The Tradeoff Between Economies of Scale and Reliability in The Electric Power Industry

Kwang-Ha Kang*
and
David Kendrick**

I. Introduction

There is a trade-off between economies of scale and reliablity standards in the electric power industry. Because of economies of scale in investment cost it is desirable to build a single large plant instead of several small ones. Because of reliability[1] it is desirable to build several small plants instead of one large plant. Garza, Manne and Valencia, Gately and Rowse have included economies of scale in capacity expansion planning in the electric power industry by using integer variables, but reserve capacities were estimated in a deterministic way in their studies. Consideration of plant size along with uncertainties in system reliability has been articulated and explored by some researchers.[2] However, this has usually been done by using simulation models. Also, some of these studies are operations rather than investment models. Only a few efforts have been made to incorporate economies of scale and uncertainty into an optimization framework. Scherer and Joe have

* Professor, Seoul National University, Seoul, Korea
** Professor, University of Taxas at Austin

1 An excellent survey of this basic concept was published in 1960 as an AIEE Committee Report.

2 Arnoff and Chambers, Baldwin, Desalvo and Limmer, Billinton, Booth, Kirchmayer, Mellor, O'Mara and Stevenson, and Vassell and Tibberts.

developed a mixed integer program expressing the reliability constraint explicitly using integer variables. Noonan and Giglio also developed a mixed integer nonlinear programming model approximating the reliability constraint by a nonlinear function.[3] However, their approaches do not include locational problems and do not examine explicitly the trade-off between economies of scale and reliability.

The purpose of this study is to develop an approach which can be applied to the combined consideration of size of plant and uncertainties in a multi-period, multi-location optimization model. A methodology is developed and this methodology is applied to a case study.

II. Investment Planning Model

A. Reliability

Sufficient generating capacity must be available so that the probability of a zero-margin[4] is less than or equal to some specified level of risk. The constraint[5] for the tth period is given by

(2.1) Prob. $(Y_t - L_t \leqslant 0) \leqslant R$ or

Prob. $(M_t \leqslant 0) \leqslant R$

where

R = maximum allowable risk,
Y = available capacity,
L = peak demand,
M = margin capacity = Y - L, and
Prob. stands for probability.

Using Chebyshev's Inequality,[6] we can convert the above equation to a deterministic function which can be used with

[3] Dillion, Edwin, Kochs and Taud also have presented a method of determining the unit commitment schedule. However, this model is an operating model rather than an investment planning model.
[4] For the definition and derivation, see Kang, 87-98.
[5] This idea comes from chance constrained programming which was developed by Charnes and Cooper.
[6] Parzen 225-228 and 378-384.

mathematical programming packages. Chebyshev's Inequality shows that for any $h > 0$,

$$(2.2) \qquad \text{Prob.}\ (\,|X - E(X)| > h\sigma_X) \leqslant 1/h^2$$

where

X = a random variable,
$E(X)$ = the expected value of X, and
σ_X = the standard deviation of X.

We can get the following reliability constraint function from equations (2.1) and (2.2).[7]

$$(2.3) \quad \overline{Y}_t - \overline{L}_t \geqslant \frac{1}{\sqrt{R}} \left(\sigma^2_{Y_t} + \sigma^2_{L_t}\right)^{1/2}$$

where $\overline{Y}$ and $\overline{L}$ denote mean values of the random variables Y and L and σ^2 denotes the variance of these random variables.

B. Formulation of the Model

The symbols used in the model can be categorized into sets and indicies, variables, and parameters, as follows:

Sets and Indicies

$i \varepsilon I$: plant type
$j, k \varepsilon J$: region
$\ell \varepsilon L$: block on the load duration curve
$t, \tau \varepsilon T$: time interval and time period
$m \varepsilon M$: transmission line
$p \varepsilon P$: mode of operation

[7] See Kang 98-101 for this derivation.

Variables

- y: investment decisions for plants (zero or one)
- u: production of electricity
- x: transmission of electricity
- h: investment decision for transmission lines (capacity)
- ϕ: cost group
- $\varkappa$: capital cost of plants
- λ: capital cost for the transmission lines
- φ: fixed operating cost
- π: variable operating cost
- ξ: total cost

Parameters

- a: availability of plant
- b: construction cost for transmission lines
- c: construction cost for unit plant
- d: demand level
- e: distances between regions
- f: fuel cost per unit output
- g: unit size of plant
- k: initial capacity
- k^p: plant capacity already under construction at time zero
- q: forced outage rate for plant
- r: transmission loss rate
- s: retirement of capacity
- v: fixed operating cost per unit of capacity
- w: construction lag period
- z: useful life of plants or transmission lines
- α: transmitting capacity per unit line
- β: escalation rate of cost
- δ: discount factor
- ρ: discount rate
- η: capital recovery factor
- θ: width of block on load duration curve
- σ^2: variance for probability distribution

The objective function and constraints of the model are as follows:

1. *Objective Function*

The total cost to be minimized is the discounted sum of capital costs (both plant and transmission line), fixed operating costs and variable operating costs (fuel cost).

