Leisure-Income Choice and the Development of a Dual Economy

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I. Introduction

Development of the closed dual economy has been an important and controversial focus of theoretical development economics since the publication of Lewis' seminal contribution.[1] Dual economy growth involves interaction between a traditional agricultural sector, employing backward production methods, and a modernizing industrial sector utilizing capital and more advanced production techniques. Reallocation of rural labor to the higher productivity industrial sector has been the essential mechanism of dual economy growth since Lewis.

Industrialization, led by the growth of manufacturing, absorbs rural labor into industrial employment. An agricultural surplus, in the form of subsistence goods, must be provided to maintain the industrial sector labor force. The pace of industrialization is

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1 The literature includes many contributions. Fei and Ranis elaborated Lewis' classicial model retaining the surplus agricultural labor assumption. Jorgenson's (1964) neoclassical model abandoned the surplus labor assumption and raised the issue of viability of dual economy growth. Zarembka (1970, 1972) considers viability from the viewpoint of non-zero price and non-unitary income elasticities of demand for food. Marino evaluates growth prospects by introducing a production function where only the usual neoclassical restrictions apply. Models by Sato and Niho, and McIntosh incorporate endogenous population growth. Stern addresses the question of optimal development of a dual economy.

related to industrial employment which is governed by the size of the agricultural surplus. There is, thus, a direct link between the growth of industry and the capacity of the agricultural sector to generate a surplus.

Since growth of the dual economy is assumed to be led by industrial growth and labor reallocation, viability is evaluated in terms of conditions affecting generation of an agricultural (food) surplus. Jorgenson (1961) addressed this issue by assuming zero price elasticity and unitary income elasticity of demand for food. The capacity of the agricultural sector to generate surplus food then depends on (i) the rate of neutral technological progress in agriculture; (ii) the population growth rate; and (iii) the elasticity of agricultural output with respect to labor.[2] Here the growth rate of the manufacturing sector has no impact whatsoever on the agricultural surplus.

Zarembka (1970, 1972) challenged this result by invoking a non-zero price elasticity of demand for food. In zarembka's model, the relative price of food depends on the relative growth rates of per capita output in the manufacturing and agricultural sectors. If per capita output growth in the manufacturing sector exceeds that in agriculture, the relative price of food will rise. A non-zero price elasticity of food will then induce consumption demand enabling the agricultural sector to supply a larger surplus for industrialization. This will accelerate labor reallocation and growth of the dual economy. This argument strengthens the dual economy tradition of emphasizing industrialization as the initial growth strategy. Zarembka, however, recognized that price elasticity of demand for food is near zero in developing countries and thus rapid industrial growth would have little effect on the size of the agricultural surplus.

In this paper we develop a dual economy model, with assumptions appropriate to developing countries, to demonstrate that unbalanced growth with a manufacturing sector bias may inhibit generation of an agricultural surplus. This would reduce industrial sector labor absorption, retarding the rate of growth of the industrial sector itself. Where these conditions prevail, a more

2 Specifically, Jorgenson's Condition is $\alpha_1 - \varepsilon\beta_1 > 0$ where $\alpha_1 =$ the rate of neutral technological progress in agriculture, β_1 = elasticity of agricultural output with respect to labor and $\varepsilon =$ an exogenous rate of growth of population.

balanced growth strategy stressing productivity improvements in agriculture would be more favorable to both overall growth and reallocation of labor.

Dual economy growth models have failed to realistically portray the farm sector by assuming labor hours are fixed. A more realistic approach, adopted here, permits variable work hours, with each farmer maximizing utility by choice between leisure and income. A number of authors, including Sen, Stiglitz, Zarembka (1972), Huang and Bhatia, have incorporated variable work hours in models of a developing country's agricultural sector and rural-urban migration. The next step is taken in this paper by incorporating variable labor hours in a neoclassical growth model of a dual economy. It is shown that if the price elasticity of demand for food is less than zero, manufacturing sector growth *may* have adverse effects on the size of the food surplus. In the empirically valid case of zero price elasticity, the effect of manufacturing sector growth is unambiguously negative.

