A Dynamic Decomposition Model of Regional Economic Development:
A Recursive Programming Approach to Developing Agriculture

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It has been well recognized that economic growth depends very much upon the performance of the agricultural sector. It is evident from the fact that in developing countries with per capita income of less than $300, 40-80% of total labor force and between 30 and 60% of the total GNP are accounted by agriculture. In addition, the non-agricultural sectors have to rely upon raw materials from agriculture and rural labor for their growth. Furthermore, the agricultural sector serves as an important domestic market, perhaps largest, for certain commodities manufactured in industrial sectors. The importance of agricultural development is further enhanced by the dynamics of demography which grows 2-3% or more per annum in developing countries. Low rates of growth of income and production in agriculture can seriously retard the growth of the non-agricultural sector.

In spite of this importance of the agriculture's roles in economic development, there have been two contrasting views on peasant behaviors of many LDC's when to design development strategies.¹ One

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¹ These two alternative views are presented in Wharton (1970) where a series of articles on both sides of this controversy are available. The latter view was pioneered by Schultz (1964).
vital assumption often made by policymakers was that peasants in developing countries were basically tradition bound, "non-rational," "uneconomic" men limited by culture and institutional restraints. On the other hand, the alternative view accepts the notion that farmers in LDC's are indeed as rational and responsive as their counterparts in industrial sectors to various economic signals. Following this latter view, several empirical studies have clearly demonstrated that agricultural production in peasant and traditional agriculture in specific LDC's is responsive to economic incentives, especially when factors such as subsistence, adjustment lags due to uncertainty, quasi-fixity of capital stocks and the state of the arts and knowledge are accounted for. What these studies suggest is that economic models based on the assumption of rational behavior and using standard analytic tools can be used effectively to explain, understand, predict, and plan the process of agricultural transformation in these developing countries.

The purpose of this paper is to show one such analytic tool — recursive programming — can be used to simulate the past development of the agricultural sector with special emphasis on diverse heterogeneity of development process due to vast differences in resource endowments, specifically farm size, at the farm level. This is done by constructing an operational regional model of agricultural development which will explicitly incorporate (a) the complex dynamic (multi-period) microeconomic decision-making process on farm-firms within their given environment under selected policy instruments, and (b) farm size groupings in arriving at regional behavioral and resource aggregates. The resulting framework of analysis is similar to the recursive programming models of regional agricultural development pioneered by Day (1963), further extended by Heidhues (1966) and recently applied to agriculture in transition in the LDC's by Singh (1971) and Mudahar (1972). The model presented here, although following directly the main methodological improvements of its predecessors, goes beyond by relaxing the usual assumptions of homogeneous farm size over which farms in a given region are aggregated to obtain a regional model for analysis. The model explicitly treats the farm size groupings by considering different farm size aggregates, with different resource availabilities and factor proportions, but facing a similar exogenously given econo-

2 "Recursive Programming" here is briefly defined as a sequential constrained optimization method involving behavioral feedbacks which take account of uncertainty, myopia, limited information and the like. As a general class of dynamic systems, it deals more with the temporal elements of decision making and less with what decisions ought to be made in terms of some optimum or normative decision rules. For the theory and empirical applications utilizing recursive programming methodologies, see Day (1963, 1967, 1970), Heidhues (1966), Singh (1971), and Day and Singh (1975).
mic environment, and competing for scarce regional resources. With the explicit introduction of farm size groupings through the decomposition principle of linear programming, it attempts to arrive at a framework capable of treating dynamically, the heterogeneous time paths of development among different farm size groups.

Next section of this paper briefly describes some fundamental components we consider essential to this transformation process, components without which such a model could not be operationally useful; Section II presents the extension of these prototype recursive programming models in order to account for farm size differences and their consequences in simulating past history of agricultural transformation; Section III presents the structure of the model; Section IV presents some important empirical results obtained for the Southern Brazil from 1960-1970 for which the model was developed to analyze its recent agricultural transformation. The paper concludes briefly with discussions on the further refinements and several policy applications for the model.

I. Fundamental Components of Modern Agricultural Transformation

While the agricultural sector in many LDC's remains at the heart of development strategies, only recently with the advent of the "green revolution" is increased attention being given to the complex nature of agricultural development. In this context, the core of the agricultural development can be viewed as resource allocation through time for individual farm-firms within an environment affected by a multitude of physical and economic variables. This environment can further be influenced by a set of policy instruments. This suggests that modern agricultural transformation in the LDC's has been mainly carried out in an environment in which (a) decision making occurs at the farm level and involves firm-household interdependence; (b) technological change, both mechanical and biological, which has often been a critical element; (c) strong government participation, either directly through the allocation of scarce resources or indirectly through established markets, has substantially directed the development process; and (d) the development of the agricultural sector has had important implications for development elsewhere in the economy and vice versa. These components define the environment within which modern agricultural transformation is taking place.³

The complexity of the modern agricultural transformation in LDC's arises first from the fact that the farm involves two distinct but interdependent decision units — the farm-firm and the farm-

³ These points were explored in great detail by Day (1963), Singh (1971), and Day and Singh (1975).
household. In this and development context, production decisions involving the allocation of scarce firm resources cannot be properly separated from investment decisions and household consumption and factor use decision. Furthermore, the importance of subsistence production to the analysis of developing agriculture needs to be emphasized. The most important implication is that developing agriculture is often characterized by subsistence production where (a) the farm-household depends upon the farm-firm for its main items of consumption so that production is mainly carried out to meet these needs and not for the market and (b) the farm-firm relies upon the household for its need of labor and other production inputs. As a result, the response to market incentives is modified considerably as household consumption requirements act as a constraint on both the product mix as well as the marketed surplus.