Objective Function

$$(2.4) \quad \xi = \sum_{t \in T} (\delta_{kt}\phi_{kt} + \delta_{\lambda t}\phi_{\lambda t} + \delta_{\psi t}\phi_{\psi t} + \delta_{\pi t}\phi_{\pi t}) \quad t \in T$$

This can be translated to

total cost = capital cost for plant + capital cost for transmission + fixed operating cost + variable operating cost

where δ_{nt} = the discount factor which is appropriate to the nth cost term, (see Kang 135-138.); and

$$(2.5) \quad \phi_{kt} = \sum_{j \in J} \sum_{i \in I} \sum_{\tau=1}^{t} \eta_i \beta_{ki} c_i y_{ji(\tau - w_i)}$$

$$(2.6) \quad \phi_{\lambda t} = \sum_{m \in M} \sum_{\tau=1}^{t} \eta_i e_m \beta_\lambda b\, \alpha_m h_{m(\tau - w_m)}$$

$$(2.7) \quad \phi_{\psi t} = \sum_{j \in J} \sum_{i \in I} v_i k_{ji} + \sum_{j \in J} \sum_{i \in I} \sum_{\tau=1}^{t} [\beta_{\psi i} v_i (g_i y_{ji(\tau - w_i)} + k^p_{ji\tau}) - v_i s_{ji\tau}]$$

$$(2.8) \quad \phi_{\pi t} = \sum_{j \in J} \sum_{i \in I} \sum_{p \in P} \beta_{\pi i} f_i \theta_p u_{jipt}$$

$$(2.9) \quad \eta_i = \frac{\sigma}{1+(1+\tilde{p})^{-z_i}}$$

where

$\tilde{\rho}$: discount rate per year
η: capital recovery factor[8].

If $(\tau - w_i)$ or $(\tau - w_m)$ is negative, then $y_{ji(\tau - w_i)}$ or $h_{m(\tau - w_m)}$ is zero, respectively.[9]

2. *Constraints*

a. *Reliability Constraint*

Sufficient generating capacity must be available so that the probability of a zero-margin must be less than or equal to some specified level of risk. The reliability constraint is as follows:[10]

$$(2.10)\quad k_{mean} + \sum_{j \varepsilon J} \sum_{i \varepsilon I} \sum_{\tau = 1}^{t} [(1-q_i)(g_i y_{ji(\tau - w_i)} + k^p_{ji\tau})$$

$$- k_{mean} \frac{s_{ji\tau}}{k}] - \bar{d}_t > \frac{1}{\sqrt{R}} [\sum_{j \varepsilon J} \sum_{i \varepsilon I} \sum_{\tau = 1}^{t}$$

$$[q_i(1-q_i)(g_i^2 y_{ji(\tau - w_i)} + (k^p_{ji\tau})^2) - \sigma^2_k \frac{s_{ji\tau}}{k}]$$

$$+ \sigma^2_k + \sigma^2_{dt}]^{1/2} \qquad \tau \varepsilon T$$

which translates into

mean initial capacity + mean net capacity increase−mean peak demand $\geqslant \frac{1}{\sqrt{R}}$ [variance of added net capacity
+ variance of initial capacity
+ variance of peak demand]$^{1/2}$

8 See Kendrick and Stoutjesdijk, 47-49 for the definition of the capital recovery factor.
9 This is due to the "construction lag."
10 For the detailed discussion, see Kang, 117-119.

where

k: total initial capacity,
k_{mean}: mean initial capacity,
$\bar{d}_t$: mean value of peak demand at time t,
σ^2_{dt}: variance of peak demand at time t
σ^2_k: variance of initial capacity,
k^p: plant capacity already under construction at time zero
R: maximum allowable risk

b. Demand Constraint

The demand for electricity in a region at time t must be satisfied, either by the current regional production or the power transmitted (less transmission loss) from the other regions.