II. A Model of Dual Economy Growth

In this section we formulate a dual economy growth model,[3] incorporating leisure-income choice in agriculture. The following sections are devoted to solution of the model and analysis of its results. A closed dual economy is assumed; food is assumed to be the only agricultural output.

Production:

We assume Cobb-Douglas production functions for both food and manufacturing output. At any point in time food output, Y, is determined by the rate of neutral technological progress in agriculture, α_1, the fixed amount of land, $\bar{L}$, and labor hours, ℓ A, (ℓ is labor hours per farmer and A is the agricultural labor force):

$$(1) \quad Y = e^{\alpha_1 t} \bar{L}^{\beta_1} (\ell A)^{1-\beta_1}$$

[3] For convenience of the reader, all symbols used in the model are listed and defined in Appendix I.

where β_1 is output elasticity of land and $1 - \beta_1$ is output elasticity of labor. Assuming $\bar{L}^{\beta_1} = 1$, we can write the manufacturing production function as:

$$(1') \quad Y = e^{\alpha_1 t}(\ell A)^{1-\beta_1}$$

Manufacturing sector output, X, is determined by the rate of neutral technological progress in industry, α_2, capital stock, K, and labor hours, $\bar{\ell}$ M, ($\bar{\ell}$ is fixed labor hours per worker and M is the industrial labor force):

$$(2) \quad X = e^{\alpha_2 t}K^{\beta_2}(\bar{\ell}M)^{1-\beta_2}$$

where β_2 is output elasticity of capital and $1\text{-}\beta_2$ is output elasticity of labor in manufacturing. Assuming $\bar{\ell}^{\,1-\beta_2} = 1$, we can write the manufacturing production function as:

$$(2') \quad X = e^{\alpha_2 t}K^{\beta_2}M^{1-\beta_2}$$

Leisure-Income Choice in Agriculture and Allocation of Labor Between Agriculture and Industry:

We assume that all agricultural workers have identical utility functions. The net utility that a farmer derives depends on his consumption of food and manufacturing output, which in turn depends on his food output, the relative price of food, q, and the disutility he derives by working on land. Farmers are thought of as a family so that income to each farmer is the average product in agriculture, Y/A. Thus, the net utility to a farmer, Z_A, is represented by:

$$(3) \quad Z_A = U(Y/A, q) - V(\ell)$$

where U reprsents utility from consumption of food and manufacturing output, V represents disutility of labor, and

$$U_1 > 0, U_{11} < 0, U_2 > 0, U_{22} < 0,$$

$U_{12} < 0, V' > 0, V'' > 0.$ (See Appendix I for definitions of U_1, U_{11}, etc.) For any given relative price, a farmer chooses between leisure and income by maximizing Z_A with respect to labor hours. The first order condition for utility maximization is:

$$(4)\quad \frac{\partial Z_A}{\partial \ell} = 0 = \frac{\partial U(\frac{Y}{A}, q)}{\partial (\frac{Y}{A})} \frac{\partial (\frac{Y}{A})}{\partial \ell} - V'(\ell)$$

or:

$$(4')\quad \frac{\partial Y}{\partial (\ell A)} = \frac{V'(\ell)}{U_1\ (\frac{Y}{A}, q)}$$

Equation (4') implies that a farmer supplies labor hours up to the point where marginal porduct of labor hours is equal to the individual rate of indifferent substitution between income and labor hours, the latter being the real cost of labor.

We assume that industrial workers have utility function, Z_M, identical to that of farmers, so that:

$$(5)\quad Z_M = U(\frac{w}{q}, q) - V(\bar{\ell})$$

where w is the wage rate in the manufacturing sector in terms of manufacturing output.