The farm-household decisions as consumers involve the choices between leisure and income expressed in the amount of family labor offered for work, between present and future income expressed in consumption and savings, and between retained and marketed output expressed in the amount of total income converted to monetary income. This, in turn, will affect the choices between labor and capital intensive technologies, between variable and quasi-fixed inputs outlays, and between commercial and subsistence outputs that the farm-firm must make as a producer. This interdependence makes it difficult to differentiate the activities of the farm-household from the farm-firm. All economic activities of the firm-household therefore have to be treated simultaneously in an integrated farmework.

The dual characteristics of the firm-household interdependencies are further complicated by the following elements prevailing in developing agriculture as emphasized by Day (1963). They include: (a) the interdependence of outputs using common inputs i.e., the multiproducts nature of agricultural production; (b) changes in both acreage and yield components in field crop production; (c) the relative interaction of input and output prices; (d) the rate of investment in quasi-fixed factors; (e) uncertainty and adjustment over time; (f) planned or programmed policy actions.

Another unique feature of agricultural development is that farming is, in general, a highly uncertain business. In this context, the importance of risk and uncertainty, learning and adoption, and multiple goals must be emphasized. The greater risk and uncertainty

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4 This interdependence and its consequences have been emphasized by Mellor (1965) and Nakajima (1965). Although empirical work emphasizing microeconomic decisions have been very limited, some notable attempts can be cited in the works done by Yotopoulos and Lau (1974), and Raj Krishna (1963). In the economic analysis of developed agriculture some attention has been given to the resulting interdependence by Heady et al (1953), Day (1963) and Heidhues (1966).
is in developing agriculture, the fewer means to control or circumvent it exist. It is highly likely that farmers are extremely cautious in adopting and learning new production techniques in order to avoid disaster in case of failure. Hence, both adoption and learning of new production techniques takes highly time consuming process. These internal constraints due to the learning process and lagged adjustments assure that (a) farmers are very reluctant to make changes in their traditional cropping patterns in response to changes in their environment unless those changes persist over time, and (b) the impact of new technologies will be distributed over time. These feature about learning and adoption behavior should clearly be incorporated in any analysis of economic development. These are considered in the flexibility constraints developed by Henderson (1969) and Day (1963).\(^5\)

There is a general agreement that economic decision making involves multiple goals and that single optimizing criteria like profit maximization are inadequate in describing the decision process. Furthermore, all goals do not have an equal priority and are often ranked according to a set of preferences.\(^6\) This has a very special significance for peasant agriculture where food requirements to meet subsistence needs or safety first criteria may be achieved ahead of profit maximization.

It is well established in the development literature that technological change is an important, if not the most important, factor responsible for economic development. When transition from a peasant to a modern agriculture gets underway, technology, mechanical and biological, has been viewed as a necessary and sometimes sufficient ingredient in any plan for increasing agricultural output and productivity, and incomes. In recent years, the importance of technological change, especially biological innovations associated with the use of high yielding varieties, water and fertilizers has received considerable attention.\(^7\) In a transitional agriculture, usually multiple technologies such as old and new varieties or draft animal versus tractor farm powers exist. Therefore we need to consider explicitly different set of inputs corresponding to existing technological choices.

\(^5\) It is unlikely though for farmers to follow the form of Monte Carlo or other sophisticated rules in making economic decisions. They are more likely to follow rules of thumb procedures summarized as strategies of cautious optimization. This type of optimizing principles can also be found in the chance-constrained models of Charnes and Cooper (1959), the loss-principle of Shackle (1958), the behavioral bounds of Cyert and March (1969), and the safety-first principle of Roy (1952).

\(^6\) For details on the multiple goals and preference orderings, see Debreu (1959) and for its application in programming model see Day and Singh (1975).

\(^7\) Biological technologies have continuously been emphasized as critical by Hopper (1965), Schultz (1964) and more recently by Hayami and Ruttan (1971).
Of course, the adoption of these new technologies are going through time-consuming learning process to diffuse over a given region. In this context, these unique features of technological change must be considered in developing a successful model to simulate the past history of economic development.

Another critical factor which influences the farm level decision making is the government participation in both the farm level resource allocation and the external environment facing farm operators. Government policy actions, especially those relating to price incentives, are viewed as affecting either the payoffs and their expectations or the resource endowments confronting decision makers at the farm level. Looking back the last decade or so, one can observe a host of ambitious policies and programs employed in many LDC's in order to accelerate their agricultural growths. They include, to name just a few, product and factor pricing policies for the purpose of import substitution and massive rural credit programs at the extremely low rate of interest in order to make new and more efficient inputs available to farmers.

