied

by an upward shift in the age specific savings function itself. The resulting interaction would be in the direction predicted in the above results. A preliminary attempt at this problem appears in Hammer.

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