The wage rate in the manufacturing sector, w, is determined competitively and is equal to the marginal product of labor. Using equation (2) the wage rate is represented as:

$$(6)\quad w = \frac{\partial X}{\partial M} = (1-\beta_2)\frac{X}{M}$$

Allocation of labor between agriculture and industry is given by the relative net utility that a worker derives by working in either sector. A farmer's opportunity cost of leaving for the city is the net utility he was deriving in agriculture (Z_A). A farmer will migrate for industrial employment as long as the net utility derived by working in the manufacturing sector (Z_M) is greater than the opportunity cost of leaving agriculture and *vice versa*. Intersectoral migration will be zero only if Z_M is equal to Z_A. Therefore, the labor allocation rule between the two sectors is:

$$(7)\quad Z_M = Z_A$$

In equilibrium, labor hours per worker in industry, $\bar{\ell}$, are not necessarily equal to labor hours per farmer, ℓ, since the latter is a choice variable. Since workers in both sectors have identical utility functions and since in equilibrium net utility derived in the two sectors will be equal, it follows that $w/q \lesseqgtr Y/A$ as $\bar{\ell} \lesseqgtr \ell$. The difference between w/q and Y/A must, however, reflect the difference between $\bar{\ell}$ and ℓ in order for Z_A to be equal to Z_M. Therefore, a different representation of equation (7) will be:

$$(8)\quad \frac{w}{q} = \phi(\bar{\ell} - \ell)\, \frac{Y}{A} = \gamma(\ell)\frac{Y}{A} \quad \text{(since } \bar{\ell} \text{ is fixed)}$$

where $\gamma(\ell)$ is the ratio of the manufacturing wage and the agricultural income per farmer, both in terms of food, i.e., $\frac{w/q}{Y/A}$; and where $0 < \gamma(\ell) \lesseqgtr 1$ as $\bar{\ell} \lesseqgtr \ell$, and $\gamma' = \frac{\partial \gamma}{\partial \ell} < 0$.

Investment:

Capitalists in the industrial sector are assumed not to consume, and hence all profits are invested. We also assume that investment activity occurs only in the manufacturing sector. With our assumption of a Cobb-Douglas production function, gross investment is:

$$(9)\quad I = \beta_2 X$$

Let the time rate of change of any variable, x, be represented as $\dot{x}$. Equations (10) and (11) then represent, respectively, net investment and the stock of capital at any instant of time:

$$(10)\quad \dot{K} = I - \eta K$$

Where η is the constant depreciation rate of capital.

$$(11)\quad K = K(0) + \int_0^t \dot{K}\, dt$$

Product Market Equilibrium:

Product market equilibrium involves equilibrium in the market for food and manufacturing goods. Applying Walras' law, equilibrium in the market for food will also ensure equilibrium in the market for manufacturing goods.[4] Therefore, only the food market equilibrium condition is specified. Since identical utility functions have been assumed for farmers and industrial workers, their demand functions will also be identical. Let us define demand for food by any worker as a function of his real income in terms of food and the relative price of food. Furthermore, the demand functions are assumed to be of the Cobb-Douglas form. Then, equality of demand and supply of food can be written as:

$$(12) \quad Y = A\left(\frac{Y}{A}\right)^{\theta} q^{-\sigma} + M\left(\frac{w}{q}\right)^{\theta} q^{-\sigma}$$

where θ is the income elasticity and σ is the price elasticity of demand for food. Question may be raised whether such a demand function follows from a utility function. We show in Appendix II that this is true under certain conditions, and these conditions apply rather well to the case of a typical developing country.

Population is assumed to grow at an exogenously given rate, ε. Assuming initial population to be equal to one, total population at any point in time is represented as:

$$(13) \quad P = e^{\varepsilon t}$$

The model is closed by stating the identity:

$$(14) \quad P = A + M$$

The model has ten endogenous variables, Y, X, ℓ, A, M, q, w, I, $\dot{K}$, K, and ten equations, (1′), (2′), (4′), (6), (8), (9), (10), (11), (12), and (14). Equation (13) determines total population independently of the system. The model is summarized in Table 1.

III. Solution of The Model

The solution of the system involves the derivation of the time

[4] By Walras' law, if n-1 markets are in equilibrium, then the n^{th} market must also be in equilibrium, which permits us to eliminate one market. In our case, we eliminate the market for manufacturing goods.