In order to understand modern agricultural transformation, last but not least, the intersectoral linkages must be accounted for. The most critical ones include (a) the demand for farm products by the non-farm and export sectors, (b) the supply of non-farm inputs, (c) opportunities for non-farm employment and non-farm investments, (d) the demand for non-farm consumer goods by the farm sector. These linkage mechanisms must be accounted for exogenously as well as endogenously while the policy variables exercised in the government participation be considered exogenously in the regional development model.

II. Farm Size Issues and Decomposition in Regional Aggregation

In addition to the fundamental components we discussed as essential to the agricultural transformation, the complexity also arises from the fact that farms and farmers are not homogeneous with respect to their relative factor endowments or in their response to economic incentives. To the extent that farmers face a similar exogenous economic environment in a relatively homogeneous zone with respect to climate and topography, their decisions are aggregatable, and these aggregates represents regional behavior and production response. However, unless the size of operational unit are also fairly homogeneous with respect to their endogenous economic environment, especially the availability of on-farm resources, ag-

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8 These intersectoral linkages have extensively been treated in the computer system simulation models to analyze agricultural sector performance. For details and its methodology, see Johnson, et al (1971)
gregation can and does lead to serious errors in regional analysis. It is therefore desirable to consider an analytical framework that minimizes the possibility of such errors by explicitly treating different farm sizes with inherent differential factor endowments.

The importance of farm size and its relation to such factors as economies of scale, risk and uncertainty, and market response has long been emphasized by many economists. Heady suggests that the difference in farm size is one of the most important factors explaining differences in the decision making process of farm firms, especially in response to various economic opportunities involving risk and uncertainty. More recently, in the study of agricultural development in the LDC's, special emphasis has been given to the nature of subsistence production and the market response in small farm peasant agriculture. Where such units exist, side by side with larger and more commercialized units, the larger farms through their greater access to technology and factor and product markets, usually reap a disproportionate share of gains when transformation gets underway.

Furthermore, the adoption of new technologies is closely associated with the relative endowments of two primary resources, land and labor. Hayami and Ruttan convincingly demonstrated that the state of relative endowments and accumulation of two primary resources, land and labor, is a critical element in determining a viable pattern of technological change in agriculture. Depending on the relative scarcity of land and labor, technological change embodied in new and more productive inputs may be induced to save labor or to save land.

In order to apply mathematical programming to industrial performance, Day has shown that production decisions for atomistic firms in an industry can be represented by a single linear programming model provided that one can observe (a) proportional variation of expected net return among all firms, and (c) common technological coefficients. Under these conditions, a single linear programming model for the aggregate is equivalent to a direct aggregation of the solutions to a set of individual firm models.

If the atomistic firms in an industry, say agriculture, are characterized by substantial differences in resource composition, the pro-

9 See Steindl (1945), Hicks (1945) and Heady (1965).
10 These points are presented in their “induced development model” developed by Hayami and Ruttan (1971). Some empirical examples are readily found: a country with high man-land ratio like Korea and Taiwan pursues high yield biological technology to increase agricultural productivity per acre whereas a country with low man-land ratio like Brazil adopts mechanical technology to cultivate land at its extensive margin at early stage of agricultural transformation.
portionality assumptions for regional aggregation breaks down. These factor disproportionalities are critical in a region like southern Brazil where the size of the operational unit may vary from less than ten hectares to several thousand hectares. Single regional aggregates, in such a case, tend to obscure rather than highlight these differences in farm endowments, farm response and policy impact. However, an explicit treatment of farm size differences by grouping similarly sized farms (farms size decomposition) enables one to capture important structural and behavioral differences among farms in the process of development.

In addition, the farm size groupings in a decomposition frameworks\textsuperscript{12} provides two additional properties. First, the interdependent structure of farm size subaggregates in competing for regional coupling resources can serve as an allocation device for guiding regional resources to different farm size groups. Second, the decomposition scheme gives a method for analyzing distributional effects of development through time such as growth and decay of a cropping activity and the disparity of income distribution by farm size. Thus, farm size and resulting differences in resources, both initially and as they become cumulative over time, need to be explicitly treated.


III. The Model

Considering the complex nature of modern agricultural transformation characterized by the fundamental components along with farm size questions discussed above, it is most difficult to develop a general model that would apply to all types of agricultural transformations underway in the LDC’s. As an attempt to consider these strategic factors essential to agricultural transformations in a single integrated framework, we have tailored a recursive programming model with a farm size decomposition to the wheat producing areas of the state of Rio Grande Do Sul in Southern Brazil.

During the decade of the sixties, this region saw considerable growth in real agricultural output and a persistent transformation of regional economy from range livestock production to intensive crop production.\textsuperscript{13} This transformation was possible through a large program of price supports for wheat producers tied to subsidized

\textsuperscript{12} In this study, the decomposition principle is used to distinguish non-aggregatable resource structure specific to each farm size group and to establish intra-farm competition mechanism for the use of regional strategic resources rather than to partition a larger matrix to solve a mathematical programming problem. For the theory of decomposition principle, see for example Lasdon (1970).

