Sequential Auctions in Informal Credit and Savings Societies:
Asian Auctions

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I. Introduction

It is quite common to find financial auctions in the informal financial market of South Asian countries, called "pia huey" (PH). The bidding procedure is similar to the English auction, that is, the highest sealed bid wins the item being auctioned off which, in our case, is the mutual funds of the PH members. Recently, the phenomenon of this type of auction has increased at a rapid rate. Among reasons given are the common regulation in interest-rate ceilings in the formal financial market and the high inflation rate; consequently, households save less in the formal financial institutions, preferring instead to participate in PH auctions which yield the market rate of interest; the individual household then can obtain the level of maximum satisfaction, while the aggregate welfare is better off with the expansion of sequential auctions in PH (Chotigeat, 1982, 1983).

The purpose of this paper is to develop a model of competitive bidding in a rotating credit and savings society in which each individual bidder's objective relies on the sequential auctions over a PH cycle. That is, the winning bidder's payoff may depend upon his personal preferences, the preferences of others, and the information of the past auctions in the PH cycle in which he is participating, the expected rate of return of the winning auction

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price, the number of bidders, the time of the bid, and the past winning-bid rates.

The paper is organized as follows. Section II describes a typical auction procedure in PH. Section III develops a model of competitive bidding for sequential auctions in a PH cycle in which each individual bidder attempts to obtain his optimal wealth. The model is based on the non-cooperative equilibrium bid function for heterogenous bidders in sealed-bid sequential auctions under three circumstances; a) under the world of certainty, a bidder has a perfect knowledge of information needed for bidding and need not know the chance of his bid winning; b) under the world of certainty, but each bidder takes the probability of winning the bid into consideration; and c) under the world of uncertainty where each bidder is risk averse. A summary of the paper and the conclusion with reference to the implications for PH as an instrumental source of funds in LDCs are set forth in Section IV.

II. The Procedure of PH Sequential Auctions

A. The General Procedure

Rotating credit and savings societies in the informal financial market have long been used by the poor to help them manage their financial affairs, acting as a mutual aid device to satisfy their savings and credit needs. A person who forms a PH in South Asian is commonly called a "taut," hereafter referred to as the PH organizer. He is typically a person of business or financial repute in the community. He organizes a PH because he needs money; the reward for his success is in borrowing money from his selected members without paying any interest, but with him rests the ultimate responsibility to all the PH members for payment of their dues. The organizer, therefore, must select the members whom he can trust and vice versa. Thus, PH is made up of a group of selected participants who make regular contributions of funds which are given to each winning member of the sealed-bid auction in turn. The PH actually combines saving and lending; its extra attraction is that the winning bidder has access to the funds at an earlier date than when saving individually in financial institutions.

The general procedure of sequential auctioning in PH for mutual funds is described as follows. Assume that there is a total of twelve persons (including the organizer) in this PH, each having a contribution share of $100 to lend out at each of twelve auctions to be held at one-month intervals (this interval is the most popular, although it could be one day, one week, two weeks, or longer).

The first auction results in each of the twelve members contributing the full price tag of $100 to a fund totalling $1,200, which is in turn given to the organizer as a reward for his organizing the PH; no bidding takes place during this round. The transaction, then, is equivalent to the organizer receiving an $1,100 interest-free loan from the remaining eleven PH members.

At the second auction, the eleven members (total members minus the organizer) each submit a sealed interest bid they are willing to pay, usually ranging between 7% and 40%. At each auction, all members who have already taken a loan are excluded from the bidding process; therefore, the respective positions of debtors and creditors are affected after each auction. The high interest bids come from members who have a greater need for the use of the money, while the low bids indicate the bidders' preferences to lend (save) to a bidder of a high rate. If there is a reluctance on the part of the bidders to bid high, the trend will be for a member to bid as quickly as possible at a rate close to (but not in excess of) the going bank rate of interest in order to take the loan and deposit it in the bank for the duration of the PH.

In this example at the second auction, if Member A wins the bidding with 40%, he will receive $100 from the organizer (each former bidding winner pays the full price tag of $100 at each auction for the rest of the cycle) and $60 = $(100-40) from each of the other eleven members, himself included, for a total of $760. In other words, the winner pays the interest rate of 40% to other members immediately at the point of borrowing (winning is like taking a loan).