Table 1

SUMMARY OF THE MODEL

(1′) $Y = e^{\alpha_1 t}(\ell A)^{1-\beta_1}$

(2′) $X = e^{\alpha_2 t} K^{\beta_2} M^{1-\beta_1}$

(4′) $\dfrac{\partial Y}{\partial(\ell A)} = \dfrac{V'(\ell)}{U_1(\frac{Y}{A}, q)}$

(6) $w = (1-\beta_2)\dfrac{X}{M}$

(8) $\dfrac{w}{q} = \gamma(\ell)\dfrac{Y}{A}$

(9) $I = \beta_2 X$

(10) $\dot{K} = I - \eta K$

(11) $K = K(0) + \int_o^t \dot{K}_{dt}$

(12) $Y = A(\frac{Y}{A})^{\theta} q^{-\sigma} + M(\frac{w}{q})^{\theta} q^{-\sigma}$

(13) $P = e^{\varepsilon t}$

(14) $P = A + M$

rate of change of the endogenous variables. We begin by substituting equation (8) in (12) and dividing both sides of (12) by P to obtain:

(12′) $\dfrac{Y}{P} = C(\dfrac{Y}{A})^{\theta} q^{-\sigma}$

where $C = 1 - \dfrac{M}{P}(1-\gamma^{\theta})$.

There is justification for treating C as a constant since $\frac{M}{P}(1-\gamma^{\theta})$ will be a small number. The value of $\frac{M}{P}$ typically remains small during development of a dual economy. The value of Υ will generally be less than one since farmers' working hours exceed those of industrial workers. Empirically, income elasticity of demand for food, θ, is less than one.

Using equations (8) and (1′) in (12′) and totally differentiating with respect to time and given intersectoral labor market equilibrium, we derive the necessary relations among the rate of change of the rural labor force, labor hours, and industrial output per industrial worker required for equilibrium in the market for food:

$$(15)\quad \alpha_1\ (1-\theta-\sigma) + [1-\beta_1(1-\theta-\sigma)]\frac{\dot{A}}{A}$$

$$+\ [(1-\beta_1)(1-\theta-\sigma)-\sigma u]\frac{\dot{\ell}}{\ell} = \varepsilon - \sigma\left(\frac{\dot{X}}{X}-\frac{\dot{M}}{M}\right)$$

where u is elasticity of Υ with respect to labor hours in agriculture.

To obtain equations (16) and (17) we totally differentiate equations (4′) and (8) respectively:

$$(16)\quad \alpha_1 + \beta_1\left(\frac{\dot{\ell}}{\ell}+\frac{\dot{A}}{A}\right) - m\left(\frac{\dot{Y}}{Y}-\frac{\dot{A}}{A}\right) + h\frac{\dot{q}}{q} = n\frac{\dot{\ell}}{\ell}$$

where $m = -(U_{11}/U_1)Y/A > 0$, $n = (V''/V')\ell > 0$,

and $h = (U_{12}/U_1)q < 0$.

$$(17)\quad \frac{\dot{q}}{q} = \frac{\dot{X}}{X} - \frac{\dot{M}}{M} - u\frac{\dot{\ell}}{\ell} - \left(\frac{\dot{Y}}{Y} - \frac{\dot{A}}{A}\right)$$

Now substituting equation (16) in (17) we dervie an expression for the rate of change of labor hours, consistent with utility maximization, in terms of the rate of agricultural technological progress, the growth rate of the agricultural labor force and the growth rate of manufacturing output per person employed in that sector:

$$(18)\quad \frac{\dot{\ell}}{\ell} = \frac{\alpha_1(1-m-h)}{\Delta_1} - \frac{\beta_1(1-m-h)}{\Delta_1}\frac{\dot{A}}{A} + \frac{h\left(\frac{\dot{X}}{X}-\frac{\dot{M}}{M}\right)}{\Delta_1}$$

where m is the elasticity of marginal utility of income with respect to real income in agriculture; h is the elasticity of marginal utility of income with respect to the relative price of food; and where $\Delta_1 = n + \beta_1(1-m-h) + m + h(1+u)$.