\textsuperscript{13} For details on the regional economic history, see Rask (1969) and Ahn (1972).
credits available for the purchase of modern capital intensive inputs. As a consequence, total output, factor productivity, factor uses and farm incomes increased substantially, bringing considerable economic prosperity to the region.

However, as shown in Table 1, there is a wide distribution of farm size in the region resulting in substantial differences in relative endowments at the farm level.

Table 1

<table>
<thead>
<tr>
<th>Hectares</th>
<th>Number of Farms</th>
<th>Percent of Farms</th>
<th>Land Used (1,000 Ha)</th>
<th>Percent of Land Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-25</td>
<td>65,054</td>
<td>67.32</td>
<td>753,155</td>
<td>13.76</td>
</tr>
<tr>
<td>26-50</td>
<td>15,007</td>
<td>16.35</td>
<td>541,066</td>
<td>9.89</td>
</tr>
<tr>
<td>51-100</td>
<td>7,485</td>
<td>7.74</td>
<td>506,092</td>
<td>9.25</td>
</tr>
<tr>
<td>101-1,000</td>
<td>7,558</td>
<td>7.82</td>
<td>2,112,846</td>
<td>38.61</td>
</tr>
<tr>
<td>1,011-10,000</td>
<td>726</td>
<td>0.77</td>
<td>1,557,784</td>
<td>23.49</td>
</tr>
<tr>
<td>Total</td>
<td>96,633</td>
<td>100.00</td>
<td>5,471,283</td>
<td>100.00</td>
</tr>
</tbody>
</table>


As evident from the Table 1, the region under consideration here is characterized by substantial differences in farm size and resource endowments. Consequently, instead of a single regional aggregate, we group all farms in the region into three farm size groups – small farms (less than 50 hectares), medium farms (51-300 hectares) and large farms (301-1,000 hectares) and assume that all farms within each group satisfy the required aggregation conditions. Further utilizing the decomposition principle of linear programming, the three farm group models are jointly treated in a single model of the region.

Seven basic components are included. These are (a) a set of farm activities representing decision variables for farms within each size group; (b) an annual objective function measuring the expected revenues from crop sales, the costs of purchased inputs and annual investment charges for resource augmenting investments; (c) a technology matrix representing the traditional and modern input-

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14 Major transformations involved the shift from the beef production on extensive natural pasture to intensive wheat, soybeans, and corn production with heavy mechanization and beef production on intensive improved pasture. By 1970, the region with 5.7 million hectares of cultivated land accounted for 42% of national wheat producing areas.
output structure of cash consumption, farm production, investment, sales, purchase and financial activities; (d) "technical" constraints representing regional resource and financial limitations; (e) "behavioral" constraints representing adaptive "safety-first" limitations for protection against mistakes of cropping and investment choices, and representing drags on investment due to "learning" and "unwillingness to change"; (f) feedback functions that relate the parameters of the current programming problem to previous decisions; and (g) exogenously given input and output prices, regional supplies of land and labor resources and exogenously estimated consumption requirements by farm size and supplies of regional wage labor, credit and non-farm quasi-fixed capital goods.

The endogenous variables explained by the model include, by farm size, the production of crops and livestock (by technology - traditional and modern); investment levels in farm power (tractors, harvestors and draft animals); working capital expenditures on machines, fertilizers, seeds, bone meal, concentrates, fuel, etc.; borrowings and savings levels and labor utilization by family and wage labor categories, by individual activity, by season and by crop. The exogenous variables not explained by the model include market prices, interest rates, supplies of land and family labor by farm size, wage labor in the region and non-farm incomes. The parameters of the model include input-output coefficients by farm size, regional depreciation rates, adoption and adjustment coefficients by machine type, flexibility coefficients by crop, and the average propensity in the region to consume out of gross sales.

Activities distinguished by farm size include production activities (wheat, soybeans, soybean-wheat rotation, corn, each at two levels of technology (traditional and modern) and beef cattle raised on either natural or improved summer and winter pastures, purchase activities (variable cash inputs such as hired labor, seeds, fertilizers, and livestock concentrates), sales activities (wheat, soybeans, corn and beef), financial activities (savings, borrowings and debt repayment) and investment activities (the purchase of capital goods, combines and draft animals and land improvement). Intermediate transfer activities allow for the use of corn and pasture for livestock production and the conversion of natural to improved pasture of crop land.

Constraints by farm size group include land, labor, power, and working capital supplies. Behavioral constraints defined within farm size groups are individual crop flexibility constraints and adoption and adjustment coefficients by machine type. Regional constraints include farm credit, wage labor by season and behavioral constraints emitting the rate of investment in mechanical power and the adoption of modern technology. The seven basic components of the model are
succinctly described in equations below.

