Estimating Location-Specific and Firm-Specific Technical Efficiency:

An Analysis of Malaysian Agriculture*

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I. The Setting

Decisions about development strategies in agriculture are in part guided by farm level performances. An important measure of such performance is relative economic efficiency, of which technical efficiency is a component. The literature provides a number of different methodologies to measure technical efficiency; of these, the frontier production function approach popularized by Aigner et al. (1977) can generally be considered as an appropriate method.[1] Lee and Tyler used a similar model to estimate average technical efficiency of Brazilian manufacturing firms. However, their approach only allows the measurement of average technical efficiency of a group of firms, and does not provide estimates of firm-specific technical efficiency for individual observations. Recently, Jondrow et al. and Kalirajan and Flinn independently used a similar method to measure firm-specific technical efficiency for individual sample observations. These in-

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1 A brief but comprehensive discussion on the evolution of frontier production function is given in Forsund et al.

dividual technical efficiency measures are more useful for policy makers than the average technical efficiency estimates. Individual firm-specific efficiency measures facilitate identification of the determinants of efficiency ratings among firms. Appropriate policies then may be formulated to decrease efficiency differentials which is important to accelerate the overall growth of firms. Using this methodology, the issue addressed in this paper is whether farmers operating in the Kemubu Irrigation Project in Malaysia, which commenced operations over a decade ago, are technically efficient.[2] This issue is addressed here with a comparison of the firm-specific technical efficiencies of these farmers with that of farmers operating outside Kemubu.

II. The Model

The theoretical notion underlying the frontier production function is that it shows the most efficient way of technical transformation of inputs into outputs. This corresponds to the terminology of the best-practice technology used by Salter. With one input and one output, the estimation of a frontier production function becomes easy. First, with the assumption of constant returns to scale, the ratio of output to input may be calculated for a number of firms and the highest of all would then represent the frontier output. Second, if the assumption of constant returns to scale is not made, then input and output of the firms may be plotted on a scatter diagram. Specifying a functional form for the relationship between input and output, a smooth curve can be drawn through the highest of these points and this would represent the frontier function.

With two inputs and one output the frontier function can be estimated as follows: First, assuming constant returns to scale, the ratio of each input to output may be plotted in a scatter diagram. Each point in the scatter diagram represents the combination of inputs x_1 and x_2 used to produce a unit of output y. A line joining the lowest of these points would be the efficient iso-

2 The Kemubu Irrigation Project is located in Kelantan state in the Northeast of Peninsular Malaysia. This irrigation scheme created an environment in which the adoption of an improved rice production technology became feasible, and led to important technological transformations within the Kemubu scheme.

quant. Second, without the assumption of constant returns to scale, specifying a functional form, a smooth curve may be drawn through the lowest of the points in scatter diagrams which would represent the frontier function. The former approach was used by Farrell to measure the technical inefficiency of firms in relation to the function thus estimated. The major criticism of this approach is that it uses only marginal observations and a vast bulk of data does not enter the estimation procedure. But, it is possible to fit a smooth curve showing frontier output, using all observations in the estimation.

Extending the above model of two inputs and one output into one output and m-inputs, the frontier production function is estimated as mentioned below.

Let the production frontier be

$$(1) \quad y^* = f(x_1, \ldots, x_m)$$

where y^* is the maximum possible output a firm can obtain by using the inputs (x_i's) in a technically efficient way. It is not unreasonable to expect that not all firms may be technically efficient and consequently, not all firms may be operating on their production frontiers. therefore, the prevailing production method which is specific to a particular individual firm at any particular time period can be written as follows:

$$(2) \quad y = f(x_1, \ldots, x_m) + u$$

where u is the firm-specific technical efficiency parameter. If the firm is technically efficient u takes the value zero, and the firm obtains the maximum possible output, y^*; u takes the value less than zero for those firms which are not technically efficient, and the firms accordingly obtain their outputs $y < y^*$. The negative value of u will vary among firms depending on their technical efficiency according to how close they are to the frontier.[3]

[3] Aigner and Chu estimated such a function using linear programming methods. The major weakness of the approach is that due to the nature of the methodology no statistical tests on the parameters could be carried out.