Now substituting equation (18) into (15), the food market equilibrium condition is re-expressed as:

$$(19)\quad \left[1 - \beta_1(1-\theta-\sigma) - \frac{\beta_1(1-m-h)\,[(1-\beta_1)(1-\theta-\sigma) - \sigma u]}{\Delta_1}\right]$$

$$\frac{\dot{A}}{A} = \varepsilon - \alpha_1\left[(1-\theta-\sigma) + \frac{(1-m-h)\,[(1-\beta_1)(1-\theta-\sigma) - \sigma u]}{\Delta_1}\right]$$

$$- (\sigma+k)\left(\frac{\dot{X}}{X} - \frac{\dot{M}}{M}\right)$$

where $k = \dfrac{h[(1-\beta_1)(1-\theta-\sigma) - \sigma u]}{\Delta_1}$

Total differentiation of equation (2′) and substitution of equations (9) and (10) yield an expression for the rate of growth of manufacturing output per person employed in that sector:

$$(20)\quad \frac{\dot{X}}{X} - \frac{\dot{M}}{M} = \alpha_2 + \beta_2^2\frac{X}{K} - \beta_2\eta - \beta_2\frac{\dot{M}}{M}$$

Following Zarembka (1970, 1972) we use the long-run value of the capital-output ratio to eliminate X/K from equation (20). The long-run capital output ratio turns out to be:[5]

$$(21)\quad \frac{K}{X} = \frac{\beta_2}{\dfrac{\alpha_2}{1-\beta_2} + \eta + \dfrac{\dot{M}}{M}}$$

[5] See Zarembka (1972), pp. 219-224, for the derivation of the long-run capital output ratio, and the distortions it use may cause.

which is constant since $\dot{M}/M$ is constant in the long-run. Using this expression at the cost of some distortions involved, but simplifying the solution greatly, we obtain:

$$(22) \quad \frac{\dot{X}}{X} - \frac{\dot{M}}{M} = \frac{\alpha_2}{1-\beta_2}$$

From equation (14) we derive:

$$(23) \quad \frac{\dot{A}}{A} = \varepsilon\left(1+\frac{A}{M}\right) - \frac{\dot{M}}{M}\,\frac{M}{A}$$

which shows the feasible values of the growth rates of agricultural and industrial labor forces for any given rate of population growth and distribution of population.

Using equations (22) and (23) in (19), we obtain equation (24). This and the remaining equations (25) through (28) comprising the model's solution are grouped together at the end of this section. Equation (24) is the model's fundamental differential equation showing the rate of growth of industrial employment, $\frac{\dot{M}}{M}$

Equation (25) showing the rate of growth of the agricultural labor force, $\frac{\dot{A}}{A}$, is obtained by substituting equation (24) in (23). The rate of growth of per capita food output, $\frac{\dot{Y}}{Y} - \frac{\dot{A}}{A}$, is given by equation (26) which is derived by using equations (1′), (18), (22) and (25). Substituting equations (22) and (25) in (18) yields an expression for the growth rate of agricultural labor hours, $\frac{\dot{\ell}}{\ell}$, in equation (27). Finally, the rate of growth of the terms of trade, $\frac{\dot{q}}{q}$, is derived in equation (28) by using equations (17), (22), (26) and (27).

$$(24) \quad \frac{\dot{M}}{M} = \varepsilon + \frac{A}{M}\,\frac{(\alpha_1 - \varepsilon\beta_1)\left\{(1+n+hu)(1-\theta-\sigma) - \sigma u(1-m-h)\right\}}{\Delta_2} + \frac{A}{M}\,\frac{(\sigma+k)\frac{\alpha_2}{1-\beta_2}\Delta_1}{\Delta_2}$$

where $\Delta_2 = \Delta_1 - \beta_1 \,(1-\theta-\sigma)\,(1+\eta+hu) + \sigma u(1-m-h)$

(25) $$\frac{\dot{A}}{A} = \varepsilon - $$

$$\frac{(\alpha_1 - \varepsilon\beta_1)\left\{(1+n+hu)\,(1-\theta-\sigma) - \sigma u(1-m-h)\right\} + (\sigma+k)\frac{\alpha_2}{1-\beta_2}\Delta_1}{\Delta_2}$$