Let \( A \) be a set of production activity indexes and let \( C \) be a set of constraint indexes. Also let \( Q = \{ \text{small, medium, large} \} \) be the set of farm size indexes. All activities are assumed to be linear, finite in number and their levels \( x_j, j \in A \) measured for the regional farm size aggregates. It is convenient to decompose activity indexes into subsets associated with individual farm size groups. Thus we let \( \{A_q, q \in Q\} \) be a partition of \( A \) where \( A_q \) is the set of activities associated with farm size \( q \). Constraining factors are identified by an index \( i \in C \). The technical coefficients \( b_{ijt}, i \in C, j \in A \) are assumed constant over time and all technology is assumed to be embodied. Positive (negative) coefficients mean a given factor is a net input (output); a zero coefficient indicates a factor not involved in the activity in question. Limitation coefficients \( c_i, i \in C \) are also defined for farm size aggregates and for the region as a whole; positive (negative) coefficients are associated with upper (lower) bounds on activity combinations, zero coefficients with balance constraints.

We also let \( \{C_t, q \in Q, C_r\} \) be a partition of \( C \) where \( C_q \) is the set of constraint indexes associated with farm size \( q \) and \( C_r \) the set of constraint indexes associated with the region as a whole.

The objective function to be maximized in each year is

\begin{equation}
q(x, a_t) = \sum_{j \in A} a_{jtt} x_j = \sum_{j \in A_q, q \in Q} a_{jtt} x_j
\end{equation}

where \( a_{jtt} \) is the anticipated net profit of activity \( j \), for the period \( t \). These represent current variable costs of the appropriate input (seeds, manure, chemical fertilizers, pesticides, animal draft, fuel, lubricants and labor costs) when \( j \) is a purchase activity, the nominal rate of interest when \( j \) is a borrowing activity, the regional time deposit rate when \( j \) is a saving activity, the expected sales price per unit of output when \( j \) is a sales activity and an investment charge estimated on a straight line depreciation basis from the current purchase price of the capital good when \( j \) is an investment activity.

\begin{equation}
\sum_{j \in A_q} b_{ijt} x_j \leq c_{it}, \quad i \in C_q, \quad q \in Q
\end{equation}

The regional constraints are

\begin{equation}
\sum_{j \in A} b_{ij} x_j = \sum_{j \in A_q, q \in Q} b_{ijt} x_j \leq c_{it}, \quad i \in C_r
\end{equation}
The objective function is maximized for each year subject to constraints (2)-(3).

In specifying model details it is convenient to decompose activity and constraints groups further. Thus we shall use the following index sets:

**Activities by farm size**

- $P_q$ production
- $P_{yr}$ final production of crop
- $y$ commodity indexes
- $H_q$ purchase
- $S_q$ sales
- $F_q$ financial
- $I_q$ investment
- $D_q$ transfer

**Constraints by farm size**

- $L_q$ land and seasonal family labor
- $K_q$ farm power capacities
- $E_q$ intermediate goods
- $G_q$ working capital
- $x^{s}_q$, $x^{l}_q$ crop flexibility

**Regional coupling constraints**

- $\omega$ regional wage labor supplies by season
- $r_c$ regional farm credit
- $B$ behavioral bounds on investment and adoption

Land is assumed to be constant, while family labor by season is assumed to grow at an exogenously given rate equal to the rate of growth of population. Hence the $c_{iy}, i \in L_q$ coefficients in (2) are exogenous variables.

Farm power constraints are endogenously generated. They are given by

\[(4) \quad \sum_{k \in K_q} b_{ik} x_j - b_{ik} x_{ki} \leq c_{it}, \quad i \in K_q\]

in which $k \in K_q$ is the investment activity in power source (or machine) $i \in K_q$, which states that current power utilization by production activities augmented by current investments must not exceed initial capacities. Current capacity is generated recursively by

\[(5) \quad c_{it} = (1 - a_t) c_{it-1} + b_{ik} x_{ki, t-1}, \quad i \in K_q\]

which states that current capacity is previous depreciated capacity
augmented by the immediately preceding year's investment.

Balance equations allow the production of intermediate outputs to be used for final outputs, as well as the transfers of additional capacities from investments to current capacities. These are completed endogenous and may be expressed by

\[(6) \quad \sum_{i \in I_q} b_{ij} x_j \leq 0, \quad i \in E_q, \quad q \in Q\]

In these constraints a positive \(b_{ij}\) means a given intermediate good is "used up" by activity \(j\), a negative \(b_{ij}\) means one "produced" by activity \(j\).

The use of working capital within each farm group is constrained in the model by current supplies augmented by current borrowings. Purchasing, savings and investments in power and machines compete for this amount. We thus have

\[(7a) \quad \sum_{j \in H_q} a_{j,t}^0 x_j + \sum_{j \in F_q} b_j x_j + \sum_{i \in I_q} a_{j,t}^0 x_j \leq c_{wq}t\]

The coefficients \(a_{j,t}^0\), \(j \in H_q\) are the current unit costs of the purchased inputs (prices of seeds, fertilizer, fuels, lubricants, wages, etc.). The \(b_j\) coefficients are equal to +1 for savings -1 for borrowing activities so the former competes for, the latter augments working capital. The \(a_{j,t}^0\), \(j \in I_q\) are the currently estimated annual capital charge for investment activities based on current prices and straight-line depreciation determined by use life. The initial supply of working capital within the farm size group is determined recursively by the equation

\[(7b) \quad c_{wq} = (1-\gamma_q) \left( \sum_{j \in S_q} a_{j,t-1}^0 x_{j,t-1} + Y_{q,t} \right) + \sum_{j \in F_q} \left[ \text{sign} a_{j,t-1}^0 + a_{j,t-1}^0 \right] x_{j,t-1}, \quad q \in Q.\]