Further, it is assumed that the frontier output y^* may vary randomly across firms or overtime for the same firm due to improvements in its technical efficiency. Therefore, a random variable, v is added to the above model.

$$(3) \quad y = f(\cdot) + u + v$$

Introduction of v in (3) also means that y is stochastic, and that v captures other random factors such as errors in measurements and deviation from the true functional relationship. The value of v therefore may either be positive, negative or zero.

The technical efficiency relative to the stochastic production frontier is therefore,

$$(4) \quad u = f(\cdot) + v - y$$

Assuming that v is distributed as $N(0, \sigma_v^2)$, and the nonpositive error u is distributed as the absolute value of a normal distribution, $|N(0, \sigma_u^2)|$, the population average technical efficiency is measured by using the estimated frontier, σ_v^2 and σ_u^2 (Aigner et. al. 1977). At this stage, the model deviates from the Aigner et al. model. In this study, besides estimating population average technical efficiency, estimates of firm-specific technical efficiency for individual observations have been calculated. These individual technical efficiency estimates are more useful than the average technical efficiency measure to analyse the distribution pattern of technical efficiency among a group of firms.

Measurement of u_i's for each observation is derived from the conditional distribution of u, given (u + v) (Kalirajan and Flinn).

$$(5) \quad E(u|u+v) = \int_{-\infty}^{0} u \cdot f(u|u+v)$$

where $\quad f(u|u+v) = \dfrac{f(u, u+v)}{f(u+v)}$

III. Estimation

For empirical estimatioin of the model, a flexible functional form, the translog, as introduced by Berndt and Christensen was chosen in the paper.

$$(6) \quad \ln y = a_0 + \sum_i a_i \ln x_i + \sum_i \sum_j b_{ij} \ln x_i \ln x_j + u + v$$

where x and y are as defined in equation (1);
u $\leqslant$ 0 and takes a half normal distribution; and
v $\lesseqgtr$ 0 and takes a normal distribution.

The density function of u and v can respectively be written as:[4]

$$(7) \quad f(u) = \frac{1}{\sqrt{\frac{1}{2}\pi}} \frac{1}{\sigma_u} \exp\left(-\frac{u^2}{2\sigma_u^2}\right) \qquad u \leqslant o$$

$$(8) \quad f(v) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_v} \exp\left(-\frac{v^2}{2\sigma_v^2}\right) \qquad -\infty \leqslant v \leqslant \infty$$

The likelihood function of y is the product of density functioin of each y_k which is equal to the density function of $(u_k + v_k)$.

By convolution formula, the joint density function of $(u_k + v_k)$ can be written as

$$(9) \quad f(u+v) = \frac{1}{\sqrt{\frac{1}{2}\pi(\sigma_u^2 + \sigma_v^2)}}$$

$$\exp\left(-\frac{(u+v)^2}{2(\sigma_u^2 + \sigma_v^2)}\right)\left(1 - F(u + v\frac{\sigma_u}{\sigma_v})\right)$$

where F(·) is the cumulative distribution functioin of the standard normal random variable.

Specifying

[4] The validity of these assumptions can be examined by using a family of convolutions. Also, it can be examined by plotting the combined residuals e, the individual technical efficiency u and the output levels.

i) $\sigma^2 = \sigma_u^2 + \sigma_v^2$

ii) $\gamma = \dfrac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2}$ where γ lies in the interval [0,1]

iii) u + v = e

The density function of y_i is:

$$(10)\quad f(y) = \frac{1}{\sigma\sqrt{\pi/2}} \exp\left(-\frac{1}{2}\frac{e^2}{\sigma^2}\right)\left(1 - F\left(\frac{e}{\sigma}\sqrt{\frac{\gamma}{1-\gamma}}\right)\right)$$

The likelihood function of the sample can now be written as:

$$(11)\quad L^*(y, \theta) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{\pi/2}} \exp$$

$$\left(-\frac{1}{2}\frac{e^2}{\sigma^2}\right) \left(1 - F\left(\frac{e}{\sigma}\sqrt{\frac{\gamma}{1-\gamma}}\right)\right)$$

where $e = \ln y - a_0 - \Sigma a_i \ln x_i - \Sigma\Sigma b_{ij} \ln x_i \ln x_j$, and θ is the parameter to be estimated and is equal to $(a_0, a', b', \sigma^2, \gamma)$.