(26) $$\frac{\dot{Y}}{Y} - \frac{\dot{A}}{A} = \frac{(1+n+hu)\left[(\alpha_1 - \varepsilon\beta_1) + \beta_1(\sigma+k)\frac{\alpha_2}{1-\beta_2}\right]}{\Delta_2}$$

$$+ \frac{(1-\beta_1)h\frac{\alpha_2}{1-\beta_2}}{\Delta_1}$$

(27) $$\frac{\dot{\ell}}{\ell} = \frac{(1-m-h)\left[(\alpha_1 - \varepsilon\beta_1) + \beta_1^{\varepsilon}(\sigma+k)\frac{\alpha_2}{1-\beta_2}\right]}{\Delta_2} + \frac{h\frac{\alpha_2}{1-\beta_2}}{\Delta_1}$$

(28) $$\frac{\dot{q}}{q} = \frac{\alpha_2}{1-\beta_2}$$

$$\frac{\left\{(\alpha_1 - \varepsilon\beta_1) + \beta_1(\sigma+k)\frac{\alpha_2}{1-\beta_2}\right\}\left\{u(1-m-h) + (1+n+hu)\right\}}{\Delta_2}$$

$$\frac{h\frac{\alpha_2}{1-\beta_2}\left\{(1-\beta_1) + u\right\}}{\Delta_1}$$

IV. Implications for Dual Economy Growth Strategy

We now apply the results obtained in the previous section to substantiate our earlier assertion that with variable labor hours in agriculture, a growth strategy biased toward the manufacturing sector may be self-defeating.

We begin by noting that the signs of Δ_1 and Δ_2 are positive. Observe that a rise in the relative price of food, ceteris paribus, has a negative impact on labor hours because of the assumptions of diminishing marginal productivity of labor hours and diminishing marginal utility of income with respect to real income defined in terms of food as well as the relative price of food.[6] Also observe that, ceteris paribus, growth of manufacturing output increases the relative price of food (see equation (17)) which subsequently reduces labor hours in agriculture. This is shown by the term $h(\dot{X}/X-\dot{M}/M)/\Delta_1$ in equation (18) or by $\frac{h}{\Delta_1} \cdot \frac{\alpha_2}{1-\beta_2}$ in equation (27). Clearly these terms will be negative since $h<0$, provided that Δ_1 is positive.

To show that Δ_2 is positive, we note from the demand function that a rise in the relative price of food reduces food demand. Ignoring any effect on labor hours, reduced food demand would induce migration of agricultural labor to the industrial sector to maintain food market equilibrium. The term $\sigma \frac{\alpha_2}{1-\beta_2} \cdot \frac{\Delta_1}{\Delta_2}$ in equation (24) captures this effect. Since this term is positive, Δ_2 must be greater than zero.

The term $\frac{h}{\Delta_1} \frac{\alpha_2}{1-\beta_2}$ in equation (27) represents the negative effect on the farmer's labor hours from a rise in the relative price of food caused by manufacturing sector growth. Any change in labor hours induces change in both demand for and sup-

6 From equation (4′) $\frac{\partial Y}{\partial(\ell A)} = \frac{V'(\ell)}{U_1(\frac{Y}{A}, q)}$

Assuming no change in A, total differentiation of (4) gives:

$$\frac{\partial \ell}{\partial q} = \frac{V'' - U_1 A \frac{\partial^2 Y}{\partial(\ell A)^2} - (\frac{\partial Y}{\partial(\ell A)})^2 U_{11}}{U_{12}} < 0$$

since $V'' > 0$, $\partial^2 Y/\partial(\ell A)^2 < 0$, $U_{11} < 0$, $U_{12} < 0$.

ply of food as shown by the term $[(1-\beta_1)(1-\theta-\sigma)-\sigma u]\frac{\dot{\ell}}{\ell}$ in equation (15). To elaborate, we rewrite the bracketed terms as $1-\beta_1-\theta(1-\beta_1)-\sigma(1-\beta_1)-\sigma u$. The first term, $1-\beta_1$, represents change in food output as labor hours change by one unit. At any given allocation of labor, therefore, average agricultural output, Y/A, changes, also causing food demand to change by $\theta(1-\beta_1)$. From equation (17), a fall in labor hours affects relative price of food in two ways. By decreasing average agricultural income, Y/A, it raises food's relative price; by increasing $\gamma(\ell)$, it tends to lower relative price. The term $\sigma\{(1-\beta_1)+u\}$ shows the effect of such a relative price change on the demand for food. Therefore, the expression $(1-\beta_1)(1-\theta-\sigma)-\sigma u$ summarizes the effect of a one unit change in labor hours on the net supply of food or food surplus. Note that this term is positive since $1-\theta-\sigma > 0$,[7] and $u < 0$.