If Member B is the winner of the bids at the third auction with 30%, he receives $100 from both the organizer and Member A, while he get $70 = $(100-30) from each of the remaining 10 members, himself included for a total of $900.

Subsequent auctions result in continually smaller numbers of bidders involved and ever-increasing loans, since the number of members having already bid and paying the full price tag of $100 increases. This is justified to the later members for they have lent out more money, risked losing it longer in the case of a PH default, and they benefit from having lent (saved) and earned interest and finally receiving a larger loan.

At the twelfth and final auction, the one remaining member who has not won a bid simply receives a loan (or his savings plus earned interest, including his savings at this meeting) equal to $1,200 = (100 x 12). The interest rate earned by this last member is usually quite high, at times reaching 100%, depending on the rates bid by the other members. Not earning this high rate of return would be not maximizing his welfare; he has waited longer, taken a greater risk of default, endured illiquidity of his savings, and been victimized by the impact of inflation. These constraints require some sort of justification; in this case, the high rate of interest earned.

Now, the rate of return for each bidder at the completion of the PH cycle can be calculated as follows. Assume, for the sake of simplicity, that each bidder wins the auction with a bidding rate of 40% throughout the cycle of PH. Thus, in Appendix 1, the first column indicates automatically the first bid of the auction belonging to the organizer; he would receive a $1,200 loan, $100 from each member including himself. If he deposits the $1,200 loan in a bank at 8% interest and withdraws it at the end of the PH cycle (11 months later), he would receive a yield of $88.

At each of the following auctions, he has to pay back the loan equivalent to the price tag until the end of the cycle; if he did not borrow, the opportunity for this money paid back (as savings) would have yielded him interest of $44 at 8%. The net gain of participating in PH as an organizer is $88 from the total saved of $1,200 (gradually saving from each auction, Column 1-S), which is equivalent to an interest rate of 16%. A rate of return of 16% is derived from the fact that to save gradually (see Column 1-S) for the total saved of $1,200 (no capital gain or loss, total loan = total saved), one earns $44 at 8% of return, while to save the same amount and earn $87.996, one must earn 16%. With an inflation rate of less than 16% annually, the PH organizer, therefore, receives a positive real rate of interest on his money (loan $1,200 = savings $1,200) by participating in PH. In other words, PH allows him to receive a $1,200 loan the first month, use it, and pay it back gradually over the next 11 months. Without participating in PH he has to save $100 per month for twelve months before having $1,200 plus interest.

The second column in Appendix 1 indicates that after the second auction, a bidder wins with an interest bid of 40%. He would receive a loan of $760, i.e., $100 from the former winning bidder (the organizer), and $60 (100-40) from the other eleven members, himself included. Again if this bidder deposits his loan in a bank at an interest rate of 8%, he would earn $50.67 at the end of the PH cycle. However, after the second auction, the second winner has to pay a price tag of $100 at each auction until the end of the PH. If all of his paid installments had been put in the bank earning 8% interest, he would have earned $41.33 (see Column 2-S). Since he has opted to give up such a chance, the PH represents an opportunity loss for him. Therefore, he actually receives a net return of $50.669-400 = -$349.331, approximately -67.713% rate of return on installment savings of $1,160 by having participated in PH and taken a loan of $1,123 at the bidding rate of 40%. Although the $1,123 borrowed is used for benefits at 8% and paid back in installments, the bidding of 40% is too high, which is why he receives the negative rate of return. In reality this is not the case, which will be shown in the next section. The explanation of Columns 3 to 12 would be the same as that of Column 2. For example, as Column 11 indicates, the bidder borrows $1,120 for one month and pays the 40% bid interest rate; at the end of the PH cycle, he would earn net interest of 89.311% on the $1,120 loan, saving gradually at each auction for a total of $800. In sum, the last row of the table indicates all the net earned interest rates of each bidder at the end of the PH cycle. The bidder who wins the bid last (the saver) receives a high net rate of interest and vice versa for the borrowers. Therefore, there is a positive relationship between net rates of interest and the length of time of borrowing.

B. The Simple Bidding Decision

An individual bidder in PH is essentially a profit maximizer, he would bid at the interest rate, b, where his wealth over the entire period of PH is optimum. This is because over the PH cycle, the bidder's wealth is involved from the beginning to the end. In Table 1, in the first row of the formula on the left, it is shown that the winner of the first auction would receive a loan plus its returns at the end of the PH cycle, while on the right side of the same row is the total amount which the winner pays for the entire PH cycle.