The maximum likelihood (ML) estimators of θ maximizing the above likelihood function are obtained by setting its first order partial derivatives with respect to the elements of θ equal to zero, and solving them simultaneously.

The next step is to estimate the firm-specific technical efficiency for the individual observations in the sample. Following equation (5), the conditional mean of u given (u + v) is

$$E(u|u+v) = \int_{-\infty}^{0} u \cdot f(u / (u+v))$$

Now,

$$f(u|u+v) = \frac{1}{\sqrt{2\pi}} \frac{\sigma}{\sigma_u \sigma_v}$$

$$\exp\left[-\frac{\sigma^2}{2\sigma_u^2\sigma_v^2}\left(U+\frac{e\sigma_u^2}{\sigma^2}\right)\right]\frac{1}{1-F}$$

where $e = u + v$ and $F(\cdot)$ is the distribution function of the standard normal variable. Therefore,

$$(12) \quad E(u|u+v) = -\frac{\sigma_u\sigma_v}{\sigma}\left[\frac{f(\cdot)}{1-F(\cdot)} - \frac{e}{\sigma}\sqrt{\frac{\gamma}{1-\gamma}}\right]$$

where $f(\cdot)$ is the standard normal density function.[5] The estimate of u is obtained from replacing e by the residual which is the difference between the estimated and actual frontiers.

IV. The Analysis

The data used in the paper were derived from a sub-sample of a large World Bank sponsored survey completed by Shand et al. The sub-sample consists of 210 and 172 rice farmers respectively operating inside and outside the Kemubu Irrigation Project in Malaysia,[6] during the year 1980. Two sub-groups of farmers were considered viz., tenant operators and owner-tenant operators.[7] It should be noted that farmers outside Kemubu do not use any irrigation at all and their farming entirely depends on rainfall (Shand et al.).

[5] The specification of γ here is different from the λ specification used by Aigner et al. (1977). γ may be called as 'intra-class correlation coefficient' of the classical random effects model (Nerlove, 362).

[6] For further details of the sample characteristics, see Shand et. al.

[7] Owner-tenant operators are farmers who own some proportion of total operational paddy area, i.e., they operate their own land and also some leased-in land.

The empirically estiamted location specific production frontier is as follows:

$$(13) \quad \ln y = a_0 + a_1 x_1 + \sum_{i=2}^{4} a_i \ln x_i + \Sigma\Sigma b_{ij} \ln x_i \ln x_j + u + v$$

where y = output of rice, per farm, in kg;
x_1 = 1 for owner-tenant operators
= 0 otherwise;
x_2 = number of days of labour used;
x_3 = quantity of fertilizer applied in kg;
x_4 = operational rice area operated in acres.

To examine whether the production frontiers for inside Kemubu and outside Kemubu are the same, the above translog frontier is first estimated pooling all the observations and second separately for the 210 farmers operating inside Kemubu and the 172 farmers operating outside Kemubu. The method used to obtain the ML estimates was that suggested by Fletcher and Powell. The likelihood ratio – test statistic is found to be 21.9722 which is significant (χ^2) at the 5% level for 11 degrees of freedom. This implies that the two production frontiers are not identical. The ML estimates of the location specific production frontiers for Kemubu farmers are given in Table 1. The estimate of the intercept term for inside Kemubu is significantly (at 10% level) larger than that for outside Kemubu.[8] This demonstrates an increase in output due to irrigation, as differences in management practices leading to various technical efficiency ratings are already incorporated into the production frontiers through the one-sided error term 'u'.

8 The test statistic to examine the difference in the two intercept terms can be constructed using $\frac{a_{01} - a_{02}}{\sqrt{\text{Var } a_{01} + \text{Var } a_{02}}}$ which is asymptotically N(0,1) under the null hypothesis. The value of the test statistic in the study is 1.7558 which is significant at the 10% level.