Conclusions about the effects of manufacturing sector growth on the agricultural surplus may now be stated. By increasing the relative price of food, manufacturing sector growth will raise the potential surplus to the extent price elasitcity of demand is non-zero. However, the rising relative food price also causes the farmer's labor hours to decrease, reducing the agricultural surplus. There net effect, therefore, depends on the relative strength of these two opposing forces.

The term $(\sigma+k)\frac{\alpha_2}{1-\beta_2}\frac{\Delta_1}{\Delta_2}$ in equations (24) and (25) illustrates these conclusions. On the one hand, $\sigma\frac{\alpha_2}{1-\beta_2}\frac{\Delta_1}{\Delta_2}$ shows the positive effect on industrial employment resulting from the relative food price increase induced by growth of manufacturing. On the other hand, $k\frac{\alpha_2}{1-\beta_2}\frac{\Delta_1}{\Delta_2}$ indicates the negative effect the food price rise has on industrial employment through the induced fall in farmer's labor hours. Therefore, whether industrial expansion will have a positive, zero or negative impact on in-

7 This follows since empirically price elasticity of demand for food is close to zero, whereas income elasticity is less than one.

dustrial employment, through the agricultural surplus, depends on the sign of $\sigma + k$.

Empirically, the price elasticity of demand for food is close to zero ($\sigma = 0$). Where this is true, term $\sigma + k$ turns out to be $h(1-\beta_1)(1-\theta)$ which is clearly negative since h is less than zero. Realistically, therefore, a development strategy biased toward industrial sector growth will have negative effects on industrial employment through the relative price-reduced labor hours mechanism. This means that if the agricultural sector cannot generate an adequate food surplus to accommodate labor transfer called for by industrial growth, the capacity will not be enhanced by promoting industrial expansion. In fact, prospects for increasing the agricultural surplus will worsen, causing reallocation of labor to slacken and perhaps even cease.

This analysis clearly points to the need for a balanced development strategy aimed at promoting growth of the agricultural surplus. If σ is close to zero, as we believe to be true, such a strategy will enhance growth of industrial employment. This is shown by the term $(\alpha_1 - \varepsilon\beta_1)\left\{(1+n+hu)(1-\theta-\sigma)-\sigma u(1-m-h)\right\}$ in equation (24). With $\sigma \doteq 0$, this term will be positive if $\alpha_1 - \varepsilon\beta_1$ is positive.[8] A balanced development strategy should be designed to affect technological conditions in agriculture; i.e., by raising α_1 or lowering β_1, or perhaps even attacking the population growth rate, ε. Successful policies of these types will raise the agricultural surplus, thus accelerating growth of the manufacturing sector.

Appendix I

Symbols Used in the Model

Y = agricultural output (food)
$\bar{L}$ = fixed amount of land

[8] Note that a positive value of $\alpha_1 - \varepsilon\beta_1$ is Jorgenson's (1967) condition for generation of an agricultural surplus and viability of the dual economy.