Table 1

Winner Auction	$Total amount of borrowing plus its return from investment/PH cycle	$Total amount of payment/PH cycle
1	$(T(1)+T(1-b_{i1})(n-1))(1+r_{ij})$	$T(n-1)+T(1-b_{i1})$
2	$(T(2)+T(1-b_{i2})(n-2))(1+r_{ij})$	$T(n-2)+T(1-b_1)+T(1-b_{i2})$
.	....	....
.	....	....
.	....	....
j	$(T(j)+T(1-b_{ij})(n-1))(1+r_{ij})$	$T(n-j)+T(1-b_1)+T(1-b_2)+ \dots +T(1-b_{ij})$

where $T=$ contribution price tag $

$b_{ij}=$ bid interest rate by the ith bidder at the jth auction,

$n=$ number of PH players, including the organizer,

$r_{ij}=$ rate of return for bidder i if he wins and uses the auction fund for (n-j-1) periods. Thus, the actual rate of interest/year $=\bar{r}_{ij}=r_{ij}M/(n-j-1)$, where $M=12$, 52, 365, or 1, if the auction is monthly, weekly, daily or yearly, respectively.

For instance,the left and right sides of the first row formula can be substituted for the first auction winner (the first following the organizer), and the results are similar to the Appendix I, Columns 2-L and 2-S, respectively.

$$
\begin{aligned}
(T(1)+(1-b_{i1})(n-1))(1+r_{i1}) &= (100(1)+100(1-.40)(12-1))(1+r_{i1}) \\
&= (760)(1+r_{i1}), \text{ as } \bar{r}_{i1}=.08, \text{ then} \\
r_{i1} &= .08(n-1-1)/M
\end{aligned}
$$

and $T(n-1)+(1-b_{i1})=100(12-1)+100(1-.40)=1{,}160$.

The second row represents the 2nd winner. Therefore, a general model of the ith winner is developed in the last row. For the ith winner to receive a benefit without concern about the probability of winning or at least breaking even from participating in PH, he must equate

$$[(T(j) + T(1-b_{ij})(n-1)] \ (1+r_{ij}) > [(T(n-j) + T(1-b_1) + T(1-b_2) + \ldots + T(1-b_{ij})]$$

At a break-even point, then,

$$(1) \quad b_{ij} = \frac{r_{ij}(n+j-1)+(j-1)+(b_1+b_2+\ldots+b_{j-1})}{n(1+r_{ij})-r_{ij}+2}$$

$$b_{ij} = f(r_{ij}, n, j, b_1, b_2, \ldots, b_{j-1})$$

Consequently, the bidding rate b_{ij} depends on the actual rate of return on the loan, the number of bidders, the auction number j, and the bidding rates before the jth auction. Observing closely, the nominator of the fraction in the equation (1) can be large if r_{ij} and n are large and many auctions have been held, yielding many b's. Meanwhile, the denominator remains constant as j increases (coming closer to the end of the PH cycle). As a result, the bidding rate is larger due to the increase of the nominator when the jth auction approximates the end of the PH cycle. Near this point, the winner of the auction has little time to invest his winning mutual fund, but he has already earned interest by paying less than a full price tag (price tag minus the interest rate bid). In other words, the late bidders are more likely to be savers (than borrowers) who receive interest in adavance; therefore, the net return rate is rather high for this group.

However, since a bidder generally would like to receive at least a real positive return, $\bar{r}_{ij}$ must be greater than or equal to the inflation rate (p). This means, at least $\bar{r}_{ij} = r_{ij}M/(n-j-1) = p$, thus $r_{ij} = \frac{p(n-j-1)}{M}$ Substituting $p(n-j-1)/M$ in equation (1), the bidding rate b now turns out to be the minimum rate that guarantees a non-negative real rate of return to the bidder, one b for each auction. This means that the bidder can adjust for inflation in his bidding.

$$(2) \quad b_{ij} = f(\bar{r}_{ij}, n, j, b_1, b_2, \ldots, b_{j-1})$$

Therefore, the model developed has shown that an individual bidder's net rate of return after completing a cycle of PH depends essentially on several factors: when the bid occurs in the sequential auction, when the interest rate bid will be paid each bidder's need, and the official market rate of interest. Furthermore, the inflation rate and tax on earned income from deposited money in financial institutions also influence the PH rate of return and the PH size.