$$(2.11)\quad \sum_{p=1}^{\ell} \sum_{i \varepsilon I} u_{jipt} + (1\text{-}r) \sum_{k \varepsilon J, k = j} x_{kj\ell t} - \sum_{k \varepsilon J, k \neq j} x_{jk\ell t} \geqslant d_{j\ell t} \qquad j \varepsilon J,\ \ell \varepsilon L,\ t \varepsilon T$$

which translates to

power generated within region + power received from others - power sent to others $\geqslant$ projected demand level.

c. Capacity Constraint

The power capacity constraint requires that production levels not exceed the power capacity available at each plant in each time period. There is no explicit energy capacity constraint.[11] Since all plants but hydroelectric plants could be operated virtually continuously if enough fuel were provided, the power capacity constraint provides an effective energy capacity constraint for coal or nuclear power plants. The capacity constraint is

11 Garza, Manne and Valencia use energy constraints in their study for Mexico.

$$(2.12)\quad \sum_{p \varepsilon p} u_{jipt} \leqslant \sum_{\tau=1}^{t} a_i(g_i y_{ji(\tau-w_i)} + k^p_{ji\tau} - s_{ji\tau}) + ak_{j_1}$$

$$t \varepsilon T,\ i \varepsilon I,\ j \varepsilon J$$

which translates to
production level ≤ maximum available capacity.

d. Transmission Capacity Constraint

The power transmitted on each line for each demand level at time t must not exceed the available capacity on that line.

$$(2.13)\quad x_{jk\ell t} + x_{kj\ell t} \leqslant \sum_{\tau=1}^{t} h_{m(\tau-w_m)} + h_{mo}$$

$$j,k \varepsilon J,\quad \ell \varepsilon L,\quad t \varepsilon T.$$

which translates to (power transmitted from j to k) + (power transmitted from k to j) ≤ (total available transmission capacity). In this inequality h_{mo} danotes the corresponding initial transmission line capacity between region k and j. Either $x_{jk\ell t}$ or $x_{kj\ell t}$ will be positive, otherwise both are equal to zero.

III. Case Study

A. Assumptions

The following assumptions were made for application of the model to the South Korea electric power industry.

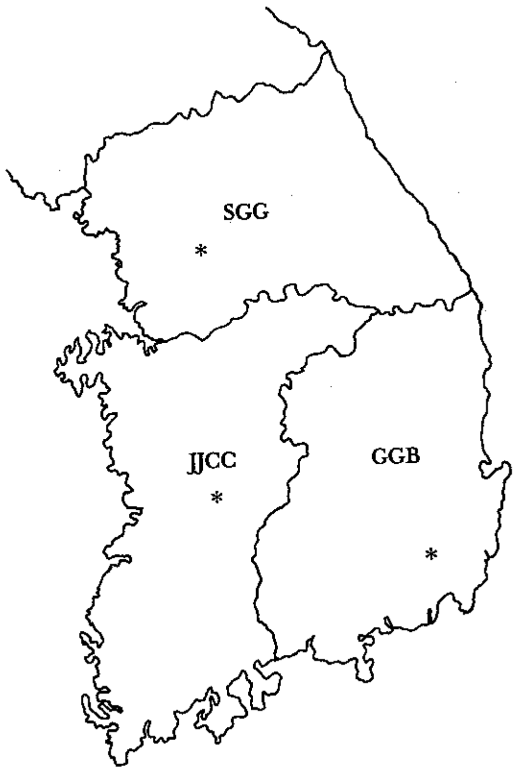
1. Generation cost: Expectations regarding escalation of capital cost, fuel cost and fixed operating cost are exogenous inputs to the model. All costs are assumed to increase at rates which are constant over the planning period.
2. Demand for electricity: Demand information is incorporated

through the use of a three year load duration curve which is composed of five rectangles. The shape of the load duration curve is unchanged over the planning period and the same load duration curve is used in the different regions. Hydroelectric production is exogenous to the model.

3. Four plant types are chosen as candidates for addition to the initial system. These are a 600MW nuclear plant, a 900MW nuclear plant, a 500MW coal plant, and a 900MW coal plant.

Figure 1

REGIONS OF SOUTH KOREA



SGG = Seoul-Gyeonggi-Gangwon.

GGB = Gyeongbuk-Gyeongnam-Busan.

JJCC = Jeonbuk-Jeonnam-Chungbuk-Chungnam.

* = weighted center point.