Table 1

MLE ESTIMATES OF LOCATION-SPECIFIC STOCHASTIC PRODUCTION FRONTIER FOR KEMUBU FARMERS

Parameters	ML Estimates		
	Pooled	Inside Kemubu	Outisde Kemubu
a_0	4.3626 (1.2912)	5.8848 (1.1840)	3.0612 (1.0883)
a_1	0.0312 (0.0061)	0.0256 (0.0056)	0.0136 (0.0048)
a_2	0.2812 (0.1208)	0.2456 (0.0810)	0.1985 (0.0620)
a_3	0.4286 (0.2012)	0.6582 (0.2006)	0.4381 (0.1563)
a_4	0.7260 (0.2418)	0.8410 (0.2253)	0.6173 (0.2180)
b_{22}	-0.1089 (0.0412)	-0.1221 (0.0380)	-0.0542 (0.0221)
b_{23}	-0.1126 (0.0382)	-0.1811 (0.0536)	-0.0873 (0.0324)
b_{24}	0.1008 (0.0426)	0.0056 (0.0123)	0.0082 (0.0024)
b_{33}	0.0381 (0.0120)	0.0690 (0.0210)	0.0430 (0.0180)
b_{34}	-0.0298 (0.0082)	-0.0897 (0.0226)	0.0011 (0.0003)
b_{44}	0.0046 (0.0018)	0.0068 (0.0021)	0.0024 (0.0010)
Log likelihood function	-48.2640	-28.1672	-31.0829
σ^2	0.3846	0.2916	0.3187
σ_u^2	0.2808	0.2449	0.2486
γ	0.7301 (0.2218)	0.8398 (0.1876)	0.7800 (0.2103)

Note: Figures in parentheses are asymptotic standard errors of estimates.

An important result (Table 1) is that the variance ratio parameter γ is comparatively large, given the interval within which it lies, and is statistically significant at the 1% level both for inside and outside Kemubu. This means that about 84% (inside Kemubu) and 78% (outside Kemubu) of the difference between the observed output and the maximum production frontier output is caused by differences in farmers' levels of technical efficiency as opposed to the conventional random variability.

Output elasticities for individual inputs are calculated from equation (13).

$$(14) \quad \frac{\partial \ln y}{\partial \ln x_i} = a_i + \Sigma b_j \ln x_j \qquad j = 2, 3, 4$$

The mean output elasticities for farmers operating inside and outside Kemubu are then estimated by substituting all input values at their sample means. The mean output elasticities for all inputs differ significantly between farms inside and outside Kemubu (Table 2). This indicates that irrigation has not only changed the mean output level but also changed the production technology.

The mean technical efficiency is calculated as:

$$E(u) = -\sigma_u \sqrt{2/\pi}$$

and the empirical results show only about 65% mean technical efficiency for the owner-tenant farmers operating inside Kemubu, and 69% for the owner-tenant farmers operating outside Kemubu. It is interesting to note that in both locations, tenant operators have a lower mean technical efficiency, namely 61% inside and 64% outside Kemubu. This means that in both locations, the owner-tenant farmers are operating relatively closer to the frontier than the pure tenant operators. Further, the results indicate that farmers operating outside Kemubu tend to be closer to their respective location specific frontier than farmers operating inside Kemubu to their respective location specific frontier. This indirectly implies that the new rice production technology which utilizes and is dependent upon irrigation has not been implemented efficiently by the sample participants.

While mean technical efficiency measures for the sample is

Table 2

MEAN PRODUCTION ELASTICITIES FOR INSIDE AND OUTSIDE KEMUBU

Input	Units of Measurement	Production Elasticities at the Mean Input Levels: Inside (e_1)	Outside (e_2)	Test Statistic (Z) for Difference in Elasticities[1]
Labour	man day	0.1518 (0.0212)	0.1089 (0.0187)	1.7229***
Fertilizer	kg	0.1201 (0.0307)	0.0583 (0.0196)	1.7119***
Area	hectare	0.4502 (0.1445)	0.3580 (0.1265)	2.4986**

Notes: ** Significant at the 5 percent level.
*** Significant at the 10 percent level.