In this expression \(a_{j,t-1}^0\), \(j \in S_q\) is the price received for commodities sold in the previous crop year, \(Y_q\) is exogenously given off-farm income, \(a_{j,t-1}^0\) is the interest received for savings and minus the interest paid on borrowing activities. Sign \(a_{j,t-1}^0\) is +1 for savings, -1 for borrowing. The parameter \(\gamma_q\) is the marginal propensity to consume on farms in group \(q\).
The production of individual commodities is bounded in each year by flexibility constraints to account for adaptive, safety-first behavior. These may be written

\[
(8a) \quad c^l_{yqt} \leq \sum_{j \in F_{yq}} x_j \leq c^u_{yqt}
\]

where \(c^l_{yqt}\) and \(c^u_{yqt}\) are respectively lower and upper bounds of production activities.

\[
(8b) \quad c^u_{yqt} = (1 + \gamma^u_{yq}) \sum_{j \in F_{yq}} x_{j,t-1}
\]

\[
(8c) \quad c^l_{yqt} = (1 - \gamma^l_{yq}) \sum_{j \in F_{yq}} x_{j,t-1}
\]

where the flexibility coefficients, \(\gamma^l_{yq}\) and \(\gamma^u_{yq}\) were parameters of the model.

Let us now describe the regional coupling constraints. Regional wage labor constraints are given by

\[
(9) \quad \sum_{i \in W} b_{ij} x_j \leq c_{it}, \quad i \in W
\]

where \(c_{it}\) is the exogenously estimated supply of regional wage labor by season.

The supply of credit is assumed to be limited to the region as a whole, but allocated efficiently among farm groups within the region. Let \(F_{bq}\) be the borrowing activities for farm size group \(q\). Then we have

\[
(10a) \quad \sum_{j \in F_{bq}} x_j \leq c_{ret}
\]

where the limitation coefficient \(c_{ret}\) is generated recursively by

\[
(10b) \quad c_{ret} = \beta \sum_{j \in F_{bq}, q \in Q} x_{j,t-1}
\]

The parameter \(\beta\) is a rule of thumb "borrowing coefficient" used by credit institutions in extending credit. Thus, the sum of regional borrowings in the current period cannot exceed a fraction of previous years gross revenues in the region.

Maximum potential investment bounds are defined for investment activities. These are defined by

\[
(11a) \quad \sum_{i \in I} x_{i,j} \leq c_{i,jt}, \quad j \in I_q
\]
Here $x_{jt}$ is investment by farm size group $q$ in the capacity associated with activity $j$. $c_{rjt}$ is the limit in year $t$ on this investment determined by the "adjustment rule"

$$
(11b) \quad c_{rjt} = \rho_j \left[ \bar{c}_{rj} - \sum_{q \in q} c_{iq} \right]
$$

where $\bar{c}_{rj}$ is the long-run desired capacity if the given capital good were used throughout the region and where $c_{iq}$ is the initial capacity in farm group $q$ of the given capital good as determined by (5b). $\rho_j$ is the "adjustment coefficient."

The final set of behavioral constraints reflect friction in adopting new technology throughout the region and are given by bounds on the use of modern technology applied to individual commodities. Let be the subset of new production activities that involve the use of new machines, seeds, and practices. Then the adoption constraints are

$$
(12a) \quad x_j \leq c_{jt}, \ j \in N
$$

where the limitation coefficients are generated recursively by

$$
(12b) \quad c_{jt} = (1 + \alpha_j) x_{jt-1}.
$$

Our description of the model is completed by returning to the objective function to describe the objective coefficients $a_{j}, j \in A$. These are as follows. The payoff for sales activities is the current observed price

$$
(1b) \quad a_{jt} = a^0_{jt}, \ j \in S_q.
$$

Those for purchasing are the current observed prices times minus one, i.e.,

$$
(1c) \quad a_{jt} = -a^0_{jt}, \ j \in H_q.
$$

$$
(1d) \quad (\text{sign } a_{jt}) a_{jt} = a^0_{jt}, \ j \in F_q.
$$

Financial activity coefficients are the "observed" interest rates with sign $a_{jt} = +1$ for savings and sign $a_{jt} = -1$ for borrowing. Investment activities coefficients are

$$
(1e) \quad a_{jt} = -a^0_{jt}, \ j \in I_q
$$

where $a^0_{jt}$ are the "observed" annual capital cost based on straight
line depreciation.

Putting the entire equations presented above into a matrix form, one can obtain the decomposition structure as shown in Figure 1. It is represented by non-empty input-output matrices along the diagonal and by null matrices in the off-diagonal zones bordered at bottom by an array of non-empty matrices representing regional resource availability and competition along with a row at the top of sub-vectors containing objective functions. Each sub-vector in the objective function corresponds to the specific technology matrix of Figure 1.