4. Five time periods of three years each are used.
5. Figure 1 is a map of South Korea showing the geographical boundaries of the three regions, each of which is both a consuming center and a source of electricity.
6. Only the capacity initially in place is considered for retirement during the planning period.
7. The existing plants are lumped into the initial system which is treated as homogeneous.
8. If plants are completed in period t, they are assumed to be available at 100 percent of capacity in that period.
9. There are no reliability constraints corresponding to first few period because of construction lag.

The sets and indicies for the case study are given in Table 1.

Table 1

SETS AND INDICIES

Set I: plant types
- 1: initial plants
- 2: 500MW coal-fired plant
- 3: 900MW coal-fired plant
- 4: 600MW nuclear plant
- 5: 900MW nuclear plant

Set J: Regions
- 1: Seoul-Gyeonggi-Gangwon (SGG)
- 2: Gyeongbuk-Gyeongnam-Busan (GGB)
- 3: Jeonbuk-Jeonnam-Chungbuk-Chungnam (JJCC)

Set L: demand blocks
- 1: summer non-peak hours
- 2: winter non-peak hours
- 3: summer peak hours
- 4: winter peak hours
- 5: highest peak hours

Set P: modes of operation
- 1: all demand levels ($\ell = 1, 2, 3, 4, 5$)
- 2: operation only for $\ell = 2, 3, 4, 5$
- 3: operation only for $\ell = 3, 4, 5$
- 4: operation only for $\ell = 4, 5$
- 5: operation for $\ell = 5$ only

Set T: time periods
- 1: 1980-82
- 2: 1983-85
- 3: 1986-88
- 4: 1989-91
- 5: 1992-94

Set M: transmission lines
- 1: transmission line between SGG and GGB
- 2: transmission line between GGB and JJCC
- 3: transmission line between JJCC and SGG

Table 2

COMPARISON OF TWO CASES

(Million Dollars)

Risk Index	Objective Value	Plant Type
0.04	13,749.9	All large plants
0.025	14,257.8	Large and small plants

B. Results

In order to analyze the trade-off between reliability and economies of scale, it is useful to vary the reliability criteria. Thus the model was solved twice with a maximum allowable risk (R) of 0.04 and 0.025.[12] Though it would have been desirable to solve the model many times and trace out the trade-off curve, it was not possible to do so because of the expense of solving the nonlinear mixed integer programming problems.

Table 2 lists the objective value for the two cases. As expected, the cost is higher when the maximum allowable risk is decreased. Also, since the smaller maximum allowable risk indicies require larger reserve requirements, it is reasonable to expect that small plants will be favored[13] in moving from 0.04 to 0.025. The optimal investment decision with a risk index of 0.025 indicates that small plants will be installed to reduce the value of the variance for capacity, thereby minimizing the capital cost by reducing the reserve requirement. The objective value in the 0.025 case is higher than in the 0.04 case, due to the diseconomies of scale in building small plants, in addition to the increased capacity itself. The increased cost of 508 million dollars due to employing a higher standard of reliability, can be traded off against the increased reliability.

12 For the input data which are used in this study, see Kang, Appendix C, 181-189.

13 Given a required capacity, the more plants we have, the less reserve capacity we need because of the smaller variance for capacity.

It is relevant to ask what might be the costs of excluding either large or small plants from consideration. The answer to this question was obtained by solving the model subject to the constraint that either no large or no small plants be constructed. Table 3 provides the solution values to the constrained problems, along with the unconstrained problem in the case with a risk index of 0.025.

Table 3

DIFFERENT CASES WITH A RISK INDEX OF 0.025

(Million Dollars)

Cases	Objective Value
Small plants only	14,389.4
Unconstrained	14,257.8
Large plants only	14,272.3

The objective value in the small-plants-only case is higher than in the unconstrained case due to the increase in capital cost because of diseconomies of scale. One interesting result from this comparison is that the total added capacity in the small-plants-

Table 4

COST COMPARISON WITH A RISK INDEX OF 0.025

(Million Dollars)

Cases	Total Added Capacity (GW)	Capital Cost	Fixed Cost	Variable Cost
Small plants only	21.5	4,515.4	723.3	8,913.2
Unconstrained	23.2	4,382.1	734.5	8,908.4
Large plants only	24.3	4,374.5	756.9	8,908.3

only case is less than in the unconstrained case due to the construction of small plants, which give more flexibility in satisfying the reliability and capacity constraints. The objective value in the large-plants-only case is also higher than in the unconstrained case because of the necessity to increase capacity due to the large variance in capacity. The capital cost in the large-plants-only case is lower in the unconstrained case due to economies of scale. However, fixed operating cost in the large-plants-only case is much higher than in the unconstrained case. The objective value in the small-plants-only case is higher than the objectie value in the large-plants-only case. From these comparisons, one can say that there exists a trade-off between the reliability standard and economies of scale. If one wants to increase the standard of reliability, it is necessary to sacrifice some benefits from economies of scale in capital costs.