[1]Test statistic, $Z = \frac{\hat{e}_{i1} - \hat{e}_{i2}}{\sqrt{Var(\hat{e}_{i1}) + Var(\hat{e}_{i2})}}$ which is N (0, 1) under the null hypothesis ($i = 1, 2, 3$), and from equation (12), $Var(\hat{e}_i) = Var\ a_i + \Sigma(\ln x_j)^2\ Var\ b_j + 2\Sigma \ln x_j\ Cov(a_i b_j) + 2\Sigma \ln x_k \ln x_j\ Cov(b_j b_k)$.

important, from a policy perspective, it is more useful to estimate technical efficiencies for individual farmers. As indicated earlier, farm-specific technical efficiencies for individual sample farmer were estimated using equation (12), and they are reported in the form of a frequency distribution in Table 3. These results show a wide variation in the level of technical efficiencies across sample farms. Individual technical efficiency ratings range from 0.4182 to 0.9206 for owner-tenant farmers operating inside Kemubu, while for those owner-tenant farmers operating outside Kemubu the range was slightly narrower, between 0.4421 and 0.8929. For pure tenants, individual technical efficiency ratings vary from 0.3927 to 0.8950 inside Kemubu, and between 0.4285 to 0.8792 for tenants outside Kemubu.

Only about 14% of sample participants inside Kemubu and 19% outside Kemubu obtained outputs which were 75% and

Table 3

FREQUENCY DISTRIBUTION OF TECHNICAL EFFICIENCY FOR INDIVIDUAL FARMS INSIDE AND OUTSIDE KEMUBU

Efficiency Interval	Number of farmers					
	Inside			Outside		
	Tenants	Owner-tenants	Sub-total	Tenants	Owner-tenants	Sub-total
0.35 – 0.40	19	0	19	0	0	0
0.40 – 0.45	6	16	22	17	12	29
0.45 – 0.50	7	9	16	8	12	20
0.50 – 0.55	10	13	23	10	8	18
0.55 – 0.60	13	15	28	9	11	20
0.60 – 0.65	13	25	38	7	16	23
0.65 – 0.70	5	18	23	5	15	20
0.70 – 0.75	7	5	12	6	4	10
0.75 – 0.80	6	10	16	5	11	16
0.80 – 0.85	2	4	6	2	8	10
0.85 – 0.90	3	2	5	2	4	6
0.90 – 0.95	0	2	2	0	0	0
0.95 – 1.00	0	0	0	0	0	0
Total	91	119	210	71	101	172

more of the maximum output estimated through their respective location specific frontiers. However, these individual technical efficiency measures give no clear explanation as to the determinants of the efficiency ratings. Useful next step would be to identify these determinants of technical efficiency at the farm level. This could not be attempted in this study owing to data limitations.

V. Conclusions

From a policy point of view, comparisions of actual individual production level with their best practice (frontier production function) production estimates provide useful insights into the production technology prevailing in the area. In contrast to most earlier studies, this paper provides a measure of technical efficiency for each observation in the sample.

The stochastic production frontier seeks to estimate the maximum output for a given set of inputs while incorporating the possibility that the frontier shifts randomly due to the influence of unmeasured forces. Also, the frontier could remain fixed with measurement error in output accounting for apparent production beyond the frontier. The assumptions about the structure and distribution of the variables u and v are very important for estimating the frontier production function. The assumption that u and v are independent is absolutely central to the analysis. Unlike most of the econometric analysis, the production frontier methodology assigns behavioural significance to the magnitudes of the error components. The assumption of a truncated normal distribution for u is not without criticism. For examle, when u is assumed to follow a half normal distribution, majority of observations are expected to lie around zero meaning that farms have high technical efficiency. This limitation may be overcome by considering u to have a truncated normal distribution whose mean is different from zero (Stevenson). However, some of the earlier studies have proved that the assumption of half normal distribution is adquate (Lee).

The analysis demonstrates that even in areas well endowed with irrigation and other input facilities, there remains scope for substantial increases in production levels. On average there appears to be 65% (= 1.0 – 0.35) and 69% (= 1.0 – 0.31) technical efficiency with respect to rice production inside and outside Kemubu, respectively. The lower rating of the former case means that production technology inside Kemubu needs more careful attention to improve performance than the traditional production technology outside Kemubu. Thus the analysis reveals that mere promulgation of recommendations for, and broad adoption of, new production technologies may not yield the long term target level of output, and specific measures need to be taken to improve technical efficiency as well.

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