The analysis above has assumed that each individual bidder is risk neutral and lives in a world of certainty, knowing his rate of return and knowing the minimum value which he can afford to bid. The bidder does not know what chance he has of winning the bid. In the next section, all the above assumptions are relaxed; that is, all bidders in PH live in a world of uncertainty; each bidder is risk averse and takes into consideration; a) his expected rate of return if he wins the price of the auction, b) the expected change of winning his bid relative to other bidders, and c) the general information which is available to every bidder, such as the previous bid rates before the jth auction in the same PH cycle, the number of bidders remaining in this particular auction, etc.

III. A Competitive Bidding Model

The competitive bidding model for heterogenous bidders in a sealed-bid sequential auction in a PH cycle is constructed by modifying the bidding model for English auctions of Cox, et al. (1982). Suppose there is an auction in which each of n members of a PH is a bidder for at most one of q auction funds offered, where $1 < q < n$, and each PH member possesses utility of money income functions of the form

$$(3) \quad U_i(Y) = Y^{k_i},$$

where k_i is the value of a random variable drawn from distribution Q on (0,1). Note that $(1-k_i)$ is the Arrow-Pratt constant relative risk aversion parameter for utility function (3). Each bidder is assumed to know his own risk aversion parameter, k_i, but does not know that of each of his rivals. Hence Q does not necessarily have a density function and can have a mass of probability of $k = 1$. As a result, this model is flexible enough to include both risk neutral and risk averse bidders, or all of the first kind or all of the second kind as a special case.

The subscript i, where i = 1, 2, 3, 4, ..., n, is used as an index for individual bidders. The value of bidder i of a unit of the jth auctioned funds is denoted by $V_{ij} = (T(1) + T(1-b_i)(n-1))(1 + r_{ij})$ (from Table 1). Assume that V_{ij} is drawn from the uniform probability distribution on the interval (0,v). Each bidder is assumed to know his own auction value before he submits his bid and to know only the probability distribution from which his rival's values were drawn.

The amount bid by bidder i at the jth auction is denoted by $P_{ij} = T(n-j) + T(1-b_1) + T(1-b_2) + \dots + T(1-b_{ij})$ (from Table 1) where b_{ij} is the bidding interest which bidder i promises to pay until the end of PH cycle. If P_{ij} derived from b_{ij} is a winning bid then the ith bidder receives the money income profit or loss, $V_{ij} - P_{ij}$. Likewise, P_{ij} is not a winning bid then he has to pay his price tag T minus rate of the winner's bid, say b_{ij}, and wait to bid the next time in which the number of bidders is then less by the number(s) of winners. Let G(P) be the probability that a bid in the amount P = f(b) will be a winning bid. Then, using (1), the expected utility to bidder i of a bid in the amount b is

$$(4) \quad U_i(P_{ij}) = G(P_{ij})(V_{ij} - P_{ij})^{k_i},$$

The first-order condition for an interior maximization (2) is

$$0 = U_i'(\dot{P}_{ij}) = G'(\dot{P}_{ij})(V_{ij} - \dot{P}_{ij})^{k_i} - K_i G(\dot{P}_{ij})(V_{ij} - \dot{P}_{ij})^{k_{i-1}}$$

Now dividing through by $(\dot{V}_{ij} - \dot{P}_{ij})$, then

$$0 = G'c(\dot{P}_{ij}) - K_i G(\dot{P}_{ij} \,/\, (V_{ij} - \dot{P}_{ij})$$

Therefore, the V inverse of the bid function is

$$(5) \quad \begin{aligned} &V_{ij} = h(P_{ij}, K_i) = P_{ij} + k_i G(P_{ij})/G'(P), \\ &\text{where } P_{ij} = f(T, j, n, b_1, b_2, \dots, b_{ij}). \end{aligned}$$