IV. Conclusion

The trade-off between reliability and economies of scale in the electric power industry has been examined. An empirically important trade-off between the standard of reliability and the economies of scale in capital costs was found. Reliability constraints play a significant role in investment decisions in the case examined. In particular, different indicies of the maximum allowable risk lead to different choices of plant size and different combination of plant sizes are adopted as an optimum solution according to the trade-off between reliability and economies of scale. The investment decisions are sensitive to the treatment of the reliability criteria in the model. Therefore, more attention should be given to the appropriate treatment of reliability constraints in electric power investment planning models.

References

AIEE Committee Report, "Application of Probability Methods to Generating Capacity Problems," *AIEE Trans.*, 79, Part III, Feb. 1961, 1165-82.

Arnoff, E.L. and John C. Chambers, "Operations Research Determination of Generation Reserves," *AIEE Trans.*, 76, Part III, June 1957, 316-28.

Baldwin, C.J., Desalvo, C.A. and H.D. Limmer, "The Effect of Unit Size, Reliability, and System Service Quality in Planning Generation Expansion," *AIEE Trans.*, 79, Part III, Feb. 1961, 1042-50.

Billinton, R., "Composite System Reliability Evaluation," *IEEE Trans Pas*, 88, April 1969, 276-80.

Booth, R.R., "Optimal Generation Planning Considering Uncertainty," *IEEE Trans Pas*, 91, Jan./Feb. 1972, 70-77.

Charnes, A. and W.W. Cooper "Chance-Constrained Programming," *Management Science*, 6, Oct. 1959, 73-79.

Dillion, T.S., K.W. Edwin, H.D. Kochs, and R.J. Taud, "Integer Programming Approach to the Probabilistic Reserve Determination," *IEEE Trans Pas*, 97, Nov./Dec. 1978, 2154-66.

Garza, Guillermo Fernandex dela, A.S. Manne, and Jose Albert Valencia, "Multi-level Planning for Electric Power Projects," *Multi-Level Planning Case Studies in Mexico*, eds., Goreux, Louis M. and A.S. Manne, North-Holland Publishing Co., 1973, 197-231.

Gately, Dermot, "Investment Planning for the Electric Power Industry: an Integer Programming Approach," Research Report 7035, Department of Economics, The University of Western Ontario.

Kang, Kwang-Ha, "An Investment Programming Model in the Electric Power Industry," Ph.D. dissertation, Department of Economics, The University of Texas at Austin, 1981.

Kendrick, D.A. and Ardy Stoutjesdijk, *The Planning of Industrial Investment Projects: A Methodology*, Johns Hopkins University Press, Baltimore, 1978.

Kirchmayer, L.K., A.G. Mellor, J.F. O'Mara, and J.R. Stevenson, "An Investigation of the Economic Size of Steam-electric Generating Units," *AIEE Trans.*, 74, Part III Aug. 1955, 600-14.

Moferi, Pier L. and Luigi Paris, "Quantitative Evaluation of Power System Reliability in Planning Studies," *IEEE Trans. Pas*, 91, March/April 1972, 611-18.

Noonan, F. and R.J. Giglio. Giglio, "Planning Electric Power Generation: A Nonlinear Mixed Integer Model Employing Bender's Decomposition," *Management Science*, 23, May 1977, 946-57.

Parzen, Emanuel, *Modern Probability Theory and Its Applications*, John Wiley & Sons, Inc., 1960.

Rowse, John Garry, "Toward Optimal Capacity Expansion for an Electric Utility: A Mixed Integer Programming Approach to Generation Planning," Ph.D. Dissertation, University of Minnesota, 1974.

Scherer, C.R. and Leland Joe, "Electric Power System Planning with Explicit Stochastic Reserve Constraint," *Management Science*, 23, May 1977, 978-85.

Vassel, Gregory S. and Nicholas Tibberts, "Analysis of Generating Capacity Reserve Requirements for Interconnected Power Systems," *IEEE Trans. Pas,* 91, March/April 1972, 638-49.