<table>
<thead>
<tr>
<th>Regional objective function</th>
<th>( Z^s(t) )</th>
<th>( Z^m(t) )</th>
<th>( Z^i(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A^1_{ij}(t) )</td>
<td>0</td>
<td>0</td>
<td>( \leq B^s(t) )</td>
</tr>
<tr>
<td>0</td>
<td>( A^m_{ij}(t) )</td>
<td>0</td>
<td>( \leq B^m(t) )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( A^i_{ij}(t) )</td>
<td>( \leq B^i(t) )</td>
</tr>
<tr>
<td>( T^s_{kj}(t) )</td>
<td>( T^m_{kj}(t) )</td>
<td>( T^i_{kj}(t) )</td>
<td>( \leq B^r(t) )</td>
</tr>
</tbody>
</table>

Figure 1. Brief Model Structure of Regional Farm Size Decomposition

The model just presented is estimated by setting up and solving a linear programming problem for a given initial year – 1960 in this case. A new set of limitation and constraint coefficients is then computed by substituting the optimal solution vector just obtained and exogenous data or trends into the feedback functions. A new objective function using exogenous input and output price data is obtained and a new linear programming problem solved. This method generates a sequence of recursive programs with model outcomes for each year.

IV. Model Results

Before going into discussion on the results, it should be stressed

15 The first row contains the objective functions for small, medium and large farm types at time period \( t \). The regional objective function is the summation of the three objective functions. The superscripts \( s, m, i, \) and \( r \) represent the small, medium, large farm types and regional binding constraints, respectively. The subscript \( j \) denotes the activities, \( i \) the resource constraints unique to each farm type, and \( k \) the regional coupling resource constraints. The \( B \) vectors are resource limitations for each farm type and the limit of regional resources.
that the model's "goodness of fit" to underlying structure of the regional development must be evaluated. Although there exist several evaluation techniques, we have been restricted by the paucity of regional and subregional data from attempting a rigorous evaluation. We compared the predicted model outcomes with observed data for the sample period 1960-70 to convince that the model captured fairly closely major historical transformations and its results are available elsewhere.

With the kinds of data needed to run the system for simulation, the model produced a series of estimates featuring (a) regional resource use, (b) technological choices, (c) factor productivities, (d) factor proportions, (e) credit use, and (f) income distribution. We present here only some highlighting portions of the results.

Total land use and cropping patterns by farm size predicted by the model are shown in Figure 2. Several salient features stand out in these model predictions. First, total crop land use has increased more than fourfold. This increase has come mostly in double-cropping. Second, the most dramatic increases are in the hectarage devoted to wheat, soybeans, and the production of beef on improved pasture systems. Third, the use of natural extensive pasture systems for livestock production, that have long characterized the region, showed a continual decline. Fourth, total corn production, a relatively minor enterprise, has remained fairly stable. Lastly, although wheat, soybean, and beef production on improved pastures increased on all farms, these increases were most dramatic on large and least dramatic on small farms.

Approximately a million hectares of open range land, a quarter of total, has been converted into intensive crop and livestock production. Range livestock production which accounted for approximately 90% of total land use on all farms in 1960 accounted for only 72, 70 and 60 percent respectively on small, medium and large farms by 1970. The open range, the way of life associated with a system of beef production on extensive pasture lands is on its way out, to be replaced by intensive and mechanized crop farming mixed with an intensive beef production system.

Although farms of different size follow similar trends in their cropping patterns, their predicted choice of technologies reveal striking differences as shown in Figure 3. Estimates of on-farm investment patterns are confined to the gross investments in quasi-fixed capacities such as draft and machine power sources. The investment patterns are implicit in the choice of technologies. There is a

16 See Johnson and Rausser (1972) for a discussion of problems in developing evaluation criteria and Day and Singh (1975) for several evaluation techniques that can be used.

17 See Ahn (1972) for details.
marked upward trend in gross new investments for draft animals on small farms with roughly a twentyfold increase over the decade, whereas only a slight increase is evident on medium farms. In contrast, gross new investments in tractors grew by 320 percent on large and 200 percent on medium farms in the same period. As a result, the on-farm quasi-fixed capital stock — that is the stock of
machinery and animals used for farm power less depreciation — increased ninefold at constant 1970 prices during the period.

As evident from Figure 3, differential resource endowments at farm level clearly affects the choice of technologies. Small farms with relatively abundant labor employ only traditional draft animal technologies and at increasing rate. On the other hand, large farms with relatively scarce labor utilize exclusively modern tractor technologies. Between these two extremes, the medium sized farms show a mixed pattern, but inclined towards the labor saving modern technologies.

The substantial increases in operating and investment capital have only partly been financed by increased on-farm cash flows and profits. The model predictions indicate that an increasing share of total cash outlays have been financed through short term credits. By virtue of the farm size decomposition, we can trace out the allocation of regional limited credits for which all farms compete. These results are displayed in Figure 4.