To arrive at equation (5), one then must assume that each i maximizes expected utility and that all i have the same probability expectation, $G(\cdot)$. It can be shown also that $P_i = f(b_i)$ is an increasing fuctnion in P_i, and that implied that for a risk neutral bidder ($k_i = 1$), the maximum of this bid is

$$(6) \quad \bar{V} = \bar{P} + G(\bar{P})/G'(\bar{P})$$

If $P \leqslant \bar{P}$ then $h(P_{ij}, k_i) < \bar{V}$ for all $K_i \in (0,1)$.
Therefore, the probability that any one bidder will bid less than some amount P, when $P \leqslant \bar{P}$, is

$$(7) \quad F(P) = \int_0^1 \int_0^{h(P_{ij}, k_i)} 1/\bar{V} \, dv \, dQ(k) = 1/\bar{V}\,(P + E(k)G(P)/G'(P)), \text{ let } q = 1/E(k)$$

The probability that any one bidder will bid less than some amount P, when $P > \bar{P}$ is

$$(8) \quad F(P) = \int_0^{R(b)} \int_0^{h(P_{ij}, k_i)} 1/\bar{V} \; dv \, dQ(k) + \int_{R(P)}^{1} \int_0^{\bar{V}} 1/\bar{V} \, dv \; dQ(k)$$
$$= 1 - (1 - P/\bar{V})(G(P)/(\bar{V} - P)G'(P)) \int_0^{R(P)} Q(r) \; dr.$$

where $(G(P)/(\bar{V} - P)G'(P)) > 1$.
The probability that n rivals of a particular bidder will bid less than P and, therefore, that a bid of P will win, is given by the probability distribution function of the nth order statistic for $F(\bullet)$:

$$(9) \quad G(P) = K \int_0^P F(Z)^{n-q-1} \; (1 - F(Z))^{q-1} \; dF(Z)$$

where $K = (n-1)!/(n-q-1)!(q-1)!$

Substituting (9) in (7) (respectively (9) in (8)), then, in conjunction with (5), this implies that (5) is the inverse of an equilibrium bid function for $P \leqslant \overline{P}$ (respectively $P > \overline{P}$). For instance, if all bidders $i \neq j$ and bid according to (5), then the probability that a bid of P^*_{ij} by i will win is $G(P^*_{ij})$, given by (7) and (9) if $P \leqslant \overline{P}$ (respectively $P > \overline{P}$), If i now choose P^*_{ij} to maximize $G(P_{ij})(V_{ij}-P_{ij}),^{k_i}$, he will bid according to (5). Therefore, if all $i \neq 1$ bid according to (5) then i can make no bid better than following (5).

Through the mathematical manipulation of equations (7) and (9) (see Appendix 2) and substituting both of them into (5) yields

$$(5')\quad V_{ij} = P_{ij}+k_i E(K)^{-1}\overline{V}(I(F(P_{ij})))^{-E(k)^{-1}} \int_0^{F(P_{ij})} I(Y)^{E(K)^{-1}} dY,$$

where $I(\cdot)$ is the Kernel of the incomplete Beta integral. Furthermore, substituting the expression for $G(P_{ij})/G'(P_{ij})$ in (5) into the manipulated (7), one obtains

$$(7')\qquad F(P_{ij}) = \overline{V}^{-1}(P_{ij} + ((V_{ij}-P_{ij})/k_i\, E(k)),$$

Therefore, the equilibrium bid function is possibly derivable from equation (5′) and (7′).

In the seqential auctions in a PH cycle. since each auction has only one mutual fund to be bid and if one assume that all bidders are risk neutral (each $= k_i = 1$) and further assume that $E(K) = 1$, then the equilibrium bid function for the single unit discriminative auction (the highest sealed-bid winning the auction) and for the first auction in a PH cycle is

$$(8')\quad P_{i0} = ((n-1)/n)V_{i0}$$

Let $L=(n-1)/n$, since in PH there are n bidders, but the organizer is not bidding even though receives an award for the first auction. Hence, the auction immediately next to it is called the 1st auction and then has $L_1 = (n-2)/(n-1)$. The second auction will have $L_2 = (n-3)/(n-2)$. Hence, L of the jth auction is $L_j = (n-j-1)/(n-j)$. Note that the value of lj is decreasing as j increasing. Substituting the values of P_i and V_i of Table I in (8), yields the interest rate bid of bidder i at the jth auction as

$$(8^*) \qquad b_{ij} = \frac{L_j(r_i+1)\ (n+j-1)-n+(b_1+b_2+\ldots+b_{j-1})}{L_j\ (r_i+1)\ (n-1)-1}$$

That is, $b_{ij}=f(L_j,\ r_{ij},\ n,\ b_1,\ b_2,\ \ldots,\ b_{j-1})$, indicating the bid b of bidder i depends on his probability to win in the jth auction, his rate of return (r_{ij}), the size of PH, and especially the information of the previous winners' bid rates of return before the jth auction in the same PH cycle.