A	= agricultural labor force
M	= industrial (manufacturing) labor force
ℓ	= labor hours per farmer
X	= industrial output
K	= capital stock
$\bar{\ell}$	= fixed labor hours per manufacturing worker
α_1	= rate of neutral technological progress in agriculture
α_2	= rate of neutral technological progress in manufacturing
ε	= exogenously given rate of growth of population
$1\text{-}\beta_1$	= output elasticity of labor in agriculture
$1\text{-}\beta_2$	= output elasticity of labor in manufacturing
β_2	= output elasticity of capital in manufacturing
Z_A	= net utility of an agricultural worker
Z_M	= net utility of a manufacturing worker
U	= utility from consumption
V	= disutility of labor
q	= relative price of food in terms of manufacturing output
U_1	= marginal utility of income defined in terms of food
U_{11}	= rate of change of maginal utility of income with respect to income defined in terms of food
U_{12}	= rate of change of marginal utility of income, defined in terms of food, with respect to the relative price of food.
U_2	= rate of change of utility, U, with respect to the realtive price of food
U_{22}	= rate of change of U_2 with respect to the relative price of food.
V'	= marginal disutility of labor.
V''	= rate of change of marginal disutility of labor with respect to labor hours
w	= manufacturing wage rate in terms of manufacturing output
w/q	= manufacturing wage rate in terms of food.
$\gamma(\ell)$	= ratio of manufacturing wage and agricultural income per farmer
γ'	= marginal change of Υ with respect to labor hours in agriculture
I	= gross investment in the manufacturing sector
$\dot{K}$	= net investment in the manufacturing sector
η	= constant rate of depreciation of capital
θ	= income elasticity of demand for food

σ = price elasticity of demand for food
P(= A + M) = total labor force (equal to total population)
u = elasticity of Υ with respect to labor hours in agriculture
m = elasticity of marginal utility of income with respect to real income in agriculture
n = elasticity of marginal disutility of labor hours in agriculture
h = elasticity of marginal utility of income with respect to relative price of food.

Appendix II

Justification for Cobb-Douglas Demand Function

Kazuo Sato uses a utility function of the form $U = \Sigma a_i q_i^{c_i}$ from which he derives a demand function of the form: $q_i = (I/P_a)^{\theta_i} (P_i/P_m)^{\sigma_i'}$

where q_i represents demand for the i^{th} commodity, θ_i and σ_i' are the corresponding income and price elasticities of demand. P_a and P_m are the price indices defined below.

Let us assume two goods: food (Y) and manufacturing output (X). Then, the price indices are: $P_a = P_Y^{\psi_1} P_X^{\psi_2}$ and $P_m = P_Y^{\mu_1} P_X^{\mu_2}$, where P_Y and P_X are the prices of food and manufacturing sector's output, $\psi_1(\psi_2)$ and $\mu_1(\mu_2)$ are proportion of income spend on food (manufacturing output) and the marginal propensity to consume food (manufacturing output).

In the present paper, we have used demand functions which are identical to Sato's under some approximations. For example, a farmer's food demand has been expressed as: $D_Y = (Y/A)^{\theta_1} q^{-\sigma}$

where Y/A is the real income per farmer in terms of food, q =

P_Y/P_X is the relative price of food and θ_1 and σ are the income and price elasticities of demand for food. For a low income country, where the proportion of income spent on food is very high, $P_a \simeq P_Y$. (In the extreme case where $\psi_1 = 1$, $P_a = P_Y$). Therefore, $I/P_a = I/P_Y = Y/A$. Also, $(P_i/P_m)^{\sigma_i'}$ in Sato can be written as $q^{\mu_2 \sigma_i'}$ (since $\mu_1 + \mu_2 = 1$) which is equivalent to $q^{-\sigma}$, where $\sigma = \mu_2 \sigma_i'$. Hence our demand function is approximately identical to Sato's.

Unlike Sato, however, we treat the price and income elasticities as constants. Sato derives the conditions under which these elasticities can be treated as approximately constants. The conditions are: (i) income elasticities are close to one, and (ii) variations in ψ_i are within close range.

Estimated values of income elasticities of demand for food vary in range. A number of studies report income elasticity of demand to be close to one, especially for the rural sector. Accepting these results as the appropriate estimation of income elasticity of demand for food, it follows that in the context of two commodities only, the income elasticity of demand for manufacturing output is approximately equal to one (since $\theta_1 \psi_1 + \theta_2 \psi_2 = 1$, and $\psi_1 + \psi_2 = 1$). Under these arguments, the first of Sato's conditions is satisfied.

The second condition follows from the fact that the proportion of income spent on food is rather high in many developing countries. Thus, even if the marginal propensity to consume for either of the commodities may be very high or low, growth in income will change the proportions only slowly, and hence the second condition is satisfied.

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