The predicted short term borrowings increase dramatically especially after 1962 when the wheat support program went into effect. At constant prices, these borrowings grew from 12.6 million Cruzerios (Cr$: Brazilian currency unit) to 278 million Cruzerios by 1970. Average credit use per hectare also increased dramatically over this period, especially on large farms. Credit use per hectare on large farms is more than 20 times greater than on small farms and 10 times greater than on medium farms. The continued availability of credits at nominal rates often far below the rates of inflation, meant that for all purposes farmers could obtain loans to finance their operations and investments at negative real rates of interest. Further, a credit limit that allocates scarce capital resources on the basis of the volume of gross revenues and ability to repay naturally tens favor larger over smaller farms.19

As a consequence, the dependence on credit to finance operations is predicted to increase dramatically. Total credit use as a percentage of total cash outlays show that by 1970 large farms depended on credit

18 Partially due to lack of data and partly due to conceptual problems in handling investments in fixed long life assets in short-run optimizing models, investment in farm building, land acquisitions and land improvements were not explicitly treated in the model. The intra-farm land market, presenting substantial theoretical hurdles in models of this nature, was not explicitly treated.

19 Of course, the shadow price of credit obtained for the decomposition model is same over all farms, implying that the working capital productivity is uniform for each farm size group. Yet the model results show that large farms are still sharing a major portion of credit due to larger land and capital endowment. Under concessional rate of interest, the banking institutions tend to favor a few large farms in order to minimize their administrative expenses which they may incur in allocating the limited amount of credit. For discussions on allocative distortions of credit under concessional interest rate scheme in developing agriculture, see Adams (1972).
for nearly 70 percent of their cash while medium farms for nearly 55 percent of their cash. Only small farms with little access to credits and at the short end of credit limits favoring larger operations, continue to finance most of their operations themselves.

It would seem apparent that this increasing dependence on credits to finance farm operations, especially on large and medium farms is directly related to the credit policies that have led to a misallocation of scarce capital resources.

The farm size decomposition also allows us to analyze the regional income distribution. In sum, the predicted growth in total output is distributed most unevenly over different farm size groups as shown in Figure 5.

During the decade of the sixties, both gross and net farm incomes at constant prices on small farms remained almost constant, the former at approximately Cr$ 1,600 and the latter at about Cr$ 1,000. Those on medium farms have grown at a slow but steady rate and are on the average at least 8 times greater than on small farms. Most disturbing of all is the growth of incomes on larger farms, with gross farm incomes increasing by 41 percent and net farm income by 112 percent between 1960-1970. This suggests that inequalities in farm incomes have substantially increased over time.

Looking at the income distribution in terms of gross and net returns to available family labor on an hourly basis, the diverging
inequalities are further exaggerated. Since this is a measure of the farm returns to fixed resources, it is a measure more indicative of the real disposable income before taxes left in the hand of farmers. Large farms earned an hourly income to their family labor some 45 times greater than on small farms in 1960. These inequalities can be directly attributed to differences in the initial resource endowments on these farms — land and capital. By 1970, however, this difference had increased to nearly 57 times. This increasing inequality in the returns to family labor are the outcome partly of the growing differences in the on-farm labor base over time and partly of the unequal impact of selective policy measures that tended to favor larger farms.

V. Further Model Refinements and Applications

Although the model captured crucial features of regional development, it has some obvious limitations which may arise to a certain extent in any attempt to quantify the process of the economic transformation. These limitations must be pointed out so that one also becomes aware of what the model does not do. These limitations include: First of all, any tool purporting to describe historical processes with quantitative measures alone is likely to be an abstraction. Second, institutions and institutional processes which are clearly at the heart of the transformation processes are altogether neglected. We have little to say about the development of institutional infrastructure that must have aided, abetted or impeded these outcomes. Third, although less partial than the analysis of production at farm level, the model is still partial and totally open so that such elements as the impacts of the industrial and foreign sectors could be additionally incorporated. Consequently, the model refinements must be addressed to improve these drawbacks. However, it should be stressed that these improvements require as their prerequisite sufficient quantity and quality data which are not readily available in many developing agricultures.

The large variety of model uses can be found in 1) static, 2) comparative static, 3) dynamic, and 4) comparative dynamic applications. Some important extended applications\(^ {20} \) can be done in the following areas: First, product supply response, derived demand for specific inputs, factor substitutions, ceteris paribus, in static as well as short run can be traced out. Furthermore, simulating policy alternatives, projections and forecasting can be attempted, utilizing the recursive nature of the model. Second, the decomposition tech-

\(^{20}\) For these comparative static and dynamic applications, see Day and Singh (1975), Ahn and Singh (1974), and Singh and Ahn (1973). For the applications of recursive programming techniques to the industrial sectors, see Abe (1969) and Tabb (1967).
nique allows us to construct a model to analyze 1) inter-regional competitive nature of agricultural production, 2) intra-enterprise competition, and 3) allocation of limited strategic resources open for all micro-economic units.

From the empirical results and these applications, we can see the great flexibility of recursive programming models of regional agricultural development. With rich and quality data base, we can build an analytical model which will enable us to capture explicitly and often in great detail those elements that are essential to our understanding of, planning for, modern agricultural change.

References