Now from the same equations (5′) and (7′), if one assumes that for each auction in a PH cycle there is only one mutual fund to be bid and all bidders are risk averse, then the heterogeneous bidders equilibrium bid function for $P \leqslant \overline{P}$ for the price auction is

$$(9') \qquad P_{io} = ((n-1)/(n-1+K_j))V_{io},\ \text{let } A_i = (n-1)/(n-1+K_i),$$

Again, in PH there are n bidders, but the PH organizer receives the first mutual fund as a reward. Hence the 1st auction has only n-1 bidders and has $A_{i1}=(n-2)/(n-2+k_i)$. The second auction will have only n-2 bidders, hence $A_{i2}=(n-3)/(n-3+k_i)$. The jth auction will have $A_{ij}=(n-j-1)/(n-j-1+k_i)$. Assume also that the risk-averse individual bidder i is the same for all sequential auctions in a PH cycle. Note that in comparing A_{ij} and L_j with the same jth auction, A_{ij} is larger, showing that the probability of winner i to win is larger when he is risk-averse. Therefore, substituting P_i and V_i of equation (5′) and (7′) in (9′), the bid rate of interest of bidder i at the jth auction is

$$(9^*) \qquad b_{ij} = \frac{A_{ij}(r_{ij}+1)(n+j-1)-n+(b_1+b_2+\ldots+b_{j-1})}{A_{ij}((r_{ij}+1)(n-1))-1}$$

That is, $b_{ij}=f(A_{ij},\ r_{ij},\ n,\ b_1,\ b_2\ldots,\ b_{j-1})$ indicating that the bidder i at jth auction will bid according to: a) his probability to win in that auction, A_{ij}, where his chance depends on his risk-aversion k_i; b) his expected rate of return on the jth auction; c) the size of PH(n); and d) the information of previous bidding rates (b_1, b_2,

$\ldots b_{j-1}$), in the same PH cycle, before the jth auction in which he is going to bid. The information of the last two points, (c) and (d), is available to every bidder. Since the PH auction is a type of first-price auction, a bidder whose information is also available to some others must have zero expected profit at equilibrium. The bidder who has access only to some information of others (the more poorly informed), but makes an independent estimate, may have a positive expected profit.

Therefore, the bidding rate of individual i at the jth auction under 3 different models (or circumstances) which we have just described above can be summarized in a general form as in Table II.

Table II

Bidding Rate at Each Auction:
b_{ij} = Individual i would bid at jth auction

Auction #	Risk Neutral Under no risk of return for investment and not include the probability to win the bid	Semi-Risk Neutral Under no risk of return for investment but include the probability to win the bid	Risk Averse Under risk of return for investment and include the probability to win the bid
1	$b_{i1} = \frac{r_{i1}(n)}{n(1+r_{i1})-r_{i1}+2}$	$b_{i1} = \frac{L_1(r_{i1}+1)(n)-n}{L_1(r_{i1}+1)(n-1)-1}$	$b_{i1} = \frac{A_{i1}(r_{i1}+1)(n)-n}{A_{i1}(r_{i1}+1)(n-1)-1}$
		where $L_1 = \frac{n-3}{n-2}$	where $A_{i1} = \frac{n-2}{n-2+k_i}$
2	$b_{i2} = \frac{r_{i2}(n+1)+b_1}{n(1+r_{i2})-r_{i2}+2}$	$b_{i2} = \frac{L_2(r_{i2}+1)(n+1)-n+b_1}{L_2(r_{i2}+1)(n-1)-1}$ where $L_2 = \frac{n-3}{n-2}$	$b_{i2} = \frac{A_{i2}(r_{i2}+1)(n+1)-n+b_1}{A_{i2}(r_{i2}+1)(n-1)-1}$ where $A_{i2} = \frac{n-3}{n-3+k_i}$
⋮	⋮	⋮	⋮
j	$b_{ij} = \frac{r_{ij}(n+j-1)+(j-1)+\sum_{j=1}^{j-1} b_j}{n(1+r_{ij})-r_{ij}+2}$	$b_{ij} = \frac{L_j(r_{ij}+1)(n+j-1)-n+\sum_{j=1}^{j-1} b_j}{L_j(r_{ij}+1)(n-1)-1}$ where $L_j = \frac{n-j-1}{n-j}$	$b_{ij} = \frac{A_{ij} = (r_{ij}+1)(n+j-1)-n+\sum_{j=1}^{j-1} b_j}{A_{ij}(r_{ij}+1)(n-1)-1}$ where $A_{ij} = \frac{n-j-1}{n-j-1+k_i}$ k_i = risk aversion of bidder i

IV. Summary and Conclusion

This paper has described the procedure of the Asian auction which is commonly found in the informal finance market, where the mutual funds of the members of PH are typically the object of the auction. The non-cooperative bidding model for heterogeneous bidders has been derived for three circumstances. The first is under condition of certainty, where each bidder is risk-neutral, knowing the exact rate of his return and not taking the probability of his winning the bid into consideration. The second is the same as the first, except that the probability of each bidder to win the auction is incorporated into the model. Finally, the model is extended to the actual day-to-day practice where all or some bidders are risk averse; each bids under his expected rate-of-return and expected rate of winning the auction among the other bidders. In all three circumstances each bidder considers the information of the previous bidding rates and the total number of auction members as the vital factors in influencing his decision on a bidding rate, especially in the later sequential auctions.

The competitive bidding just analyzed seems to show that the winning bid rate of each auction reflects the general market rate. Thus, expansion of this type of auction is commonly found in recent years in South Asian countries because the rate of interest provided by the financial institutions is fixed (interest-rate ceilings), and with high inflationary prices, interest-rate ceilings are very low in real terms. The excess demand for credit in formal financial institutions has become crucial; the savers do not want to save in the financial institutions because of the low rate of return. On the other hand, demand for credit at the institutions is high because of the low rate charged, but only borrowers with callateral can obtain the loans, and this leaves others without callateral without loans. They, therefore, have to find other sources of funds. Participation in sequential acutions in PH seems to provide for such a need and the bidding rate seems to reflect the market rate of interest which is lower than the interest charged by the money lenders. Therefore, if the regulation of the interest-rate ceilings is not going to change and the financial liberalization program continues at a slow rate, then it seems to be appropriate for the governments to cultivate the sequential auctions in PH as a legal form and recognize the role of PH in providing a source of capital funds for stimulating economic growth.

References

Chotigeat, T., "Savings Mobilization Via Rotating Credit in LDCs," Paper presented at the Caribbean Economic Growth Conference, November, 1982.

———, "Interest-rate Ceilings, Rotaging Credit and Savings Societies, and Economic Welfare in LDCs," Discussion Paper, January, 1983.

Cox, J. C., Smith, V. L. and J. M. Walker, "Auction Market Theory of Heterogeneous Bidders," *Economic Letters,* 9, 1982, 319-325.

Holt, C. A. Jr., "Competitive Bidding for Contracts under Alternative Auction Procedures," *Journal of Political Economy,* 88, 1980, 433-445.

Milgrom, P. R. and R. J. Weber, "A Theory of Auctions and Competitive Bidding," *Econometrica,* 50, 1982, 1089-1122.

Oren, S. S. and M. H. Rothkopf, "Optimal Bidding in Sequential Auctions," *Operations Research,* 23, 1975, 1081-1090.

Riley, J. G. and W. F. Samuelson, "Optimal Auctions," *American Economic Review,* 71, 1981, 381-392.

Vickrey, W., "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance,* 16, 1961, 8-37.

APPENDIX 1

PH: Deduction of Interest at the Moment of Bidding
All win at 40%

Winner # / Auction #	Organizer		2		3		4		5		6		# months earn int.
	L	S	L	S	L	S	L	S	L	S	L	S	
1	1200	100		100		100		100		100		100	11
2		100	760	60		60		60		60		60	10
3		100		100	800	60		60		60		60	9
4		100		100		100	.840	60		60		60	8
5		100		100		100		100	880	60		60	7
6		100		100		100		100		100	920	60	6
7		100		100		100		100		100		100	5
8		100		100		100		100		100		100	4
9		100		100		100		100		100		100	3
10		100		100		100		100		100		100	2
11		100		100		100		100		100		100	1
12		100		100		100		100		100		100	0
Total	1200	1200	760	1160	800	1220	840	1080	880	1040	920	1000	
Capital gain (L-S)	0		-400		-320		-240		-160		-80		
If invested at 8%	87.996	44	50.669	41.333	48	38.933	44.797	36.8	41.07	34.933	36.8	33.333	
% of benefit at the end of the PH	16		-67.713		-55.890		-42.435		-27.236		-10.368		

APPENDIX 1 (Continued)

PH: Deduction of Interest at the Moment of Bidding
All win at 40%

Winner # / Auction #	7		8		9		10		11		12		# months earn int.
	L	S	L	S	L	S	L	S	L	S	L	S	
1		100		100		100		100		100		100	11
2		60		60		60		60		60		60	10
3		60		60		60		60		60		60	9
4		60		60		60		60		60		60	8
5		60		60		60		60		60		60	7
6		60		60		60		60		60		60	6
7	960	60		60		60		60		60		60	5
8		100	1000	60		60		60		60		60	4
9		100		100	1040	60		60		60		60	3
10		100		100		100	1080	60		60		60	2
11		100		100		100		100	1120	60		60	1
12		100		100		100		100		100	1160	60	0
Total	960	960	1000	920	1040	880	1080	840	1120	800	1160	760	
Capital gain (L–S)	0		80		160		240		320		400		
If invested at 8%	32	32	26.67	30.933	20.80	30.133	14.396	29.6	7.47	29.333	0	29.333	
% of benefit at the end of the PH	8		27.587		48		68.756		89.311		109.092		

Appendix 2

Let $X = F(z)$. Hence the equation (9) becomes

$$\text{(a)}\quad G(P_{ij}) = K\int_{F(0)}^{F(P_{ij})} x^{n-q-1}(1-x)^{q-1}\,dx = KI\,(F(P_{ij}))$$

where $I(\cdot)$ is the kenel of the incomplete Beta integral.

(b) Hence, $G'(P) = KI'(F(P))\,(dF/dP)$.
We can rewrite (7) as
$F-(1/\overline{V})P = (1/\overline{V})E(k)I(F)/I'(F)(dF/dP)$, or

$$\text{(c)}\quad \frac{dP}{dF} + \frac{PI'(F)}{E(k)I(F)} = \frac{\overline{V}FI'(F)}{E(R)I(F)}$$

Multiply (b) by the integrating factor $I(F)^{\theta}$ and let $\theta = 1/E(k)$, We then arrive

$$\text{(d)}\quad \begin{aligned} &I(F)^{\theta}(dp/dF) + \theta\,\overline{V}FPI'(F)I(F)^{\theta-1} = \theta\overline{V}FI'(F)I(F)^{\theta-1}, \text{ or} \\ &(d/dF)\,(PI(F)^{\theta}) = \theta\overline{V}FI'(F)I(F)^{\theta-1} \end{aligned}$$

We can integrate (d) and arrive as

$$\text{(e)}\quad PI(F)^{\theta} = \theta\overline{V}\int_{0}^{F} YI'(Y)I(Y)^{\theta-1}\,dY + C.$$

If $P = 0$, then $F = 0$, then $C = 0$.

Now, we can integrate (e) by part and divide it by $IF(F)^{\theta}$

$$\text{(f)}\quad P = \overline{V}(F - I(F)^{-\theta}\int_{0}^{F} I(Y)^{\theta}\,dY).$$

This shows that the inverse of $F(P)$ is (f) and allows $G(P) = KI(F(P))$ to be determined from (c).

By substituting (f) into the equilibrium inverse bid function (equation(5) in the text). We obtain

$$\text{(g)} \quad V_{ij} = P_{ij} + k_i \frac{I(F(P_{ij}))}{I'(F(P_{ij}))F'(P_{ij})}$$

Differentiate(h) and substituting the resulting expression for $I'(F)F'$ into (g) we get

$$V_{ij} = P_{ij} + k_i \, \theta \overline{V}(I(F(P_{ij}))^{-\theta} \int_0^{F(P_{ij})} I(Y)^{\theta} \, dY,$$

where $\theta = 1/E(k)$.